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**Method of Constructing the
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United States, 1949**



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Method of Constructing the Abridged Life Tables for the United States, 1949

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This report outlines the method used in constructing the abridged life tables for the continental United States for the years 1946 to 1949, inclusive. The life table for the total population for 1949 is shown in table 2 as an illustration of this method. The first annual life tables based on reported births and deaths, and estimates of population at midyear, were for 1945. Certain adjustments were made in the calculation of life tables for that year¹ because of the effects of the war, but were not used in subsequent years.

Method of construction

As the usefulness of the annual series of abridged life tables which was initiated in 1945 depends, in large measure, on promptness in publishing them as soon as the tabulations of deaths for each year become available, and as great accuracy and refinement cannot be expected in life tables based on estimated populations projected over a period of several years since the taking of the last census, it is desired to use as simple and expeditious a method as possible. The method selected is known as the method of construction by reference to a standard table. This process is somewhat less laborious than the excellent short method of Reed and Merrell,² and does not have the wide applicability of that method, being limited to those cases where the "proportion dying" for the various age groups in the life table under construction are known to be very similar to those of another life table already existing (referred to as the "standard" table). In the calculation of the abridged life tables for 1946-49,³ the published life tables for white males and for white females in 1939-41⁴ were used as the standard tables, while in the computations for nonwhites, unpublished abridged life tables for nonwhite males and nonwhite females in 1939-41 were so employed.

This method, like the Reed-Merrell method, is based on an assumed relationship between the functions design-

ated by the symbols n^m_x and n^q_x . The function n^m_x is the conventional age-specific death rate: that is, the quotient of the number of deaths between exact ages x and $x+n$ during the year or other period of observation, by the population between these exact ages at the middle of the period of observation. In the case of these abridged life tables, therefore, it is the quotient of the deaths in a given year between exact ages x and $x+n$ by the estimated population between those exact ages, on July 1 of that year. The function n^q_x has been defined as the proportion of the survivors to exact age x in the hypothetical life table cohort, starting from 100,000 births, who die before reaching exact age $x+n$. In the customary actuarial notation, it is given by the formula:

$$n^q_x = (l_x - l_{x+n}) \div l_x$$

For convenience, n^h_x will be defined as the ratio $n^q_x \div n^m_x$. The assumption underlying the abbreviated method of life table construction used here is that, in each age interval x to $x+n$ the ratio n^h_x may be regarded as having the same value in the life table under construction as in the standard table. The convenience of this method in the construction of a series of annual life tables will now be apparent, since the values of n^h_x can be calculated once and for all (in this case, from 1939-41 data), and can then be used each year, until revised "standard" tables become available, as adjustment factors to convert the age-specific death rates n^m_x into values of n^q_x on which an abridged life table can be based. As will be explained later, the first year of life, and the final age group 85 years and over, were given special treatment.

The death statistics used in calculating the values of n^m_x for the abridged life tables were obtained from the regular tabulations of the National Office of Vital Statistics, and are published in the annual volumes entitled "Vital Statistics of the United States." They include only deaths occurring in the continental United States. The deaths for which the age was unstated may be distributed proportionally among the various age groups. The same results may be obtained by using only the number of deaths at each stated age in calculating the n^m_x values. The n^m_x values are then adjusted by multiplying them by a factor, which is the quotient of the total of all deaths divided by the total deaths at stated ages. The estimated populations on July 1 were obtained from the Bureau of the Census, and exclude the estimated number of persons in the armed forces overseas.⁵

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¹For a description of the method employed in 1945 see "United States Abridged Life Tables, 1945," National Office of Vital Statistics, Vital Statistics—Special Reports, vol. 23, No. 11, 1947.

²Reed, Lowell J., and Merrell, Margaret, "A Short Method for Constructing an Abridged Life Table," American Journal of Hygiene, vol. 30, No. 2, pp. 33-62, September 1939; reprinted as Vital Statistics—Special Reports, vol. 9, No. 54, pp. 681-712, 1940.

³Published in the annual reports of this office, "Vital Statistics of the United States," for each year.

⁴Bureau of the Census, "United States Life Tables and Actuarial Tables, 1939-1941," 1946.

⁵Bureau of the Census, "Estimates of the Population of Continental United States, by Age, Color, and Sex: July 1, 1947 to 1949," Current Population Reports, Series P-25, No. 39, with the exception of age groups 75 years and over, which are unpublished data supplied by the Bureau of the Census.

After the values of ${}_nq_x$ had been obtained, the l_x and ${}_n d_x$ columns were computed in the conventional manner, which may be expressed by the formulas:

$${}_n d_x = l_x \times {}_n q_x, \quad l_{x+n} = l_x - {}_n d_x.$$

In seeking a convenient abbreviated method of obtaining the values of ${}_n l_x$, it was noted that a first approximation of this is given by⁶

$${}_n l_x = \frac{1}{2}n(l_x + l_{x+n}).$$

It was, therefore, assumed that in each race-sex subdivision the values of the ratio

$$\frac{{}_n l_x}{l_x} = \frac{{}_n l_{x+n}}{l_{x+n}}$$

could be considered the same in the abridged life table as in the corresponding 1939-41 life table being used as a standard table. This assumption also is convenient for use in a series of annual life tables, since the same $\frac{{}_n l_x}{l_x}$ factors can be used year after year.⁶ The values of $\frac{{}_n l_x}{l_x}$ were, of course, obtained by summing the ${}_n l_x$ column, starting with the oldest age group. In other words, $\frac{{}_n l_x}{l_x} = \frac{{}_n l_{x+n} + {}_n l_x}{l_x}$. The average remaining lifetime was then obtained by division: $e_x = \frac{{}_n l_x}{l_x}$.

Values of ${}_n h_x$ and ${}_n j_x$ based on the 1939-41 United States Life Tables are given in table 1.

Since all those who reach age 85 in the life table cohort must die sometime after that age, it follows that ${}_{\infty} d_{85} = l_{85}$, and therefore ${}_{\infty} q_{85} = 1$. Hence, the assumption that the value of ${}_{\infty} h_{85}$ is the same in the table under construction as in the standard table is not admissible. Instead the ratio r_{85} , defined as the quotient of the value of ${}_{\infty} m_{85}$ based on the actual data by the corresponding value for the stationary population of the life table, was assumed to be the same in the table under construction as in the standard table. In the actual population, ${}_{\infty} m_{85}$ is merely the general death rate for the age group 85 years and over; in the life table population, it is $l_{85} \div e_{85}$, which is the reciprocal of the average remaining lifetime e_{85} . Thus, for the year 1949, for example, e_{85} can be computed by the formula:

$$e_{85}^{1949} = \frac{{}_{1939-41} e_{85} \times {}_{1949} m_{85}}{{}_{1949} m_{85}}$$

The life table can then be completed, since

$$r_{85} = l_{85} \times e_{85}.$$

The standard table values (1939-41) are: $e_{85} = 4.31$ and ${}_{\infty} m_{85} = 0.225857$.

The value of q_0 , the proportion of liveborn infants dying before reaching age 1, is computed from birth and death statistics, being taken as equal to the adjusted infant death rate. A method of adjusting the

⁶The formula ${}_n l_x = \frac{{}_n d_x}{n} \div {}_n m_x$, sometimes employed, was found not to be satisfactory here, as in this case it reduces to ${}_n l_x = l_x \times {}_n h_x$, and the values of ${}_n h_x$ were found (1) to exceed n in a few instances, and (2) to fluctuate somewhat irregularly from age group to age group. The use of this procedure would therefore result in (1) ${}_n l_x$ being greater than n times l_x (an impossibility) in certain cases, and (2) values of ${}_n l_x$ not always decreasing with advancing age (equally impossible). This situation results, of course, from the differences existing between the values of ${}_n m_x$ obtained from actual death and population statistics and those computed from the hypothetical life table population, and would not arise under a method, like the Reed-Merrell method, in which ${}_n q_x$ is obtained from ${}_n m_x$ by a mathematical formula.

infant death rate for the changing number of births is described in a previous publication.⁷ The adjustment is made by allocating the deaths of infants occurring during a given year to the year in which the infants were born. The infant deaths so allocated are then related to the births occurring in the respective year of birth. The expression for computing the adjusted infant mortality rate per 1,000 live births may be written:

$$\text{Adjusted rate} = \left[\frac{D(1-f)}{E} + \frac{Df}{E'} \right] \times 1,000,$$

where,

D = number of infant deaths occurring in the given year.

f = ratio of deaths occurring in the given year among infants born in the preceding year to the total infant deaths of the given year. This is referred to as the "separation factor."⁸ In 1949, $f = 0.14244$.

E = number of births occurring in the given year.

E' = number of births occurring in the preceding year.

The stationary population in the first year of life was obtained by the formula $l_0 = l_0 - (1-f)d_0$.

Explanation of the columns of the life table

COLUMN 1—AGE INTERVAL (x to $x+n$).—The age interval, shown in column 1, is the interval between the two exact ages indicated. For instance, "20-25" means the 5-year interval between the twentieth birthday and the twenty-fifth.

COLUMN 2—POPULATION (${}_n P_x$).—This column shows the estimated midyear population for the indicated age interval. For this example, the estimated midyear population of the United States, excluding armed forces overseas, for 1949 was obtained from the Bureau of the Census release, "Estimates of the Population of Continental United States, by Age, Color, and Sex: July 1, 1947 to 1949," Current Population Reports, Series P-25, No. 39. The population under 1 year is not used in life table computation. Births for 1948 and 1949 were used in computing q_0 . These figures have been published in "Vital Statistics of the United States," for the respective years.

COLUMN 3—DEATHS (${}_n D_x$).—This column shows the number of deaths for the age interval and year. In this table, the figures are registered deaths in 1949, excluding fetal deaths and deaths among the armed forces overseas. These figures are published in "Vital Statistics of the United States, Part I, 1949," table 9, pp. 112 and 113.

COLUMNS 4 AND 5—DEATH RATES (${}_n m_x$).—The age-specific death rate shown in column 4 is the central death rate for the age interval. In column 5, these rates have been adjusted proportionately for deaths for which age was not reported. The adjustment was made by multiplying each rate in column 4 by the ratio of total deaths to deaths at stated ages ($1,443,607 \div 1,442,119 = 1.001032$).

COLUMN 6—CONVERSION FACTOR (${}_n h_x$).—This column is derived from a "standard" table, in this instance,

⁷Bureau of the Census, "Effects of Changing Birth Rates Upon Infant Mortality Rates," Vital Statistics—Special Reports, vol. 19, No. 21, p. 401, 1944.

⁸The separation factor does not vary greatly from year to year and moderate variation in the separation factor affects the infant mortality rate only slightly. The method of calculation is described in the publication cited in footnote No. 7.

the life table for the total population of the United States for 1939-41. These factors are shown in table 1 and are discussed on pages 257 and 258.

COLUMN 7—PROPORTION DYING (${}_nq_x$).—This column shows the proportion dying during the indicated age interval among those alive on the birthday which marks the beginning of the interval. For example, the proportion dying in the age interval 20-25 is 0.0074. In other words, if the age-specific mortality rates prevailing in 1949 should continue in effect over a 5-year period, then, out of every 1,000 persons alive and exactly 20 years old at the beginning of the period, 7.4 would die before reaching their twenty-fifth birthday. The "proportion dying" in the first year of life, q_0 , is computed from the formula shown on page 258. For example, substituting the values for 1949,

$$q_0 = \frac{111,531 (1 - .14244) + 111,531 (.14244)}{3,559,529 + 3,535,068} = .031364.$$

This figure was then adjusted for age "not stated" (see column 5).

COLUMN 8—NUMBER SURVIVING (l_x).—This column shows the number of persons who would survive to the exact age marking the beginning of each age interval out of a cohort of 100,000 live births, among whom the proportions dying in each age interval throughout their lives are exactly those shown in column 7. Thus, table 2 shows that out of 100,000 babies born alive, 96,860 will complete the first year of life and enter the second; 96,279 will begin the sixth year; 95,128 will reach age 20; and 15,833 will live to age 85.

COLUMN 9—NUMBER DYING (${}_nd_x$).—This column shows the number dying in each successive age interval out of 100,000 live births. Out of 100,000 babies born alive 3,140 die in the first year of life, 581 in the succeeding 4 years, 704 in the 5-year period between exact ages 20 and 25, and 15,833 die after reaching age 85. Each figure in column 9 is the difference between two successive figures in column 8.

COLUMN 10—SUM OF ($l_x + l_{x+n}$).—This column supplies one factor needed in computing column 12, ${}_nL_x$, which is equal to ${}_n j_x (l_x + l_{x+n})$. Each figure in this column is obtained by adding the number of persons alive at the beginning of any age interval, to the number alive in the next higher age interval (column 8).

COLUMN 11—CONVERSION FACTOR (${}_nj_x$).—This column is derived from a "standard" table, in this instance, the life table for the total population of the United States for 1939-41. These factors are shown in table 1 and are discussed on pages 257 and 258.

COLUMNS 12 AND 13—STATIONARY POPULATION (${}_nL_x$ and f_x).—Suppose that a group of 100,000 individuals like that assumed in columns 8 and 9 is born every year, and that the proportions dying in each such group in each age interval throughout the lives of the members are exactly those shown in column 7. If there were no migration and if the births were evenly distributed over the calendar year, the survivors of these births would make up what is called a stationary population—

stationary because in such a population the number of persons living in any given age group would never change. When an individual left the group, either by death or by growing older and entering the next higher age group, his place would immediately be taken by some one entering from the next lower age group. Thus, a census taken at any time in such a stationary community would always show the same total population and the same numerical distribution of that population among the various age groups. In such a stationary population supported by 100,000 annual births, column 8 shows the number of persons who, each year, reach the birthday which marks the beginning of the age interval indicated in column 1, while column 9 shows the number of persons who die each year in the indicated age interval.

Column 12 shows the number of persons in the stationary population in the indicated age interval. For example, the figure given for the total population in the age interval 20-25 is 473,975. This means that in a stationary population supported by 100,000 annual births and with proportions dying in each age group always in accordance with column 7, a census taken on any date would show 473,975 persons between exact ages 20 and 25.

Column 13 shows the total number of persons in the stationary population (column 12) in the indicated age interval and all subsequent age intervals. For example, in the stationary population referred to in the last illustration, column 13 shows that there would be at any given moment a total of 4,845,004 persons who have passed their twentieth birthday. The population at all ages 0 and above (in other words, the total population of the stationary community) would be 6,764,778.

COLUMN 14—AVERAGE REMAINING LIFETIME (e_x^o).—The average remaining lifetime (also called the complete expectation of life) at any age is the average number of years remaining to be lived by those surviving to that age, on the basis of a given set of age-specific rates of dying. In order to arrive at this value, it is first necessary to observe that the figures in column 12 of the life table can also be interpreted in terms of a single life table cohort, without introducing the concept of the stationary population. From this point of view, each figure in column 12 represents the total time (in years) lived between the two indicated birthdays by all those reaching the earlier birthday among the survivors of a cohort of 100,000 live births. Thus, the figure of 473,975 for persons in the age interval 20-25 is the total number of years lived between the twentieth and the twenty-fifth birthdays by the 95,128 (column 8) who reach the twentieth birthday out of 100,000 born alive. The corresponding figure in column 13 (4,845,004) is the total number of years lived after attaining age 20 by the 95,128 reaching that age. This number of years divided by the number of persons (4,845,004 divided by 95,128) gives 50.9 years as the average remaining lifetime of persons at age 20.

SYMBOLS USED IN SPECIAL REPORTS

Class or item not applicable (three dots) -----	...
Data not available (three dashes)-----	---
Quantity is zero, in frequency tables (one dash)-----	-
Quantity is zero, in rate or percent tables (one cipher)-----	0
If rate or percent is more than 0 but less than 0.05-----	0.0
If both frequency and population base are zero in rate or percent tables (one dash)-----	-

TABLE 1. CONVERSION FACTORS BASED ON LIFE TABLES FOR THE POPULATION OF THE UNITED STATES, 1939-41

AGE INTERVAL (YEARS)	Total population	WHITE		NONWHITE	
		Male	Female	Male	Female
$n^h x = n^q x + n^m x$					
1-5-----	4.0204	4.0077	4.0081	4.1191	4.1136
5-10-----	4.9706	4.9583	4.9841	4.9368	4.9406
10-15-----	5.0388	5.0368	5.0362	5.0374	5.1859
15-20-----	4.9510	4.9600	4.9557	4.9987	4.8758
20-25-----	4.9480	4.9406	4.9654	4.8909	4.9189
25-30-----	4.9830	4.9893	4.9846	4.9194	4.9368
30-35-----	4.9800	4.9682	4.9743	4.8167	4.8444
35-40-----	4.9512	4.9478	4.9614	4.9568	5.0073
40-45-----	4.9274	4.9395	4.9657	4.8260	4.7292
45-50-----	4.9246	4.8903	4.9241	4.8378	4.9778
50-55-----	4.8470	4.8656	4.9094	4.5545	4.5824
55-60-----	4.8011	4.7701	4.8622	4.6006	4.6512
60-65-----	4.7049	4.6657	4.7643	4.5421	4.5826
65-70-----	4.5936	4.5299	4.6706	4.4731	4.5613
70-75-----	4.3918	4.3251	4.4704	4.2719	4.3810
75-80-----	4.0693	3.9819	4.1373	4.0773	4.2304
80-85-----	3.6589	3.5642	3.7211	3.7524	4.0490
$n^j x = n^l x \div (l_x + l_{x+n})$					
1-5-----	1.9967	1.9970	1.9973	1.9931	1.9943
5-10-----	2.4995	2.4995	2.4995	2.4988	2.4989
10-15-----	2.5004	2.5003	2.5002	2.5001	2.5001
15-20-----	2.5008	2.5007	2.5005	2.5010	2.5013
20-25-----	2.5005	2.5003	2.5004	2.5013	2.5008
25-30-----	2.5004	2.5003	2.5004	2.5006	2.5003
30-35-----	2.5007	2.5007	2.5005	2.5006	2.5006
35-40-----	2.5012	2.5013	2.5008	2.5009	2.5009
40-45-----	2.5019	2.5022	2.5014	2.5014	2.5012
45-50-----	2.5028	2.5033	2.5021	2.5023	2.5020
50-55-----	2.5039	2.5047	2.5031	2.5021	2.5022
55-60-----	2.5051	2.5059	2.5047	2.5013	2.5017
60-65-----	2.5062	2.5067	2.5068	2.4999	2.5002
65-70-----	2.5083	2.5078	2.5102	2.4992	2.4993
70-75-----	2.5090	2.5067	2.5125	2.4987	2.4992
75-80-----	2.4974	2.4912	2.5035	2.4967	2.4980
80-85-----	2.4573	2.4437	2.4680	2.4828	2.4924

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TABLE 2. COMPUTATION OF ABRIDGED LIFE TABLE FOR

AGE INTERVAL		Estimated population July 1, 1949 within age interval ¹	Deaths in 1949 within age interval	Death rate unadjusted Col. 3 Col. 2	Death rate adjusted for age not stated Col. 4 × 1.001032	Conversion factor (See table 1)	Proportion of persons alive at beginning of age interval dying during interval Col. 5 × Col. 6
Period of life between two exact ages stated in years							
x to $x+n$		n^p_x	n^d_x	...	n^m_x	n^h_x	n^q_x
(1)		(2)	(3)	(4)	(5)	(6)	(7)
1	0-1-----	...	111,531	0.0314
2	1-5-----	12,534,000	18,687	0.001491	0.001493	4.0204	.0060
3	5-10-----	13,452,000	8,738	.000650	.000651	4.9706	.0032
4	10-15-----	11,161,000	6,979	.000625	.000626	5.0388	.0032
5	15-20-----	10,610,000	11,988	.001130	.001131	4.9510	.0056
6	20-25-----	11,716,000	17,495	.001493	.001495	4.9480	.0074
7	25-30-----	12,074,000	19,929	.001651	.001653	4.9830	.0082
8	30-35-----	11,293,000	23,374	.002070	.002072	4.9800	.0103
9	35-40-----	10,816,000	33,085	.003059	.003062	4.9512	.0152
10	40-45-----	9,900,000	45,316	.004577	.004582	4.9274	.0226
11	45-50-----	9,087,000	63,652	.007005	.007012	4.9246	.0345
12	50-55-----	8,153,000	86,558	.010617	.010628	4.8470	.0515
13	55-60-----	7,340,000	113,445	.015456	.015472	4.8011	.0743
14	60-65-----	6,042,000	139,949	.023163	.023187	4.7049	.1091
15	65-70-----	4,471,000	162,325	.036306	.036343	4.5936	.1669
16	70-75-----	3,211,000	171,867	.053524	.053579	4.3918	.2353
17	75-80-----	2,085,000	166,768	.079985	.080068	4.0693	.3258
18	80-85-----	1,042,000	128,853	.123659	.123787	3.6589	.4529
19	85 and over ² -----	461,000	111,580	.242039	.242289	...	1.0000

¹These estimates are projections based on the 1940 census and do not agree with estimates for 1949 prepared after the 1950 census of population.

²For method of computing values at these ages, see text on p. 258.

THE TOTAL POPULATION OF THE UNITED STATES, 1949

Number surviving to exact age x out of 100,000 born alive Col. 8 (Line above) — Col. 9 (Line above)	Number dying in age interval Col. 7 X Col. 8	Sum of Col. 8 for this line and Col. 8 (line below)	Conversion factor (See table 1)	STATIONARY POPULATION		Average years of life remaining to survivors at age x <u>Col. 13</u> Col. 8	
				In age interval Col. 10 X Col. 11	In this and all subsequent intervals Sum of Col. 12 for this line and all below		
l_x	n^d_x	$l_x + l_{x+n}$	n^j_x	n^l_x	f_x	e_x	
(8)	(9)	(10)	(11)	(12)	(13)	(14)	
100,000	3,140	97,307	6,764,778	67.6	1
96,860	581	193,139	1.9967	385,641	6,667,471	68.8	2
96,279	308	192,250	2.4995	480,529	6,281,830	65.2	3
95,971	307	191,635	2.5004	479,164	5,801,301	60.4	4
95,664	536	190,792	2.5008	477,133	5,322,137	55.6	5
95,128	704	189,552	2.5005	473,975	4,845,004	50.9	6
94,424	774	188,074	2.5004	470,260	4,371,029	46.3	7
93,650	965	186,335	2.5007	465,968	3,900,769	41.7	8
92,685	1,409	183,961	2.5012	460,123	3,434,801	37.1	9
91,276	2,063	180,489	2.5019	451,565	2,974,678	32.6	10
89,213	3,078	175,348	2.5028	438,861	2,523,113	28.3	11
86,135	4,436	167,834	2.5039	420,240	2,084,252	24.2	12
81,699	6,070	157,328	2.5051	394,122	1,664,012	20.4	13
75,629	8,251	143,007	2.5062	358,404	1,269,890	16.8	14
67,378	11,245	123,511	2.5083	309,803	911,486	13.5	15
56,133	13,208	99,058	2.5090	248,537	601,683	10.7	16
42,925	13,985	71,865	2.4974	179,476	353,146	8.2	17
28,940	13,107	44,773	2.4573	110,021	173,670	6.0	18
15,833	15,833	63,649	63,649	4.0	19