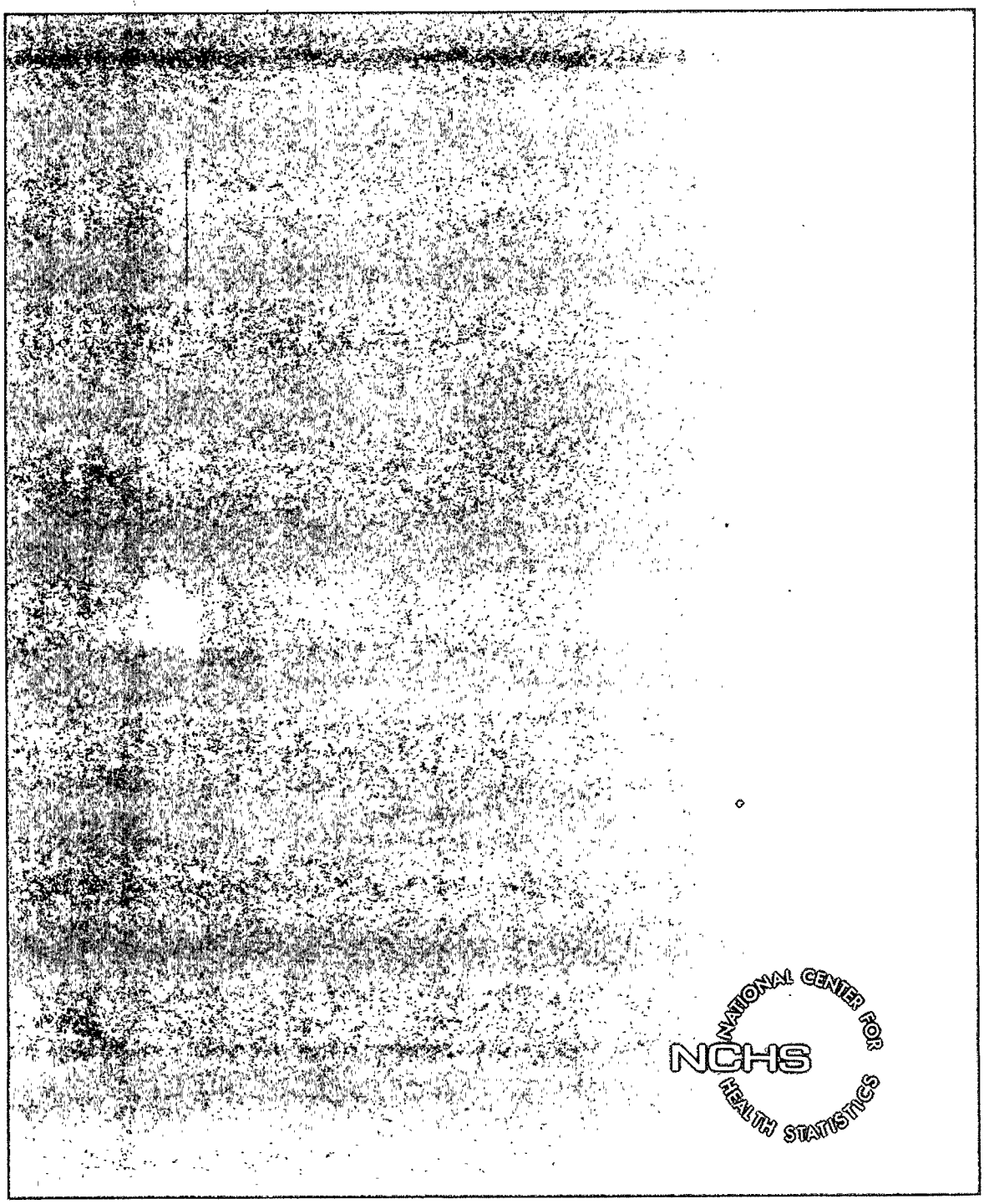
U.S. DECENNIAL LIFE TABLES FOR 1969-71

Volume I, Number 3

Methodology of the National and State Life Tables for the United States: 1969-71



There will be five reports in Volume I. Number 1 is published, and Numbers 2, 4, and 5 will be published shortly.

There are 51 reports in Volume II. These contain the State life tables for 1969-71, and they are published.

U.S. DECENNIAL LIFE TABLES FOR 1969-71

Volume I, Number 3

Methodology of the National and State Life Tables for the United States: 1969-71

DHEW Publication No. (HRA) 75-1150

U.S. DEPARTMENT OF
HEALTH, EDUCATION, AND WELFARE
Public Health Service
Health Resources Administration
National Center for Health Statistics
Rockville, Maryland 20852
May 1975

For sale by the Superintendent of Documents, U.S. Government Printing Office
Washington, D.C. 20402 - Price 45 cents

Stock Number 017-022-00381-3

There is a minimum charge of $1.00 for each mail order

NATIONAL CENTER FOR HEALTH STATISTICS

DOROTHY P. RICE, *Director*

ROBERT A. ISRAEL, *Deputy Director*
JACOB J. FELDMAN, Ph.D., *Associate Director for Analysis*
GAIL F. FISHER, *Associate Director for the Cooperative Health Statistics System*
ELIJAH L. WHITE, *Associate Director for Data Systems*
GEORGE P. FAILLA, *Associate Director for Management*
PETER L. HURLEY, *Associate Director for Operations*
JAMES M. ROBEY, Ph.D., *Associate Director for Program Development*
ALICE HAYWOOD, *Information Officer*

DIVISION OF VITAL STATISTICS

JOHN E. PATTERSON, *Director*
ALICE M. HETZEL, *Deputy Director*
T. N. E. GREVILLE, Ph.D., *Actuarial Adviser*
ROBERT J. ARMSTRONG, M.S., *Chief, Mortality Statistics Branch*
MICHAEL J. ZUGZDA, *Chief, Statistical Resources Branch*

CONTENTS

METHODOLOGY OF THE NATIONAL AND STATE LIFE TABLES FOR THE UNITED STATES: 1969-71

T. N. E. Greville, Ph.D., *Division of Vital Statistics*

INTRODUCTION

This report describes the methodology employed in the preparation of the decennial life tables for 1969-71 for the United States and for the 50 individual States and the District of Columbia. These are based on the 1970 census and the deaths of 1969-71. The methodology involved in the development of the 1969-71 life tables for the United States by causes of death is highly specialized and is described in the report containing those tables.[1] It will be assumed that the reader is acquainted with the definitions of the usual life-table functions as given in other reports of this series. The calculations to be described were performed on the IBM 360/50 computer of the National Center for Health Statistics.

Before the programming was initiated, some ideas on methodology were circulated informally to a few demographers and actuaries, and comments and suggestions were invited. Thanks are due to those who graciously responded. Most methodological decisions were made by the writer of this report. The basic methodology followed in obtaining life-table mortality rates for ages under 5 was unchanged from the preceding decennial life tables (based on the 1960 census and deaths of 1959-61), and originated with Zenas M. Sykes, now of the Population Dynamics Department, Johns Hopkins University.

[1] "United States Life Tables by Causes of Death, 1969-71," report number 5 of this volume.

The mortality rates at ages 85 and over based on the experience of the Social Security Medicare Program were developed by Steven F. McKay of the Office of the Actuary, Social Security Administration, under the supervision of Francisco Bayo, Deputy Chief Actuary, who also devised the procedure for blending the mortality rates derived from census populations and registered deaths into those based on Medicare experience.

The publications of decennial life tables based on the 1960 census included life tables not only for the United States and for States but also for the nine geographic divisions and for metropolitan and nonmetropolitan areas within these divisions. Little interest was expressed in the life tables for geographic divisions and few requests for them were received. The metropolitan and nonmetropolitan areas are aggregates of counties regarded as having, respectively, urban and rural characteristics. The life tables for these areas had been conceived as a substitute for life tables for urban and rural areas, which can no longer be prepared because tabulations of deaths are not obtainable for urban and rural areas as currently defined. However, the distinctive characteristics of urban and rural areas proved to be so diluted in these aggregates of counties that the differences in the life-table quantities between metropolitan and nonmetropolitan areas were insignificant. Accordingly, life tables for geographic divisions and for metropolitan and nonmetropolitan areas were omitted from the plans for decennial life tables based on the 1970 census.

PRELIMINARY ADJUSTMENT OF DATA

The census populations used in the construction of the life tables are not exactly the "official" census data[2] but are data adjusted by the U.S. Bureau of the Census. Misunderstanding by some respondents of certain items in the self-enumeration form used in the 1970 census is believed to have produced (1) an overstatement of the number of centenarians at the expense of all other age groups and (2) an overstatement of the population of races other than white or Negro at the expense of the white population.[3] Thanks are due to Jacob Siegel and Jerome Glynn of the Bureau of the Census for furnishing magnetic tapes containing special population data adjusted for these errors.

A further relatively minor adjustment relates to the fact that the tabulations of deaths include a relatively small number for which the age is not reported. The assumption was made that these deaths were distributed among the various age groups in the same proportions as the deaths for which age was reported. To this end an adjustment factor was computed for each population category for which a life table was to be constructed. This factor was obtained by dividing the total number of deaths reported for the given category for the 3-year period 1969-71 by the total less the number for which age was not reported. The number of deaths reported in each age group for the given category was then multiplied by the adjustment factor.

In the preparation of these decennial life tables no specific allowance was made for possible incompleteness in the enumeration of the population or in registration of births or deaths. In the calculation of previous decennial life tables the use of birth statistics (rather than population data) in calculating the denominators of the mortality rates at ages under 2 has been justified largely on the ground that the census populations under age 2 were believed to be underenumerated. However, there are other advantages of using the methodology based on birth data, since it may be expected to produce a more accurate estimate of the average population during the 3-year period than is provided by the population enumerated on the census date. Accordingly, its use was continued in connection with 1969-71 life tables.

In fact, rough calculations based on comparison of the enumerated populations under age 2 (adjusted for the two errors described above) with the corresponding populations estimated from births and deaths suggest that any underenumeration of the white population in this age range in the 1970 census is less than 1 percent, while for the population other than white it may be of the order of 10 percent. For the latter population group, therefore, the employment of birth statistics could be regarded as fulfilling the objective of avoiding the use of population data subject to underenumeration.

No adjustments have been made in the underlying census data for misreporting of age. This is the first time in several decades that such adjustments have not been considered necessary in the construction of the decennial life tables. For example, in the 1960 census there was substantial evidence of overreporting of 1900 as a year of birth in the population other than white, and, accordingly, that population was redistributed by age between ages 55 and 64, inclusive, before construction of the life tables was began.

DATA USED FOR CALCULATION OF LIFE-TABLE VALUES

The underlying data used in the preparation of each of the 1969-71 decennial life tables consisted of (1) reported deaths occurring in the 3-year period classified by age at death, (2) population data by age on the census date April 1, 1970, corrected as previously described for overstatement of the number of centenarians and overstatement of the population of races other

[2] As published, for example, in U.S. Bureau of the Census, "United States Summary," *Census of Population, 1970, General Population Characteristics,* final report PC(1)-B1, Washington, U.S. Government Printing Office, 1972, and corresponding publications for States.

[3] U.S. Bureau of the Census: Estimates of the population of the United States by age, sex, and race, April 1, 1960, to July 1, 1973. *Current Population Reports.* Series P-25, No. 519. Washington. U.S. Government Printing Office, Apr. 1974. The types of errors mentioned are described on p. 8. Population data consistent with those used in preparing the life tables in this report appear in table 6, p. 70.

Table 1. Comparison of corrected enumerated populations on April 1,1970, and comparable estimated populations on July 1, 1970, by color and sex: United States resident population in thousands

Item	White		All other	
	Male	Female	Male	Female
Corrected enumerated population, April 1, 1970 . . .	86,906	91,192	12,020	13,118
Estimated population, July 1, 1970	87,125	91,426	12,078	13,182
Percentage increase	0.25	0.26	0.48	0.49

SOURCE: Tables 2 and 6, pp. 29 and 70, in reference cited in footnote 3.

than white and Negro, and (3) total registered births for each of the years 1967 to 1971, inclusive. These data were available separately by sex and race, but only the subdivision by color was used in the construction of life tables for the States and the District of Columbia.

Populations and deaths were available by single years of age through age 5 and by 5-year age groups from 5 to 99 with the final age group 100 and over. There is a slight overlap here in that the population at age 5 was available separately and was also included in the age group 5-9. In each case the age referred to is the age in completed years: that is, the exact age on the individual's last birthday. In addition, deaths occurring at ages under 1 year were available for four subdivisions of the first year of life: under 1 day, 1-6 days, 7-27 days, and 28-364 days. Life-table values were calculated for these subdivisions of the first year (but not published in the case of the State life tables) and by single years of age throughout the remainder of the life span.

With regard to the census data, actuarial theory would suggest that the populations to be used in the calculations should be those of the central date of the 3-year period, that is, July 1, 1970. However, the enumerated populations as of April 1, 1970 (adjusted as previously described) were used just as if they were July 1 populations. Estimates of the latter, consistent with the population data used in the life-table calculations, are contained in the census report last cited.[4] Table 1 shows that the percentage differences between the two sets of population figures are very small. In view of the approximations involved and the various possibilities of error in the underlying data, it was not considered necessary to correct all the national and State populations for the lapse of time between April 1 and July 1, 1970.

[4]Table 2, p. 29, in reference cited in footnote 3.

NUMBERS OF SURVIVORS AT AGES 2 AND UNDER

At ages under 2 the first life-table quantities to be calculated were the values of ${}_td_x$, the number of deaths occurring between exact ages x and $x+t$ in the life table cohort commencing with ℓ_0 live births. This was calculated by the formula

$$ {}_td_x = \ell_0 \, {}_tD_x / {}_tE_x $$

where ${}_tD_x$ denotes the number of deaths (adjusted as described earlier for nonreporting of age) occurring in 1969-71 between exact ages x and $x+t$, and ${}_tE_x$ denotes the appropriate denominator as indicated in table 2. These denomi-

Table 2. Denominators ${}_tE_x$ used in calculating ${}_td_x$ for ages under 2 years

Age interval x to $x+t$	Denominator of ${}_td_x$
0-1 day.....	$\frac{1}{730}(B_{1968}+730B_{1969}+730B_{1970}+729B_{1971})$
1-7 days.....	$\frac{1}{730}(8B_{1968}+730B_{1969}+730B_{1970}+722B_{1971})$
7-28 days.....	$\frac{1}{730}(35B_{1968}+730B_{1969}+730B_{1970}+695B_{1971})$
28-365 days..	$\frac{1}{730}(393B_{1968}+730B_{1969}+730B_{1970}+337B_{1971})$
1-2 years.....	$\frac{1}{2}(B_{1967}+2B_{1968}+2B_{1969}+B_{1970})$

NOTE: B_z denotes the reported number of births occurring during the calendar year z for the population category (by sex, color, and geographic area) involved.

nators are based on the assumption of uniform distribution over the year of the births of 1967, 1968, 1969, 1970, and 1971. In each case ℓ_0 is taken as 100,000. The appearance of overlapping in the designations of the age intervals is due to the fact that exact ages are involved.

The unrounded values of ${}_td_x$ were then used to calculate values of ℓ_x up to age 2 by successive application of the formula

$$\ell_{x+t} = \ell_x - {}_td_x$$

MORTALITY RATES AT AGES 2-94

The life-table mortality rate q_x is the fraction or proportion of a group of persons at exact age x who are expected to die before attaining age $x+1$: If m_x denotes the ratio d_x/L_x, commonly called the central death rate, then it is well known[5] that on the assumption of uniform distribution of deaths over the year of age

$$(1) \qquad q_x = \frac{m_x}{2+m_x}$$

This approximation is sufficiently accurate when the life table is by single years of age. Formula (1) was the basis of the calculation of mortality rates at ages 2-94. Completion of the calculations depends, therefore, on the ability to calculate central death rates m_x at these ages. For this purpose different methods were used at ages 2-4 and at ages 5-94, as will now be described.

Central Death Rates at Ages 2-4

If D_x denotes the adjusted number of deaths in a population category at age x (in completed years) occurring in 1969-71 and P_x denotes the population at age x in the middle of the period, then approximately[6]

$$(2) \qquad m_x = D_x/3P_x$$

As previously noted, populations of April 1, 1970, were actually used.

However, since the deaths occurring in a given single year of age during 1969-71 were drawn from three consecutive annual cohorts of the population, it was considered that the accuracy of the calculation of these m_x values would be improved by replacing $3P_x$ in the denominator of (2) by the sum of the populations at age x-1, x, and x+1. Thus the formula becomes

$$(3) \qquad m_x = \frac{D_x}{P_{x-1}+P_x+P_{x+1}}$$

The combination of formulas (1) and (3) is equivalent to the single formula

$$q_x = \frac{D_x}{P_{x-1}+P_x+P_{x+1}+\frac{1}{2}D_x}$$

which was used for x=2, 3, and 4.

Mortality Rates at Ages 5-94

The combination of formulas (1) and (2) is equivalent to

$$(4) \qquad q_x = \frac{D_x}{3P_x+\frac{1}{2}D_x}$$

which was used for ages 5-94, with values of D_x and P_x obtained by interpolation from data by 5-year age intervals. The procedure of interpolating populations and deaths separately has a long history. Apparently it was first used by Dr. John Tatham in the preparation of English Life Table No. 5 covering the period 1881-90 and published in 1895.[7] It was also used in the construction of the United States life tables based on the censuses of 1900 to 1930, inclusive. When life tables are being mass-produced, as is the case with the life tables for the United States and for the 50 States and the District of Columbia, the Tatham method has an operational ad-

[5] Spiegelman, M.: *Introduction to Demography* (revised edition). Cambridge, Mass. Harvard University Press, 1968. p. 121.

[6] Ibid. pp. 121 and 123.

[7] Tatham, J.: *Supplement to the Fifty-Fifth Annual Report of the Registrar-General of Births, Deaths, and Marriages in England and Wales, 1881-1890, Part I.* London. Her Majesty's Stationery Office, 1895.

Table 3. Beers' interpolation coefficients for subdividing quinquennial sums to obtain estimated numbers by single years of age ("minimized fifth-difference formula with smoother ends")

Coefficients to be used at ages 5-9

Age in years for which interpolated number is required	Coefficient of quinqennial sum commencing at age:				
	0	5	10	15	20
5	0.0404	0.2000	-0.0344	-0.0128	0.0068
6	.0093	.2268	-.0402	.0028	.0013
7	-.0108	.2272	-.0248	.0112	-.0028
8	-.0198	.1992	.0172	.0072	-.0038
9	-.0191	.1468	.0822	-.0084	-.0015

Coefficients to be used at ages 10-94

Age in years for which interpolated number is required	Coefficient of quinquennial sum commencing at age:				
	5m-10	5m-5	5m	5m+5	5m+10
5m	-0.0117	0.0804	0.1570	-0.0284	0.0027
5m+1	-.0020	.0160	.2200	-.0400	.0060
5m+2	.0050	-.0280	.2460	-.0280	.0050
5m+3	.0060	-.0400	.2200	.0160	-.0020
5m+4	.0027	-.0284	.1570	.0804	-.0117

SOURCE: Beers, H. S.: Reply to the discussion of his paper "Six-Term Formulas for Routine Actuarial Interpolation," *Record of the American Institute of Actuaries* 34: 52-61, 1945. This table is on p. 60.

vantage in that the interpolated deaths at a given age for any class is exactly the sum of those for the subclasses of which it is composed, and a similar statement applies to populations. For example, at any age the interpolated deaths for total whites are the sum of those for white males and white females, and the interpolated deaths for the United States are exactly the sum of those for the 50 States and the District of Columbia.

The interpolation of both deaths and populations was performed by means of interpolation coefficients due to H. S. Beers (table 3). In certain age intervals the headings of this table are not to be taken precisely at their face value. In interpolating at ages 90-94, the numbers (of deaths or population) at ages 100 and over were used as if they applied to ages 100-104.

Moreover, in interpolating at ages 5-14, the value used for the "quinquennial sum starting at age 0" was not the actual number reported at ages 0-4 but a fictitious value obtained in the manner to be described. Because of the mortality peak in infancy, the use of the actual numbers at ages 0-4 probably would not yield plausible values. There is also no reason to expect that the interpolated values for ages 5-9 would join smoothly with the numbers reported at ages 2-4 if actual numbers for the age group 0-4 were used.

Accordingly, the artifice was used of employing ficititious quinquennial sums for the age interval 0-4 so determined that the sum of the interpolated values at ages 2-4 would be equal to the number reported in this 3-year age interval (adjusted for nonreporting of age in the case of deaths). The extension to ages 2-4 of the coefficients in table 3 would yield the values in table 4.

If W_x denotes the quinquennial sum commencing with age x and V denotes the sum of the interpolated numbers for ages 2-4 (which we shall require to be equal to the reported number), these coefficients give

$$V = .4072W_0 + .2416W_5 + .0080W_{10} - .0896W_{15} + .0328W_{20}$$

Table 4. Extension of table 3 to ages 2-4

Age in years for which interpolated number is required	Coefficient of quinquennial sum commencing at age:				
	0	5	10	15	20
2	0.1924	0.0064	0.0184	-0.0256	0.0084
3	.1329	.0844	.0054	-.0356	.0129
4	.0819	.1508	-.0158	-.0284	.0115

SOURCE: Same as table 3.

Solving for W_0, the fictitious quinquennial sum, gives

$$W_0 = 2.45580V - .59332W_5 - .01965W_{10} + .22004W_{15} - .08055W_{20}$$

This formula was used to compute the fictitious value.

The interpolated populations and deaths were used in formula (4) to calculate the mortality rates.

At ages 85-94 the mortality rates obtained as described here were blended with those derived (as explained in the next paragraph) from experience of the Medicare program. Thus the rates actually used in the construction of the life tables were obtained by the formula

$$q_x = \frac{1}{11}[(95-x)q_x^C + (x-84)q_x^M]$$

where q_x denotes the life-table mortality rate at age x, q_x^C is the mortality rate calculated by formula (4), and q_x^M is the corresponding rate based on Medicare experience.

MORTALITY RATES AT AGES 95 AND OVER

Mortality rates at ages 95 and over were based on experience of the Medicare program and were provided by the Office of the Actuary, Social Security Administration. In addition, at ages 85-94 mortality rates based on this experience were blended with those based on census populations and registered deaths, as mentioned in the preceding paragraph. Therefore mortality rates based on Medicare experience were required at all ages 85 and above.

These mortality rates were differentiated by sex and color but not by geographic area. Thus in the life-table mortality rates for the United States and for the States, there is no distinction by geographic area at ages 95 and over, and the influence of geographic area on the rates diminishes with increasing age at ages 85-94. For the United States (but not for States) life tables for Negroes have been calculated in addition to those for the population other than white (which includes Negroes). For this purpose the rates based on Medicare experience for persons other than white have been regarded as applying to Negroes, since separate data for Negroes are not available from the Medicare experience.

The mortality rates based on Medicare experience used in the construction of the life tables are shown in table 5. The procedure by which these were obtained consists of a series of steps. First, "crude" mortality rates for ages 66 to 102, inclusive, for white males, white females, males other than white, and females other than white were computed directly from the data on deaths and enrollments. The data used for this purpose were limited to the "HI-insured" group (i.e., insured for Hospital Insurance).[8] In general terms, this excludes persons who were "blanketed into" the Medicare program even though they had no covered employment or only a minimal amount of such employment under the Social Sercurity program or the Railroad Retirement

[8] For explanation of the terminology, see Bayo, F., "Mortality of the Aged," *Transactions of the Society of Actuaries* 24: 1-24, 1972.

Table 5. Graduated mortality rates by sex and color: Medicare experience of 1969-71, United States

Age in years	Total			White			All other		
	Both sexes	Male	Female	Both sexes	Male	Female	Both sexes	Male	Female
85	0.125754	0.148881	0.111682	0.126330	0.149782	0.112126	0.118491	0.138166	0.106004
86	.136504	.160132	.122458	.137316	.161302	.123126	.126499	.146336	.114060
87	.147922	.172211	.133858	.149009	.173683	.134794	.134859	.154819	.122431
88	.160128	.185147	.146009	.161527	.186973	.147243	.143467	.163397	.131060
89	.173079	.198688	.159004	.174825	.200961	.160552	.152170	.171782	.139869
90	.186743	.212610	.172853	.188895	.215461	.174757	.160757	.179777	.148630
91	.201035	.226760	.187421	.203722	.230379	.189805	.168975	.187311	.156985
92	.215620	.240780	.202410	.219088	.245468	.205512	.176541	.194346	.164577
93	.230113	.254386	.217414	.234703	.260548	.221557	.183222	.200919	.171090
94	.244175	.267403	.232019	.250242	.275501	.237528	.189215	.207034	.176824
95	.257446	.279621	.245837	.265302	.290145	.252978	.194813	.212705	.182201
96	.269588	.290903	.258545	.279571	.304307	.267625	.200002	.217949	.187187
97	.280239	.301353	.269803	.292835	.317839	.281330	.204795	.222784	.191797
98	.289768	.311110	.279957	.305133	.330852	.294127	.209213	.227233	.196049
99	.298687	.320175	.289489	.316626	.343244	.306146	.213275	.231317	.199960
100	.306962	.328572	.298361	.327365	.354792	.317421	.217001	.235058	.203550
101	.314615	.336327	.306590	.337359	.365535	.327941	.220414	.238480	.206839
102	.321675	.343471	.314200	.346627	.375497	.337723	.223533	.241605	.209847
103	.328172	.350038	.321219	.355198	.384706	.346789	.226380	.244453	.212594
104	.334136	.356061	.327678	.363103	.393198	.355168	.228976	.247048	.215098
105	.339602	.361574	.333607	.370375	.401009	.362891	.231338	.249407	.217379
106	.344602	.366614	.339040	.377052	.408179	.369993	.233486	.251551	.219453
107	.349168	.371212	.344009	.383169	.414747	.376510	.235437	.253497	.221337
108	.353332	.375403	.348548	.388763	.420754	.382478	.237208	.255262	.223048
109	.357124	.379217	.352686	.393872	.426238	.387935	.238814	.256862	.224599

program.[9] It is believed that ages have been more accurately determined for the HI-insured group. In fact, it was found that some mortality rates would be lowered as much as 10-15 percent if the noninsured were included, presumably because of overstatement of age among them.[10] Table 6 shows by age and sex the percentage of Medicare beneficiaries who were HI-insured as of January 1, 1970.

Also excluded from the calculations were persons of unknown color, estimated to have been roughly 3 percent.[11]

Crude mortality rates for the remaining five categories of the population were obtained as weighted averages of the four basic columns, using as weights the proportions of the enrolled population in the categories involved on January 1, 1970. In calculating these proportions, the entire population enrolled for Medicare (not merely the HI-insured) was used, except that, of course, persons of unknown color had to be excluded in obtaining proportions involving color.

The crude rates for each of the nine population categories from age 66 to 102, inclusive, were then graduated (i.e., smoothed) by a Whittaker-Henderson Type B formula.[12] Such a formula involves minimizing the quantity

Table 6. Percentage of Medicare beneficiaries who were HI-insured, by age and sex: United States, January 1, 1970

Age in years	Male	Female
65-69	96.4	93.5
70-74	94.8	90.8
75-79	93.6	87.0
80-84	91.4	78.5
85-89	85.0	63.6
90-94	73.8	45.4
95-99	56.0	27.1
100 and over	41.8	16.2

[9] For a discussion of the "blanketing in" legislation (commonly known as the Prouty Amendment), see Cohen, W. J., Ball, R. M., and Myers, R. J., "Social Security Payments to Noninsured Persons," *Social Security Bulletin* 29 (9): 3-9, Sept. 1966.

[10] It should be noted that if a noninsured person had clearly reached age 72 before 1968, it would make no difference in his Medicare benefits whether his age was 80, 90, or 100. Thus, for many of the very old, there was little incentive for a careful determination of age.

[11] Page 7 of reference cited in footnote 8.

[12] Miller, M. D.: *Elements of Graduation.* Chicago. Society of Actuaries, 1946. ch.5.

$$(5) \qquad \sum_{x=\alpha}^{\beta} W_x (q''_x - q_x)^2 + k \sum_{x=\alpha}^{\beta-2} (\Delta^z q_x)^2$$

where W_x is a designated positive weight and Δ^z denotes the zth finite difference, z being commonly chosen as 2 or 3. The first summation is a measure of the departure of the smoothed rates from the crude rates, and the second summation measures the roughness of the smoothed rates (i.e., the smaller this quantity, the smoother these rates). The constant k indicates the degree of importance the user attaches to smoothness as against fidelity to the observed data.

Minimization of the preceding expression leads to a system of linear equations to be solved for the smoothed mortality rates q_x. It has been suggested[13] that on theoretical grounds the weights W_x should be taken as the reciprocals of the variances of the smoothed mortality rates q_x, that is,

$$W_x = E_x / [q_x (1 - q'_x)]$$

where E_x denotes the "exposed-to-risk." In fact, it may be pointed out that with this choice of weights the first term of expression (5) becomes the value of chi-squared to be used in the chi-squared test of goodness of fit. However, the use of such weights would lead to a system of non-linear equations in the variables q_x that could not easily be solved. This difficulty may be obviated by using the crude rate as an approximation to the smoothed rate in the computation of weights so that the approximate formula becomes[14]

$$W_x = E_x / [q''_x (1 - q''_x)]$$

In graduating the mortality rates at ages 66-102 based on Medicare experience, z was taken as 2 and k as 5,000 so that the expression to be minimized becomes

$$\sum_{x=66}^{102} W_x (q''_x - q_x)^2 + 5{,}000 \sum_{x=66}^{100} (\Delta^2 q_x)^2$$

It was considered desirable to have the graduated rates smoothly approach 1, while never reaching 1. Accordingly the graduated rates at certain of the oldest ages were rejected and replaced by rates obtained by a method of extrapolation that will now be described. The last rate accepted was that at the youngest age y such that $q_y / q_{y-1} < 1.04$. For the different population categories, this occurred at various ages between 90 and 100, inclusive. Thereafter the mortality rates were extrapolated to age 111 by the formula

$$(6) \qquad \frac{q_{x+1}}{q_x} - 1 = .9\left(\frac{q_x}{q_{x-1}} - 1\right)$$

The constant .9 was chosen because it was found in many trials of curve fitting that the ratio in question tended to fall between .88 and .92 at ages around 100.

As the underlying data were available in such a form that the mortality rate heretofore referred to as q_x was in reality $q_{x-\frac{1}{2}}$, interpolation was performed to get the correct rates for integral ages from 85 to 109, inclusive, using the approximate formula

$$(7) \qquad q_x \cong -\frac{1}{16} q_{x-\frac{3}{2}} + \frac{9}{16} q_{x+\frac{1}{2}} + \frac{9}{16} q_{x+\frac{1}{2}} - \frac{1}{16} q_{x+\frac{3}{2}}$$

This formula would be exact if the four consecutive rates in the right member were exactly fitted by a third-degree polynomial.

[13] For example, in Peterson, R. M., "Group Annuity Mortality," *Transactions of the Society of Actuaries* 4: 246-307, 1952 (especially p. 288), and in Camp, K., *The Whittaker-Henderson Graduation Processes*, New York, published by the author, 1950. p. 15.

[14] In the graduation of the mortality rates based on Medicare data for 1969-71 the reciprocal of the approximate standard deviation (i.e., the square root of the expression given in this formula) was actually used as the weight. However, it has been ascertained that use of the more appropriate weights would make little difference in the resulting mortality rates.

CALCULATION OF THE REMAINING LIFE-TABLE VALUES

Mortality rates q_x were now available for all ages 2 to 109, inclusive. From these, using double precision, numbers of survivors ℓ_x were calculated for ages 3 to 110, inclusive, by the formula

$$\ell_{x+1}=\ell_x-\ell_x q_x \tag{8}$$

where x ranges from 2 to 109. Values of ℓ_x were now available for all integral ages from 0 to 110, inclusive, as well as for certain ages between 0 and 1 (1 day, 7 days, and 28 days).

Values of q_x were available for ages 2 and above but not for ages under 2. Accordingly q_0 and q_1 were calculated by the formulas

$$q_0=1-\ell_1/\ell_0$$

and

$$q_1=1-\ell_2/\ell_1$$

Moreover, for the United States, mortality rates ${}_tq_x$ for subdivisions of the first year of life were calculated by the formula

$${}_tq_x=1-\ell_{x+t}/\ell_x$$

Such values of ${}_tq_x$ were not published for States.

Values of $\mathring{e}_{110}$ for each of the nine color-sex categories were furnished by the Office of the Actuary of the Social Security Administration. These were calculated as follows. The extrapolation of $q_{x-½}$ values by means of formula (6) was continued to age 147, and values of q_x up to age 145 were obtained by interpolation, using formula (7). Values of ℓ_x for x=111 to 145, inclusive, were calculated by formula (8). If it is assumed that the force of mortality is constant between ages x and x+1, then it follows that

$$L_x=\int_0^1 \ell_{x+t}\,dt=f_x\ell_{x+1}+(1-f_x)\ell_x$$

where

$$f_x=\frac{1}{q_x}+\frac{1}{ln\,(1-q_x)}$$

These formulas were used to compute L_x for ages 110 to 145, inclusive, and T_x was computed sequentially from age 145 back to age 110 by

$$T_x=T_{x+1}+L_x$$

with T_{146} taken equal to zero. The final step was to compute $\mathring{e}_{110}=t_{110}/\ell_{110}$. These values are shown in table 7.[15]

With the values of $\mathring{e}_{110}$ available, T_x for the ages included in the life tables was computed by the formulas

$$T_{110}=\ell_{110}\mathring{e}_{110}$$

and

$$T_x=T_{x+1}+½\,(\ell_x+\ell_{x+1})$$

proceeding from age 109 back to age 0. For the subdivisions of the first year of life, the corresponding formula is

$$T_x=T_{x+t}+\frac{t}{2}(\ell_x+\ell_{x+t})$$

where t is the appropriate fraction of a year corresponding to the interval involved. The values of t used are as follows:

---

[15] Values of the life annuity $\ddot{a}_{110}$ needed in the calculation of the tables in report number 2 of this volume, "Actuarial Tables Based on United States Life Tables, 1969-71," were calculated in a similar manner.

Age	*t*
0	1/365
1 day	6/365
7 days	21/365
28 days	337/365

The average remaining lifetime (or "expectation of life") $\mathring{e}_x$ was calculated at all ages 0 to 109, including ages between 0 and 1, by the formula

$$\mathring{e}_x = T_x / \ell_x$$

The values of q_x, ℓ_x, T_x, and $\mathring{e}_x$ were then rounded to the number of decimal places shown in the published life tables. In other words, ℓ_x and T_x were rounded to the nearest integer, q_x to 5 decimal places, and $\mathring{e}_x$ to 2 decimal places. Because ℓ_x and q_x were independently rounded, calculation of q_x from the published (rounded) values of ℓ_x by $q_x = \ell_{x+1} / \ell_x$ may not always agree with the published value in the fifth decimal place.

Finally, d_x and L_x were obtained by differencing the rounded values of ℓ_x and T_x, respectively. In other words, the formulas were

$$d_x = \ell_x - \ell_{x+1}$$

and

$$L_x = T_x - T_{x+1}$$

SPECIAL ADJUSTMENTS IN THE STATE LIFE TABLES

For each of the 50 States and the District of Columbia, life tables were calculated for each of the nine sex-color groups shown in table 7. However, in some States not all of the nine tables were published, because it was considered that the amount of data for one color was too small to produce reliable results. If for either color fewer than 1,600 male or female deaths at all ages were registered in the given State for the 3-year period 1969-71, the tables for that color were not published. The number 1,600 was chosen because with 1,600 deaths the coefficient of variation of the crude death rate is approximately 2½ percent. As a result of applying this criterion, life tables for persons other than white were not published for the States of Alaska, Arizona, Colorado, Delaware, Idaho, Iowa, Kansas, Maine, Minnesota, Montana, Nebraska, Nevada, New Hampshire, New Mexico, North Dakota, Oregon, Rhode Island, South Dakota, Utah, Vermont, Washington, West Virginia, Wisconsin, and Wyoming, and life tables for white persons were not published for Alaska and Hawaii. In only four instances (Arizona, Kansas, Washington, and West Virginia) was the number of deaths of persons other than white fewer than 1,600 for one sex (females in all four cases) and not for the other. Also for whites in both Alaska and Hawaii the number of female deaths (but not the number of male deaths) was fewer than 1,600.

In most of the State life tables special adjustments were made at certain ages to correct or mitigate anomalous behavior of the life-table values that may be attributed to the small numbers involved. After each life table to be subjected to such adjustment had been calculated and printed out, the q_x values for individual years of age (including subdivisions of the first

Table 7. Values of $\mathring{e}_{110}$ extrapolated from Medicare experience by color and sex: United States, 1969-71

Color and sex	Values of $\mathring{e}_{110}$
Total	
Both sexes	2.200410
Male	2.043468
Female	2.228283
White	
Both sexes	1.932497
Male	1.743802
Female	1.965001
All other	
Both sexes	3.585175
Male	3.303186
Female	3.840739

year of life) were examined and certain tests of consistency applied. The position was taken that all the other life-table functions are completely determined by the q_x values and no tests need to be applied to them.

It was considered that, in each life table, the q_x values should decrease from age 0 to about age 10 or 11 and then increase to the early twenties. They should increase again from about age 30 to the end of the table. Strict increase in mortality rates with increasing age was not required between ages 20 and 30, because a slight dip in the mortality curve in this age range (due to violent and accidental deaths) is a feature of many of the life tables. Abrupt age-to-age changes in q_x values (indicated by large second differences) were also examined.

In general, such adjustments were made directly only to the life tables for white males, white females, males other than white, and females other than white. After the data underlying these four tables had been adjusted to remove anomalies from the life tables, the adjusted data were combined in various ways to produce the remaining five tables. It was assumed that if the four basic components were free from anomalies, this would also be true of the various combinations. In general, the data for population groups for which life tables were not published (e.g., males and females other than white in a number of States) were not adjusted. It was assumed that if the life tables for white persons were free from anomalies and the deaths of persons other than white were too few to warrant publication of the life tables, there would be no anomalies in the life tables for total males and total females. For three States (Montana, Nevada, and Wyoming) this assumption was not justified, and adjustments were found to be necessary in the data for persons other than white, even though the corresponding life tables were not published, in order to avoid anomalies in the life tables for total persons, total males, and total females. In Alaska, for which the only life tables published were those for total persons, total males, and total females, adjustments were necessary in the underlying data for both white and other persons.

For each pair of published life tables for males and females of a given color in a given State it was considered that the q_x value for females at each age should be less than the corresponding value for males. When life tables for both colors were published, the q_x value for white persons at each age should be less than the corresponding value for persons other than white of the same sex up to about age 70. If the values for other than white persons do become lower at about age 70 or later, they should remain lower. In other words, corresponding mortality curves for whites and others should not be permitted to cross and recross a number of times. This criterion was not applied, however, to the State of California, where the population other than white is composed predominantly of ethnic groups having mortality rates closely comparable to those of the white population.

In every instance in which an adjustment was considered necessary, it was effected by redistributing by age the numbers of deaths in two or more adjacent age groups, so the total number at all ages remained unchanged. In using this type of adjustment, the intention was to change the local shape of the mortality curve while preserving the overall mortality level. In many cases, the numbers of deaths in the age groups involved were redistributed by age in proportion to the corresponding numbers for the same sex and color for the United States. When this procedure failed to remove the observed anomalies, deaths were redistributed by age in a more arbitrary manner.

Sometimes several trial runs had to be made for a given State before satisfactory q_x values were obtained. As the National Center for Health Statistics is in the Parklawn Building in Rockville, Maryland, while its computer installation is in Research Triangle Park, North Carolina, the process was greatly facilitated by using a remote terminal in the Parklawn Building to make the adjustments in the numbers of deaths under the CRJE (Computer Remote Job Entry) system which was then in operation. In this way the author of this report was able to make changes in the numbers of deaths in disk storage in North Carolina and then have the calculation of q_x values rerun.

When appropriate redistributions by age of the numbers of deaths had been decided on, the process of computation of the various life-table

Table 8. Number of published State[1] life tables in which special adjustments were made, by color, sex, and selected age intervals: United States, 1969-71

Age interval between exact ages in years	Number of life tables in which special adjustments were made[2]			
	White		All other[3]	
	Male	Female	Male	Female
0-2	15	12	10	9
2-5	37	33	20	20
5-20	25	11	14	13
20-50	5	5	4	3
50-95	0	0	12	6
Total number of life tables published	49	49	27	27

[1]Including the District of Columbia but excluding Alaska. Note that life tables for other than white persons, but not for white persons, have been published for Hawaii.

[2]In some instances the age interval involved in a single redistribution of deaths by age included parts of two or more of the age intervals shown in the table. Thus the sum of the entries in any column, in general, exceeds the total number of separate redistributions made.

[3]Does not include Montana, Nevada, and Wyoming, where minor adjustments were made, even though the life tables were not published.

functions, as previously described, was carried out with the redistributed deaths. Moreover, the redistributed deaths for the four basic demographic categories (white males, white females, males other than white, and females other than white) were appropriately combined to produce the redistributed deaths by age for the five remaining categories (total population, total males, total females, total white, and total other than white), and computation of life tables for the latter categories was completed. The life tables for the United States were not corrected to reflect redistributions by age of deaths in the States.

Table 8 gives some idea of the number of special adjustments made. Alaska is excluded because its deaths were not differentiated by color in making the redistributions by age. For both males and females in Alaska some adjustments were made in each of the age intervals shown in the table except 0-2. Further details are given in the appendix, which shows, for each sex and color in each State, the ages at which adjustments were made.

More adjustments were required in the age interval 2-5 years than in any other. This is because deaths by single years of age were used at ages 2, 3, and 4, and these numbers are small and subject to severe statistical fluctuations. Only 10 out of 49 State life tables for white males, and only 16 out of 49 for white females, did not require special adjustments at any age. All but one (New York) of the published State life tables for males other than white and all but four (Mississippi, New York, Pennsylvania, and Tennessee) of those for females other than white required some special adjustments. There were two States (Minnesota and New York) for which no adjustments of any kind were required.

APPENDIX. AGES IN YEARS AT WHICH SPECIAL ADJUSTMENTS WERE MADE IN 1969-71 STATE LIFE TABLES, BY SEX AND COLOR

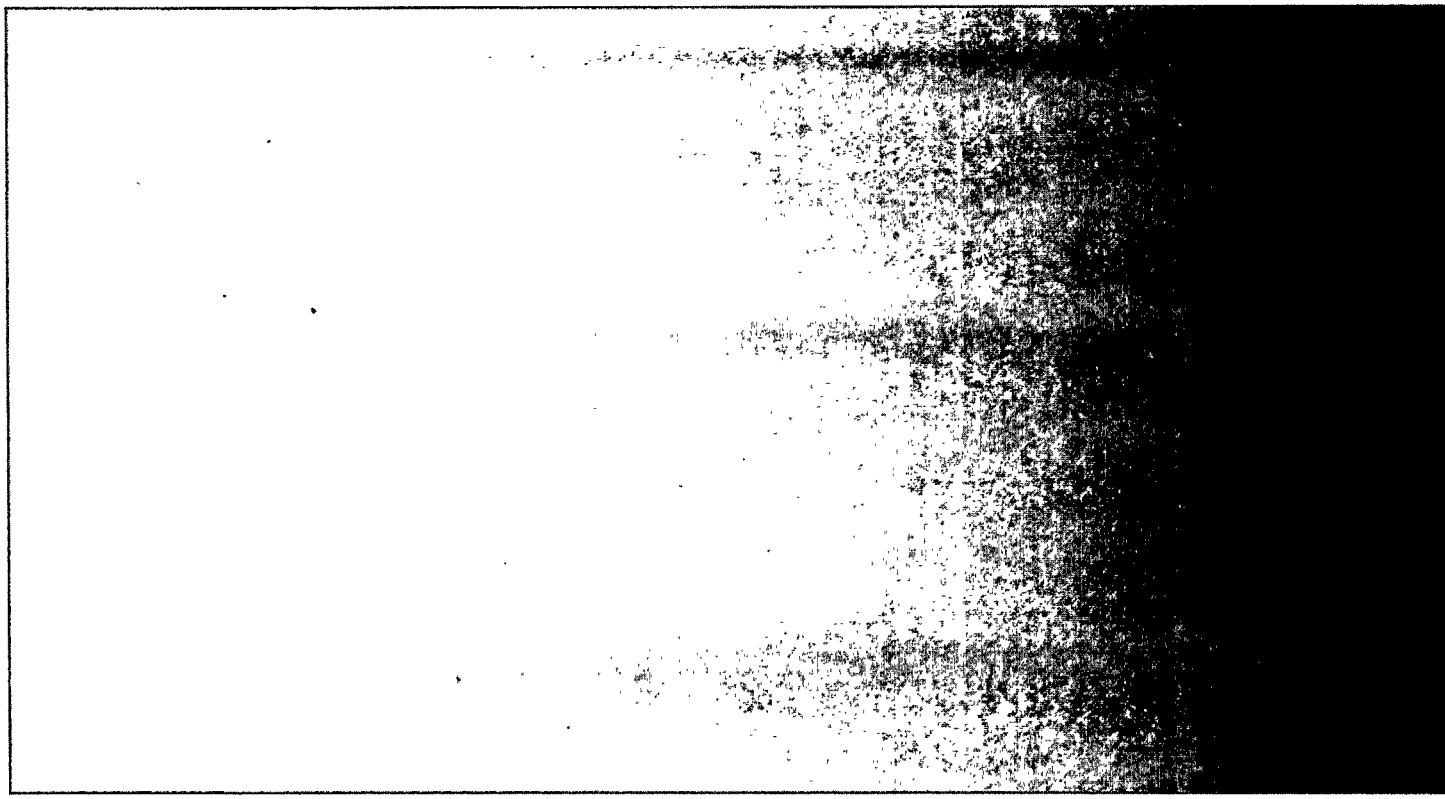
State	Male		Female	
	White	All other	White	All other
Alabama	1-2	3-9	1-4	4-9
Alaska[1]	3-4,15-24,85-94		3-9,15-34, 40-49,80-89	
Arizona	4-9		2-3	
Arkansas		0-9,65-74	1-3	1-2,5-14,25-44
California	2-3	4-9,75-84		1-2
Colorado	3-4			
Connecticut	1-9	2-9,40-54,65-89	2-19	1-2,4-19,30-44,75-84
Delaware	4-9		3-9	
District of Columbia	0-19		0-2,4-9	
Florida	1-2	3-9,70-89		70-74,80-94
Georgia	15-24	70-74,80-89	3-9	3-9,70-84
Hawaii		2-3,75-84		2-9
Idaho	1-9		3-4,30-39	
Illinois	4-9	0-1	3-4	0-1,4-9
Indiana		2-4		4-19
Iowa	0-9			
Kansas	4-14,20-24		2-9	
Kentucky	4-9	1-4	1-9	1-3,5-14
Louisiana		3-4	2-4	4-9
Maine	3-4		2-3	
Maryland	4-9	1-2		1-2
Massachusetts	3-9	1-9,70-74,80-84	3-4	5-14,55-64
Michigan		0-1,4-19		3-4
Minnesota[2]				
Mississippi	2-9,15-24	15-24,65-74	3-4	
Missouri	2-9	2-3,75-84		4-9
Montana	1-3		1-4,25-34	4-9
Nebraska	2-4		2-4	
Nevada	4-9,20-29	3-4	2-9	4-9
New Hampshire	0-9		1-4,10-19	
New Jersey		4-9,30-39	2-3	20-29
New Mexico	1-2,4-9			
New York[2]				
North Carolina	3-4	4-9		3-9
North Dakota			1-4	
Ohio	2-9	4-9		1-2
Oklahoma		0,5-19		1-4,50-59
Oregon	4-9		3-4	
Pennsylvania	0-1,4-9	0-1	1,4	
Rhode Island	0-4		0-4,35-44	
South Carolina	2-9	0-1,3-9,70-84	4-9	3-4,70-89
South Dakota	3,5-9			
Tennessee	0,2-14	4-9,15-24,75-84	1-2	
Texas		75-84		3-4
Utah	4-9		4-9	
Vermont	3-19		2-3,25-29,35-44	
Virginia	0-1,30-39	2-4	4-9	2-3
Washington	3-4		2,4	
West Virginia	1-3		1-4	
Wisconsin	2-3		2-4	
Wyoming	0-4	3-4	0-4,35-44	

[1]Deaths in Alaska were not differentiated by color in making adjustments.
[2]No special adjustments were required for the States of Minnesota and New York.

☆ U. S. GOVERNMENT PRINTING OFFICE : 1976 O - 575-959

FILE

DHEW Publication No. (HRA) 75-1150



NCHS

U.S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE
PUBLIC HEALTH SERVICE
Health Resources Administration
National Center for Health Statistics
5600 Fishers Lane
Rockville, Maryland 20852

OFFICIAL BUSINESS
Penalty for Private Use $300

POSTAGE AND FEES PAID
U.S. DEPARTMENT OF HEW

H E W 390

U.S.MAIL

THIRD CLASS
BLK. RT.

FILE