## U.S. Decennial <br> Life Tables for 1989-91

Volume 1, Number 2, Methodology of the National and State Life Tables From the CENTERS FOR DISEASE CONTROL AND PREVENTION/National Center for Health Statistics

U.S. DEPARTMENT OF HEALTH AND HUMAN SERVICES

Centers for Disease Control and Prevention National Center for Health Statistics

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U.S. DEPARTMENT OF HEALTH AND HUMAN SERVICES

Centers for Disease Control and Prevention
National Center for Health Statistics
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# Methodology of the National and State Life Tables: 1989-91 

by Robert J. Armstrong, M.S., Division of Vital Statistics


#### Abstract

This report describes the methodology employed in preparing the 1989-91 decennial life tables for the United States, the 50 individual States, and the District of Columbia. Preliminary adjustments that were made to the data before the life tables were constructed are discussed. Also the different methodologies used at very young ages and at very old ages are explained separately.

There were two changes in the methodology of these life tables from those of the previous tables (1979-81). The technique used to smooth the death rates in the State tables was changed and the method of terminating both the national and State life tables, which was developed by the Office of the Actuary, Social Security Administration, was also changed. Because of the small numbers of deaths involved, the smoothing process used in the 1979-81 tables was changed. Instead of moving whole deaths from one age group to another, the new process allowed moving tenths of deaths. This produces death rates that are more consistent with the level of the observed death rates but still allows the rates to be smoothed. The method of terminating the life tables was changed because the previous method resulted in unrealistically low death rates at ages over 95 years. The 1989-91 rates are appreciably higher than those in the 1979-81 life tables.


## Introduction

This report describes the methodology employed in preparing the 1989-91 decennial life tables for the United States, the 50 individual States, and the District of Columbia. The following tables were produced for the United States total population, total males, total females, white, white males, white females, other than white, other than white males, other than white females, black, black males, and black females. The same tables were produced for the States provided that they were based on at least 700 deaths. These tables are based on the 1990 U.S. census populations and deaths occurring during the 1989-91 period. The methodology involved in developing the 1989-91 life tables for the United States by causes of death, which is highly specialized, is described in the report containing those tables (1). In addition, definitions of the usual life table functions may be found in other reports of this series.

Death rates at ages 85 years and over are based on the experience of persons who were fully insured under the

Medicare program. Between ages 85 and 94 years these rates are blended into the rates derived from census populations and deaths. Medicare rates are used exclusively beginning at age 95 .

The two changes in the methodology of these life tables from those of 1979-81 involve the technique used to smooth the deaths at the State level and the method of terminating the life tables, which was developed by the Office of the Actuary. Because of the small numbers of deaths involved, the smoothing process used in the 1979-81 tables was changed. Instead of moving whole deaths from one age group to another, the new process allowed moving tenths of deaths.

Death rates among the extreme elderly produced in previous decennial life tables were thought to be too low. Accordingly, death rates for the extremely elderly used in the 1989-91 life tables have been allowed to increase at a faster rate than in previous decennial life tables. Also the 1990 actuarial tables will be produced by the Office of the Actuary, Social Security Administration, as they were for 1970 and 1980.

## Preliminary adjustment of the data

Some preliminary adjustments were made to the data before the life tables were constructed. The census populations used in the denominators of the death rates are not the "official" census data (2), but are data adjusted by the U.S. Bureau of the Census. In order to bring the definition of age and race in the census base populations into conformity with definitions used in vital statistics, race was modified by the Census Bureau to conform to definitions of race specified by the Office of Management and Budget (OMB). For age, adjustments were made to correct biases that resulted either from misstatement of age or year of birth, or displacement of age reporting from the census reference date. The modification procedures used in the 1980 and 1990 censuses are fundamentally different. For the 1980 census, modification was carried out on distributions of the population classified by age, sex, race, Hispanic origin, and county of residence. For 1990, modification was carried out through reassignment of age and race on individual records. While the modification procedure tried to ensure consistency between the two censuses, some discontinuities nevertheless emerged in the process (2).

A further relatively minor adjustment relates to the very small number of deaths (a total of 1,700 out of nearly 6.5
million deaths during the 3-year period) for which age was not reported.

The assumption was made that these deaths were distributed among the various age groups in the same proportions as the deaths for which age was reported. To this end, an adjustment factor was computed for each population category for which a life table was to be constructed. This factor was obtained by dividing the total number of deaths reported for the given category for the 3-year period 1989-91 by the total number less the number for which age was not reported. The number of deaths reported in each age group for the given category was then multiplied by the adjustment factor.

In preparing these decennial life tables, no specific allowance was made for possible incompleteness in the enumeration of the population or in the registration of births or deaths. In calculating previous decennial life tables, the use of birth statistics (rather than population data) in calculating the denominators of the mortality rates at ages under 2 years was justified largely on the basis that the census populations under age 2 years were believed to be underenumerated. In addition, there are other advantages of using the methodology based on birth data, since it may be expected to produce a more accurate estimate of the average population during the 3 -year period than is provided by the population enumerated on the census date. Accordingly, its use was continued in the 1989-91 life tables.

## Data used for calculating life table values

The underlying data used in preparing each of the 1989-91 decennial life tables consisted of: a) deaths occurring in the 3-year period classified by age at death, b) population data by age on the census date April 1, 1990, corrected as previously described, and c) births for each of the years 1987 to 1991, inclusive. These data were treated separately by sex and race.

Populations and deaths were available by single years of age through age 5 years and by 5-year age groups from 5 to 99 years with the final age group being 100 years and over. A slight overlap occurred because the population was needed at age 5 separately and as part of the age group 5-9 years. In each case the age referred to is in completed years-that is, the exact age on the individual's last birthday. In addition, deaths occurring at ages under 1 year were available for four subdivisions of the first year of life: Under 1 day, 1-6 days, 7-27 days, and 28-364 days. Life-table values were calculated for these subdivisions of the first year of life (but not published in the case of the State life tables) and for single years of age throughout the remainder of the life span.

With regard to the census data, actuarial theory suggests that the populations to be used in the calculations should be those of the central date of the 3-year period, that is, July 1, 1990. However, the enumerated populations as of April 1, 1990 (adjusted as previously described), were used as if they were July 1 populations. This was done for two reasons. First, estimates of the latter are available for the Nation as a whole, but not for the individual States. Second, because the percent
differences between the two sets of population figures are very small at the national level, it was not considered necessary to produce new estimates for each of the States to reflect the lapse of time between April 1 and July 1, 1990. The national population estimates as of July 1, 1990, are contained in a report of the U.S. Bureau of the Census (2).

## Number of survivors at ages 2 years and under

At ages under 2 years, the first life table quantities to be calculated were the values of ${ }_{t} d_{x}$, the number of deaths occurring between exact ages $x$ and $x+t$ in the life table cohort commencing with $l_{0}$ live births. This was calculated by the formula

$$
{ }_{t} d_{x}=\frac{l_{0} D_{x}}{{ }_{t} E_{x}}
$$

where ${ }_{t} D_{x}$ denotes the number of deaths (adjusted as described earlier for nonreporting of age) occurring in 1989-91 between exact ages $x$ and $x+t$, and ${ }_{t} E_{x}$ denotes the appropriate denominator as indicated in table A. These denominators are based on the assumption of uniform distribution over the year of the births of 1987, 1988, 1989, 1990, and 1991. In each case $l_{0}$ is taken as 100,000. The appearance of overlapping in the designations of the age intervals occurs because exact ages are involved.

The unrounded values of ${ }_{t} d_{x}$ were then used to calculate values of $l_{x}$ up to age 2 years by successive application of the formula

$$
l_{x+t}=l_{x}-{ }_{t} d_{x}
$$

## Death rates at ages 2-94 years

The life table death rate $q_{x}$ is the fraction or proportion of a group of persons at exact age $x$ who are expected to die before attaining age $x+1$. If $m_{x}$ denotes the ratio $d_{x} / L_{x}$, commonly called the central death rate, then it is well known, on the assumption of uniform distribution of deaths over the year at age $x$, that

$$
\begin{equation*}
q_{x}=\frac{2 m_{x}}{2+m_{x}} \tag{1}
\end{equation*}
$$

Table A. Denominators of ${ }_{t} E_{x}$ used in calculating ${ }_{t} d_{x}$ for ages under 2 years

| Age interval $x$ to $\mathrm{x}+1$ | Denominator of $\mathrm{d}_{x}$ |
| :--- | :---: |
| $0-1$ day $\ldots \ldots \ldots$ | $\frac{1}{730}\left(\mathrm{~B}_{1988}+730 \mathrm{~B}_{1989}+730 \mathrm{~B}_{1990}+729 \mathrm{~B}_{1991}\right)$ |
| $1-7$ days $\ldots \ldots \ldots$ | $\frac{1}{730}\left(8 \mathrm{~B}_{1988}+730 \mathrm{~B}_{1989}+730 \mathrm{~B}_{1990}+722 \mathrm{~B}_{1991}\right)$ |
| $7-28$ days $\ldots \ldots \ldots$ | $\frac{1}{730}\left(35 \mathrm{~B}_{1988}+730 \mathrm{~B}_{1989}+730 \mathrm{~B}_{1990}+695 \mathrm{~B}_{1991}\right)$ |
| $28-365$ days $\ldots \ldots \ldots$ | $\frac{1}{730}\left(393 \mathrm{~B}_{1988}+730 \mathrm{~B}_{1989}+730 \mathrm{~B}_{1990}+337 \mathrm{~B}_{1991}\right)$ |
| $1-2$ years $\ldots \ldots$ | $\ldots \ldots$ |

NOTE: $B_{z}$ denotes the reported number of births occurring during the calendar year $z$ for the population category ( by sex, race, and geographic area) involved.

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This approximation is sufficiently accurate when the life table is by single years of age. Formula [1] was the basis of the calculation of death rates at ages 2-94 years. Completion of the calculations depends, therefore, on the ability to calculate central death rates $m_{x}$ at these ages. For this purpose different methods were used at ages 2-4 years and at ages 5-94 years, as will now be described.

## Central death rates at ages 2-4 years

If $D_{x}$ denotes the adjusted number of deaths in a population category at age $x$ (in completed years) occurring in 1989-91, and $P_{x}$ denotes the population at age $x$ in the middle of the period, then (3),

$$
\begin{equation*}
m_{x}=\frac{D_{x}}{3 P_{x}} \tag{2}
\end{equation*}
$$

As previously noted, the populations actually used were those of April 1, 1990.

However, because the deaths occurring in a single year of age during 1989-91 were drawn from three consecutive annual cohorts of the population, it was considered that the accuracy of these $m_{x}$ values would be improved by replacing $3 P_{x}$ in the denominator of formula [2] by the sum of the populations at ages $x-1, x$, and $x+1$. Thus the formula becomes

$$
\begin{equation*}
m_{x}=\frac{D_{x}}{P_{x-1}+P_{x}+P_{x+1}} \tag{3}
\end{equation*}
$$

The combination of formulas [1] and [3] is equivalent to the single formula

$$
q_{x}=\frac{D_{x}}{P_{x-1}+P_{x}+P_{x+1}+\frac{1}{2} D_{x}}
$$

which was used for $x=2,3$, and 4 .

## Death rates at ages 5-94 years

The combination of formulas [1] and [2] is equivalent to

$$
\begin{equation*}
q_{x}=\frac{D_{x}}{3 P_{x}+\frac{1}{2} D_{x}} \tag{4}
\end{equation*}
$$

which was used for ages 5-94 years, with values of $D_{x}$ and $P_{x}$ obtained by interpolation from data by 5 -year age intervals. The procedure of interpolating populations and deaths separately has a long history. It was first used by Dr. John Tatham in preparing English Life Table No. 5 covering the period 1881-90 and published in 1895 (4). The Tatham method was also used in the construction of the U.S. life tables based on the censuses of 1900-30, inclusive. When life tables are mass-produced, as in the United States, the 50 States, and the District of Columbia, the Tatham method of osculatory inter-
polation has an operational advantage in that the interpolated deaths at a given age for any class is exactly the sum of those for the subclasses of which it is composed, and a similar statement applies to populations. For example, at any age the interpolated deaths for total white persons is the sum of those for white males and white females, and the interpolated deaths for the United States is exactly the sum of those for the 50 States and the District of Columbia.

For the 1989-91 life tables, the interpolation of both deaths and populations was performed by means of the interpolation coefficients developed by H. S. Beers (table B). In certain age intervals the headings of this table are not to be taken precisely as stated. In interpolating at ages 90-94 years, the numbers (of deaths or populations) at ages 100 years and over were used as if they applied to ages $100-104$ years. Moreover, in interpolating at ages 5-14 years, the value used for the "quinquennial sum starting at age 0 " was not the actual number reported at ages $0-4$ years but a fictitious value. Because of the mortality peak in infancy, the use of the actual numbers at ages $0-4$ years probably would not yield plausible values. There is no reason to expect that the interpolated values for ages 5-9 years would join smoothly with the numbers reported at ages 2-4 years if actual numbers for the age group 0-4 years were used.

As a result, a fictitious quinquennial sum was used for the age interval $0-4$ years. The numbers were chosen so that the sum of the interpolated values at ages $2-4$ years would be equal to the number reported in this 3 -year age interval (adjusted for nonreporting of age in the case of deaths).

If $W_{x}$ denotes the quinquennial sum commencing with age $x$ and $V$ denotes the sum of the interpolated numbers for ages $2-4$ years (which shall be required to be equal to the reported number), these coefficients give

$$
\begin{aligned}
V= & 0.4072 W_{0}+0.2416 W_{5}+0.0080 W_{10}-0.0896 W_{15} \\
& +0.0328 W_{20}
\end{aligned}
$$

Solving for $W_{0}$, the fictitious quinquennial sum, gives

$$
\begin{aligned}
W_{0}= & 2.45580 V-0.59332 W_{5}-0.01965 W_{10}+0.22004 W_{15} \\
& -0.08055 W_{20}
\end{aligned}
$$

This formula was used to compute the fictitious value for the age interval $0-4$ years. The interpolated populations from the census and deaths from the National Center for Health Statistics were used in formula [4] to calculate the death rates up to age 94 years.

At ages 85-94 years, the death rates obtained here were blended with those derived (as explained in the next section) from experience of the Medicare program. Thus, the rates actually used in the construction of the life tables were obtained by the formula

$$
q_{x}=\frac{1}{11}\left[(95-x) q_{x}^{C}+(x-84) q_{x}^{M}\right]
$$

where $q_{x}=$ life table death rate at age $x$
$q_{x}^{C}=$ death rate calculated with formula [4]
$q_{x}^{M}=$ corresponding rate based on Medicare experience.

Table B. Beers' interpolation coefficients for subdividing quinquennial sums to obtain estimated numbers by single years of age (minimized fifth-difference formula with smoothed end)

| Age |  | Quinquennial sum beginning at age |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0 years | 5 years | 10 years | 15 years | 20 years |
|  |  | Coefficients for 2-4 years |  |  |  |  |
| 2 years |  | 0.1924 | 0.0064 | 0.0184 | -0.0256 | 0.0084 |
| 3 years |  | 0.1329 | 0.0844 | 0.0054 | -0.0356 | 0.0129 |
| 4 years |  | 0.0819 | 0.1508 | -0.0158 | -0.0284 | 0.0115 |
|  |  | Coefficients for 5-9 years |  |  |  |  |
| 5 years |  | 0.0404 | 0.2000 | -0.0344 | -0.0128 | 0.0068 |
| 6 years |  | 0.0093 | 0.2268 | -0.0402 | 0.0028 | 0.0013 |
| 7 years |  | -0.0108 | 0.2272 | -0.0248 | 0.0112 | -0.0028 |
| 8 years |  | -0.0198 | 0.1992 | 0.0172 | 0.0072 | -0.0038 |
| 9 years |  | -0.0191 | 0.1468 | 0.0822 | -0.0084 | -0.0015 |


|  | Quinquennial sum beginning at age |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Age | $5 m-10$ years | $5 m-5$ years | 5 m years | $5 m+5$ years | $5 m+10$ years |
|  | Coefficients for 10-94 years |  |  |  |  |
| 5 m years | -0.0117 | 0.0804 | 0.1570 | -0.0284 | 0.0027 |
| $5 \mathrm{~m}+1$ years | -0.0020 | 0.0160 | 0.2200 | -0.0400 | 0.0060 |
| $5 \mathrm{~m}+2$ years | 0.0050 | -0.0280 | 0.2460 | -0.0280 | 0.0050 |
| $5 \mathrm{~m}+3$ years | 0.0060 | -0.0400 | 0.2200 | 0.0160 | -0.0020 |
| $5 \mathrm{~m}+4$ years | 0.0027 | -0.0284 | 0.1570 | 0.0804 | -0.0117 |

SOURCE: H.S. Beers: Reply to the discussion of his paper six-term formulas for routine actuarial interpolation. Rec. Amer. Inst. Act. 34:60, 1945.

## Death rates at ages 95 years and over

As in the 1979-81 life tables, death rates at ages 95 years and over were based solely on experience of the Medicare program and were provided by the Office of the Actuary, Social Security Administration. Medicare data were used at ages 95 years and over because they were considered more accurate than conventional death rates, which have problems in the accuracy of the reporting of age among the extremely elderly. As mentioned in the preceding section, death rates at ages 85-94 years based on Medicare data were blended with those based on census populations and registered deaths. Therefore, death rates based on Medicare experience were required at all ages 85 years and over.

These death rates were differentiated by sex and race but not by geographic area. The Office of the Actuary provided, for the first time, death rates among the elderly black population. Previously it had been necessary to apply the rates for the other than white population to the black population. Since no distinction is made by geographic area at ages 95 years and over, the life table death rates for the United States and for the States are identical and the influence of geographic area on the rates diminishes with increasing age at ages 85-94 years.

Death rates based on Medicare experience used in the construction of the life tables are shown in table C. The procedure by which these were obtained consists of a series of steps. First, "crude" death rates for ages 66-105 years, inclusive, for white males, white females, males other than white, and females other than white were computed directly from the data on deaths and enrollments. The data used for this
purpose were limited to the "HI-insured" group (that is, insured for Hospital Insurance). In general terms, this excludes persons who were "blanketed into" the Medicare program even though they had no covered employment or only a minimal amount of such employment under the Social Security program or the Railroad Retirement program (5). It is believed that ages have been more accurately determined for the HI -insured group. Table D shows by age and sex the percent of Medicare beneficiaries who were HI-insured as of January 1, 1990. Also excluded from the calculations were persons of unknown race, estimated to have been roughly 3 percent.

Crude death rates for the remaining five categories of the population (total male, total female, total white, total other than white, and the total population) were obtained as weighted averages of the four previously determined categories, using as weights the proportions of the enrolled population in the specified category on January 1, 1990. In calculating these proportions, the entire population enrolled for Medicare (not merely the HI -insured) was used, except that, of course, persons of unknown race had to be excluded in obtaining proportions involving race. A comparison between the Medicare proportions and the available age grouping proportions in the 1990 census showed little difference.

The crude rates for each of the nine population categories for ages 66-105 years, inclusive, were then graduated (that is, smoothed) by a Whittaker-Henderson Type B formula (6). Such a formula involves minimizing the quantity

$$
\sum_{x=\alpha}^{\beta} W_{x}\left(q_{x}^{\prime \prime}-q_{x}\right)^{2}+k \sum_{x=\alpha}^{\beta-z}\left(\Delta^{z} q_{x}\right)^{2}
$$

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Table C. Graduated mortality rates from Medicare experience by sex, race, and age at last birthday: United States, 1989-91

| Age | Total |  |  | White |  |  | Other than white |  |  | Black |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Both sexes | Male | Female | Both sexes | Male | Female | Both sexes | Male | Female | Both <br> sexes | Male | Female |
| 85 years | 0.09713 | 0.12622 | 0.0836 | 0.09688 | 0.12637 | 0.08337 | 0.09929 | 0.1253 | 0.08552 | 0.10079 | 0.12956 | 0.08699 |
| 86 years | 0.10588 | 0.13623 | 0.09245 | 0.10583 | 0.13659 | 0.09246 | 0.10664 | 0.1337 | 0.09269 | 0.10759 | 0.13729 | 0.09391 |
| 87 years | 0.11559 | 0.14704 | 0.10233 | 0.11575 | 0.1476 | 0.10258 | 0.11461 | 0.14281 | 0.10038 | 0.11503 | 0.14566 | 0.10139 |
| 88 years | 0.12625 | 0.1587 | 0.11317 | 0.12656 | 0.15942 | 0.1136 | 0.12328 | 0.15263 | 0.10871 | 0.12318 | 0.1547 | 0.10949 |
| 89 years | 0.13777 | 0.17113 | 0.12491 | 0.13823 | 0.17204 | 0.12553 | 0.13242 | 0.163 | 0.11763 | 0.13187 | 0.16428 | 0.11817 |
| 90 years | 0.15005 | 0.18424 | 0.13756 | 0.15074 | 0.18541 | 0.13843 | 0.14183 | 0.17375 | 0.12702 | 0.14097 | 0.17429 | 0.12734 |
| 91 years | 0.16316 | 0.19808 | 0.15115 | 0.16423 | 0.19961 | 0.15237 | 0.15149 | 0.18475 | 0.13689 | 0.15047 | 0.1846 | 0.13703 |
| 92 years | 0.17726 | 0.21274 | 0.16576 | 0.17878 | 0.2147 | 0.16738 | 0.16156 | 0.19588 | 0.14736 | 0.16045 | 0.19504 | 0.14733 |
| 93 years | 0.19244 | 0.22819 | 0.18139 | 0.19437 | 0.2306 | 0.18338 | 0.17231 | 0.20705 | 0.15859 | 0.17102 | 0.20549 | 0.15833 |
| 94 years | 0.20853 | 0.24412 | 0.19788 | 0.21081 | 0.24695 | 0.2002 | 0.18379 | 0.21812 | 0.17061 | 0.18219 | 0.2158 | 0.17003 |
| 95 years | 0.22502 | 0.26004 | 0.21475 | 0.2276 | 0.26329 | 0.21737 | 0.19586 | 0.22903 | 0.18338 | 0.19386 | 0.22659 | 0.18244 |
| 96 years | 0.24126 | 0.27536 | 0.23143 | 0.24414 | 0.27914 | 0.23434 | 0.2083 | 0.24048 | 0.19682 | 0.2059 | 0.23792 | 0.19556 |
| 97 years | 0.25689 | 0.28943 | 0.24775 | 0.26009 | 0.29399 | 0.25091 | 0.22089 | 0.2525 | 0.21089 | 0.21821 | 0.24982 | 0.20946 |
| 98 years | 0.27175 | 0.3039 | 0.26375 | 0.27538 | 0.30869 | 0.26715 | 0.2337 | 0.26513 | 0.22557 | 0.23087 | 0.26231 | 0.22414 |
| 99 years | 0.28751 | 0.3191 | 0.27957 | 0.29135 | 0.32413 | 0.28318 | 0.24726 | 0.27838 | 0.23911 | 0.24426 | 0.27542 | 0.23758 |
| 100 years | 0.30418 | 0.33505 | 0.29635 | 0.30824 | 0.34033 | 0.30017 | 0.2616 | 0.2923 | 0.25346 | 0.25843 | 0.2892 | 0.25184 |
| 101 years | 0.32182 | 0.35181 | 0.31413 | 0.32612 | 0.35735 | 0.31818 | 0.27677 | 0.30692 | 0.26866 | 0.27342 | 0.30365 | 0.26695 |
| 102 years | 0.34049 | 0.3694 | 0.33298 | 0.34504 | 0.37522 | 0.33727 | 0.29282 | 0.32226 | 0.28478 | 0.28927 | 0.31884 | 0.28297 |
| 103 years | 0.36024 | 0.38787 | 0.35296 | 0.36505 | 0.39398 | 0.3575 | 0.30981 | 0.33837 | 0.30187 | 0.30605 | 0.33478 | 0.29994 |
| 104 years | 0.38113 | 0.40726 | 0.37413 | 0.38622 | 0.41368 | 0.37895 | 0.32778 | 0.35529 | 0.31998 | 0.3238 | 0.35152 | 0.31794 |
| 105 years | 0.40324 | 0.42762 | 0.39658 | 0.40862 | 0.43436 | 0.40169 | 0.34679 | 0.37306 | 0.33918 | 0.34258 | 0.36909 | 0.33702 |
| 106 years | 0.42663 | 0.449 | 0.42038 | 0.43232 | 0.45608 | 0.42579 | 0.3669 | 0.39171 | 0.35953 | 0.36245 | 0.38755 | 0.35724 |
| 107 years | 0.45137 | 0.47145 | 0.4456 | 0.4574 | 0.47888 | 0.45134 | 0.38818 | 0.4113 | 0.3811 | 0.38348 | 0.40693 | 0.37867 |
| 108 years | 0.47755 | 0.49503 | 0.47233 | 0.48393 | 0.50282 | 0.47842 | 0.4107 | 0.43186 | 0.40397 | 0.40572 | 0.42727 | 0.40139 |
| 109 years | 0.50525 | 0.51978 | 0.50068 | 0.512 | 0.52797 | 0.50712 | 0.43452 | 0.45345 | 0.42821 | 0.42925 | 0.44864 | 0.42548 |

where $x=$ index for single years of age
$\alpha=$ minimum age
$\beta=$ maximum age
$W_{x}=$ designated positive weight
$q_{x}^{\prime \prime}=$ crude rate
$q_{x}=$ smoothed rate
$\Delta^{z}=z$ th finite difference, $z$ being commonly chosen as 2 or 3 .

The first summation is a measure of the departure of the smoothed rates from the crude rates, and the second summation measures the roughness of the smoothed rates (that is, the smaller this quantity, the smoother these rates). The smoothing constant $k$ indicates the degree of importance the user attaches to smoothness in relation to closeness of fit to the observed data.

Minimization of formula [5] leads to a system of linear equations to be solved for the smoothed death rates $q_{x}$. It has been suggested $(8,9)$ that on theoretical grounds the weights $W_{x}$ should be taken as the reciprocals of the (binomial) variances of the smoothed death rates $q_{x}$, that is,

$$
W_{x}=\frac{E_{x}}{q_{x}\left(1-q_{x}\right)}
$$

where $E_{x}$ denotes the "exposed to risk." In fact, it may be pointed out that with this choice of weights the first term of expression [5] becomes the value of the chi-square statistic that is used in the chi-squared goodness-of-fit test. However, the use of such weights would lead to a system of nonlinear equations in the variables $q_{x}$ that could not easily be solved.

This difficulty may be overcome by using the crude rate as an approximation of the smoothed rate in the computation of weights so that the approximate formula becomes

$$
W_{x}=\frac{E_{x}}{q_{x}^{\prime \prime}\left(1-q_{x}^{\prime \prime}\right)}
$$

In graduating the death rates at ages 66-105 years based on Medicare experience, the finite difference $z$ was taken as 3 and the smoothing constant $k$ as 5,000 , so that the expression to be minimized becomes

$$
\sum_{x=66}^{105} W_{x}\left(q_{x}^{\prime \prime}-q_{x}\right)^{2}+5,000 \sum_{x=66}^{102}\left(\Delta^{3} q_{x}\right)^{2}
$$

A comparison was done between letting $k$ equal 140,000 , as was done in the 1979-81 life tables, and letting it equal 5,000,

Table D. Percent of HI-insured Medicare beneficiaries by age and sex: United States, January 1, 1990

| Age | Percent |  |
| :---: | :---: | :---: |
|  | Male | Female |
| 65-69 years | 98.4 | 97.7 |
| 70-74 years | 98.5 | 97.6 |
| 75-79 years | 98.1 | 97.0 |
| 80-84 years | 97.7 | 96.4 |
| 85-89 years | 96.2 | 94.4 |
| 90-94 years | 91.9 | 90.9 |
| 95-99 years | 83.0 | 83.4 |
| 100 years and over | 44.8 | 35.8 |

the value used by the Office of the Actuary, Social Security Administration. The difference between the two expressions was minimal and 5,000 was chosen for $k$ because a smaller smoothing coefficient leaves the graduated numbers closer to the original values calculated from the data.

For a variety of reasons, including misreporting of age, tabulated death rates at very high ages tend to be understated. In order to correct for this understatement the graduated rates at certain of the oldest ages were rejected and replaced by rates obtained by a method of extrapolation that is used in the Annual Social Security Trustees Reports. For each sex, a minimum percent change from $q_{x-1}$ to $q_{x}$ was required. The minimum percent changes used were, 5.8 percent for the total population, 5.0 percent for males, and 6.0 percent for females. These values are consistent with those used in the 1994 Trustees Report. The level of $q_{x}$ for females was not allowed to rise to a level higher than that for males of the same age and race. The age at which the minimum percent change was first imposed varied from age 94 to 99 years. Through this procedure $q_{x}$ reached a level of 1.000 at a terminal age that varied from 121 to 126 years.

As the underlying data were available in such a form that the death rate referred to as $q_{x}$ was in reality $q_{x-0.5}$, interpolation was performed to produce rates for integral ages from 85 to 109 years, inclusive, using the approximate formula

$$
\begin{equation*}
q_{x} \cong-\frac{1}{16} q_{x-1.5}+\frac{9}{16} q_{x-0.5}+\frac{9}{16} q_{x+0.5}-\frac{1}{16} q_{x+1.5} \tag{6}
\end{equation*}
$$

This formula would be exact if the four consecutive rates in the right member were exactly fitted by a third-degree polynomial.

## Calculation of the remaining life table values

Death rates $q_{x}$ were now available for all ages 2-109 years, inclusive. From these, using double precision, numbers of survivors $l_{x}$ were calculated for ages 3-110 years, inclusive, by the formula

$$
\begin{equation*}
l_{x+1}=l_{x}-l_{x} q_{x} \tag{7}
\end{equation*}
$$

where $x$ ranges from 2 to 109 years. Values of $l_{x}$ were now available for all integral ages from 0 to 110 , inclusive, as well as for certain ages between 0 and 1 year ( 1 day, 7 days, and 28 days).

Values of $q_{x}$ were available for ages 2 years and over but not for ages under 2 years. Accordingly $q_{0}$ and $q_{1}$ were calculated by the formulas

$$
\begin{aligned}
& q_{0}=1-\frac{l_{1}}{l_{0}} \\
& q_{1}=1-\frac{l_{2}}{l_{1}}
\end{aligned}
$$

Moreover, for the United States, death rates ${ }_{t} q_{x}$ for subdivisions of the first year of life were calculated by the formula

$$
{ }_{t} q_{x}=1-\frac{l_{\mathrm{x}+t}}{l_{x}}
$$

Such values of ${ }_{t} q_{x}$ were not published for the States.
Values of ${ }^{o} e_{110}$, the expectation of life at age 110 years, for each of the 12 race-sex categories were furnished by the Office of the Actuary, Social Security Administration. These were calculated by extrapolating $q_{x-0.5}$ values by the method previously described, to age 132 years, and obtaining values of $q_{x}$ up to age 130 years by interpolation using formula [6]. Values of $l_{x}$ for $x=111$ to 130 years, inclusive, were calculated by formula [7]. Values of $L_{x}$, the number of person-years lived between age $x$ and $x+1$, for age 110-130 years, inclusive, were obtained from the general formula

$$
L_{x}=0.5\left(l_{x}+l_{x+1}\right)
$$

$T_{x}$, the number of person-years lived after age $x$, was computed sequentially from age 130 years back to age 110 years by

$$
T_{x}=T_{x+1}+L_{x}
$$

with $T_{131}$ taken equal to zero. The final step was to compute

$$
{ }^{o} e_{110}=\frac{T_{110}}{l_{110}}
$$

These values are shown in table E.
With the values of ${ }^{o} e_{110}$ available, $T_{x}$ for the ages included in the life tables was computed by the formulas

$$
T_{110}=l_{110}{ }^{o} e_{110}
$$

and

$$
T_{x}=T_{x+1}+0.5\left(l_{x}+l_{x+1}\right)
$$

proceeding from age 109 years back to age 0 . For the subdivisions of the first year of life, the corresponding formula is

$$
T_{x}=T_{x+t}+\frac{t}{2}\left(l_{x}+l_{x+t}\right)
$$

where $t$ is the appropriate fraction of a year corresponding to the interval involved. The values of $t$ used are as follows:

| Age | $t$ |
| :---: | :---: |
| 0 | 1/365 |
| 1 day | 6/365 |
| 7 days | 21/365 |
| 28 days | 337/365 |

The average remaining lifetime (or "expectation of life") ${ }^{o} e_{x}$ was calculated at all ages from 0 to 109 years, including ages between 0 and 1 year, by the formula

$$
{ }^{o} e_{x}=\frac{T_{x}}{l_{x}}
$$

The values of $q_{x}, l_{x}, T_{x}$, and ${ }^{o} e_{x}$ were then rounded to the number of decimal places shown in the published life tables. In other words, $l_{x}$ and $T_{x}$ were rounded to the nearest integer, $q_{x}$ to five decimal places, and ${ }^{o} e_{x}$ to two decimal places. Because $l_{x}$ and $q_{x}$ were independently rounded, calculation of $q_{x}$ from the published (rounded) values of $l_{x}$ by

$$
q_{x}=1-\frac{l_{x+1}}{l_{x}}
$$

may not always agree with the published value in the fifth decimal place. Finally, $d_{x}$ and $L_{x}$ were obtained by differencing the rounded values of $l_{x}$ and $T_{x}$, respectively:

$$
d_{x}=l_{x}-l_{x+1}
$$

and

$$
L_{x}=T_{x}-T_{x+1}
$$

## Calculation of standard errors of the life table functions

The 1989-91 U.S. decennial life tables are the second set of decennial life tables to show standard errors for certain life table functions. Specifically, the standard errors for the probabilities of dying and for the life expectancies are shown. It is important to consider that these standard errors reflect only stochastic variation and are based on an assumption that the age-specific deaths follow a binomial distribution. Stochastic variation is not the only source of error for life table functions; measurement error, such as age misstatements on death certificates or on census reports, also affects the accuracy of the life table functions. While the extent of measurement error on life table functions has not been quantified, it is generally thought that measurement errors could be larger than stochastic errors. Because the life tables for the United States and for the published States are based on relatively large numbers of deaths, the standard errors presented are rather small.

For ages less than 85 years, a binomial distribution assumption yields the following estimate for the variance of $q_{x}$ :

$$
\begin{equation*}
S^{2}\left(q_{x}\right)=\frac{q_{x}^{2}\left(1-q_{x}\right)}{D_{x}^{*}} \tag{8}
\end{equation*}
$$

where $D_{x}^{*}$ is the age-specific number of deaths, smoothed by interpolation and adjusted for the number of deaths with age not stated.

For ages 85-109 years, Medicare data were used to estimate the probabilities of dying; equation [8] cannot be used. An empirical investigation led to estimates of $S^{2}\left(q_{x}\right)$ for these ages as well as for $S^{2}\left({ }^{o}{ }_{110}\right)$. For the variances of the life expectancies at ages $0-109$ years, an equation from Chiang (9), with a slight modification, was used, namely:

Table E. Values of ${ }^{\circ} e_{110}$ extrapolated from Medicare experience by race and sex: United States, 1989-91

|  | Race and sex | ${ }^{\circ}{ }_{110}$ |
| :---: | :---: | :---: |
| Total |  |  |
| Both sexes |  | 1.29 |
| Male |  | 1.27 |
| Female |  | 1.30 |
| White |  |  |
| Both sexes |  | 1.27 |
| Male |  | 1.24 |
| Female |  | 1.28 |
| All other |  |  |
| Both sexes | . . . . . . | 1.34 |
| Male |  | 1.50 |
| Female |  | 1.30 |
| Black |  |  |
| Both sexes |  | 1.58 |
| Male |  | 1.52 |
| Female | . . . . . . . . | 1.59 |

$$
\begin{equation*}
S^{2}\left({ }_{e}^{o} e_{x}\right)=\frac{l_{110}^{2} S^{2}\left(e_{110}\right)+\sum_{y=x}^{109} l_{y}^{2}\left({ }^{o} e_{y+1}+0.5\right)^{2} S^{2}\left(a_{y}\right)}{l_{x}^{2}} \tag{9}
\end{equation*}
$$

## Special adjustments in the U.S. and State life tables

For each of the 50 States and the District of Columbia, life tables were calculated for each of the 12 race-sex groups shown in the U.S. report. However, in some States not all of the 12 tables were published, because it was considered that the amount of data for one race was too small to produce reliable results. If for any racial group fewer than 700 male or 700 female deaths at all ages were registered in the given State for the 3 -year period 1989-91, the tables for that race group were not published. The number 700 was chosen after experimenting with data from the 1969-71 period to determine the minimum number of deaths that could be smoothed into a reliable life table. As a result of applying this criterion, life tables for persons other than white were not published for 11 States: Idaho, Iowa, Maine, Montana, New Hampshire, North Dakota, Rhode Island, South Dakota, Utah, Vermont, and Wyoming. Life tables were not published for the black population for these same States plus the seven States of Alaska, Hawaii, Minnesota, Nebraska, Nevada, New Mexico, and Oregon. In three of the above 18 States (Minnesota, Nevada, and South Dakota) the number of deaths of black persons or of persons other than white was fewer than 700 for one sex (females in all cases) and not for the other. Life tables for white persons were published for every State.

In most of the State life tables, special adjustments were made at certain ages to correct or mitigate anomalous behavior of the life table values that may be attributed to the small
numbers involved. After each life table to be subjected to such adjustment had been calculated and printed out, the $q_{x}$ values for individual years of age were examined and certain tests of consistency applied. The other life table functions are completely determined by the $q_{x}$ values, so no tests needed to be applied to them.

Because of the small numbers of deaths involved, the smoothing process used in the 1979-81 tables was changed. Instead of moving whole deaths from one age group to another, the new process allowed moving tenths of deaths. For example, under the 1979-81 methodology if in a given State the number of deaths to white females at ages 1-4 years was 4 and smoothing at these ages was needed, it was necessary to arbitrarily add at least 6 deaths to obtain a new total of 10 . This was the minimum number that would allow the death rates to decline from age 1 to age 4 . The 10 deaths were distributed 4 to age 1,3 to age 2,2 to age 3 , and 1 to age 4 . Thus, the smoothing process raised the level of the observed death rates by 150 percent, and although death rates for white males were required to be higher at every age than for white females, the rates for white males were distorted even more. The new process often allowed the smoothing to be done without increasing the number of deaths in the age interval and allowed the rates to be smoothed without radically changing their level.

It was considered that in each life table, the $q_{x}$ values should decrease from age 0 to about age 10 or 11 years and then increase to the early twenties. They should increase again from about age 30 years to the end of the table. Strict increase in death rates with increasing age was not required between 20 and 30 years, because a slight decrease in the mortality curve in this age range (due to declining numbers of deaths from violent and accidental causes) is a feature of many of the life tables. Abrupt age-to-age changes in $q_{x}$ values (indicated by relatively large second differences) were also examined.

Such adjustments were made directly only to the life tables for white males, white females, males other than white, females other than white, black males, and black females. (No adjustments were made directly to the the total population or the various subpopulations.) After the data underlying these six tables had been adjusted to remove anomalies, the adjusted data were combined in various ways to produce the remaining six tables. It was assumed that if the six basic components were free from anomalies, this would also be true of the various subpopulations. For some States the data for population groups for which life tables were not published (for example, males and females other than white in 11 States and black males and females in an additional 7 States) were not adjusted. It was assumed that if the life tables for white persons were free from anomalies and the deaths of persons other than white were too few to warrant publication of the life tables, there would be no anomalies in the life tables for total males and total females. For 7 of the 18 States (Montana, North Dakota, Rhode Island, South Dakota, Utah, Vermont, and Wyoming) this assumption was not justified, and adjustments were found to be necessary in the data for persons other than white or black, even though the corresponding life tables were not published, to avoid anomalies in the life tables for
total persons, total males, and total females. These adjustments were most frequently made at ages $1-4$ years.

For each pair of published life tables for males and females of a given race in a given State it was considered that the $q_{x}$ values for females at each age should be less than the corresponding value for males. When life tables for the population other than white were published, the $q_{x}$ values for white persons at each age should be less than the corresponding value for persons other than white of the same sex up to about age 70 years. This requirement was relaxed between the early teens and the early twenties. At these ages the rates for whites were permitted, but not required, to be higher than those for persons other than white. When the values for persons other than white do become lower at about age 70 years or over, they should remain lower. In other words, corresponding mortality curves for white persons and others should not be permitted to cross and recross a number of times. This criterion was not applied, however, to the States of Hawaii and California, where the population other than white is composed primarily of ethnic groups having death rates closely comparable to those of the white population. Similarly, when life tables for the black population were published, the $q_{x}$ value for persons other than white at each age should be less than the corresponding value for black persons of the same sex. This should be true at all ages.

In every instance in which an adjustment was considered necessary, it was effected by redistributing by age the numbers of deaths in two or more usually adjacent age groups, so that the total number of deaths at all ages remained unchanged. In using this type of adjustment, the intention was to change the local shape of the mortality curve while preserving the overall mortality level. In some cases, the numbers of deaths in the age groups involved were redistributed by age in proportion to the corresponding numbers for the same sex and race for the United States. This procedure was used most frequently at ages 1-4 years. When this process failed to remove the observed anomalies, deaths were redistributed by age in a more arbitrary manner.

Sometimes several trial runs had to be made for a given State before satisfactory $q_{x}$ values were obtained. For each trial run small changes were made in the numbers of deaths; then $q_{x}$ values were recalculated. When redistributions by age of the numbers of deaths resulted in appropriate $q_{x}$ values, the process of computation of the various life table functions, as previously described, was carried out with the $q_{x}$ values based on the redistributed numbers of deaths.

The redistributed deaths for the six basic demographic categories (white males, white females, males other than white, females other than white, black males, and black females) were then combined to produce the redistributed deaths by age for the six remaining categories (total population, total males, total females, total white, total other than white, and total black), and computation of the life tables for the latter categories was completed. If these life tables contained anomalies, some additional redistribution of deaths was done in the individual race-sex groups. The life tables for the United States were not corrected to reflect redistributions by age of deaths in the States.

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Table F. Number of published State and District of Columbia life tables with special adjustments by race, sex, and selected age intervals: United States, 1989-91

| Age interval between exact ages | White male | White female | All other male | All other female | Black male | Black female |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0-2 years | 49 | 49 | 39 | 39 | 33 | 32 |
| 5-29 years | 46 | 51 | 36 | 39 | 28 | 32 |
| 30-49 years | 21 | 31 | 20 | 28 | 13 | 22 |
| 50 years and over | 5 | 6 | 34 | 31 | 28 | 27 |

The U.S. life tables required some minor smoothing, as did the 1979-81 tables. The unsmoothed national data for both the other than white and black populations showed higher death rates for females than for males at ages 10 and 11. This was corrected by moving a few deaths out of the age group 10-14 years into adjacent age groups for females other than white and black females. This necessitated some minor shift in the number of deaths for white females at ages 15-24 years. The number of deaths for males other than white and black males were raised slightly in the age group 10-14 years and lowered in adjacent age groups. These changes did not necessitate changes in the deaths of white males.

Table F gives some idea of the number of special adjustments made in the State life tables. Further details are given in the appendix, which shows, for each sex and race group in each State, the ages at which these adjustments were made.

By far, more adjustments were required in the age interval $1-44$ years than in any other. First, at ages $1-4$ years, deaths by single years of age were used, and these numbers are small and subject to severe statistical fluctuations. Second, the U.S. life tables were themselves smoothed at ages 5-14 years for males other than white and black females; at ages 5-19 years for females other than white; at ages 10-19 years for black males; and at ages 15-24 years for white females. This was an indication that most State life tables would have to be smoothed at these same ages. Of all the published life tables, only the one for white males in Ohio did not require special adjustments at any age.

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## Appendix

Ages at which special adjustments were made in the 1989-91 State life tables by sex and race

| State | Male |  |  | Female |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | White | All other | Black | White | All other | Black |
| Alabama . . . | 1-9, 15-34 | 1-14, 20-29, 65-84 | 1-29 | 4-39 | 1, 3-29, 60-79 | 1, 3-9, 20-44, 70-79 |
| Alaska | $\begin{aligned} & 0-9,15-24,35-44 \\ & 80-89 \end{aligned}$ | 1-3, 15-49, 80-99 |  | 0-34, 75-94 | 0-54, 75-94 |  |
| Arizona | 1, 3, 4, 10-19 | 1-44, 50-99 | 1-3, 5-99 | 1, 4, 10-44, 85-89 | 2-14, 20-24, 30-94 | 0-29, 35-54, 60-84 |
| Arkansas | 1-34 | $\begin{aligned} & 1-9,15-19,25-29 \\ & 70-89 \end{aligned}$ | 1-9, 20-29, 70-89 | 1-3, 10-44 | 3-29, 35-39 | 1, 3-29, 35-39 |
| California | 5-14 | 2-19 | 1-24 | 5-24 | 4-24 | 1-24 |
| Colorado | 2-34, 40-49 | 1-49 | $\begin{aligned} & 1-14,20-24,30-54 \\ & 65-89 \end{aligned}$ | 1, 4, 10-39 | $\begin{aligned} & 1,3,4,10-24,30-49 \\ & 65-74 \end{aligned}$ | 1-14, 20-49, 65-79 |
| Connecticut | 0-3, 5-29 | 1-9, 20-29, 70-89 | $\begin{aligned} & 1-4,10-14,20-24, \\ & 45-54,65-89 \end{aligned}$ | 4-29, 35-39 | 0-49, 75-84 | 0-9, 15-49, 75-84 |
| Delaware | 1, 2, 4-24, 30-39 | 1-44 | 1-24, 30-49 | 1, 2, 4-44 | 1-44, 65-84 | 1-44, 65-84 |
| District of Columbia | 1-24, 35-59 | 1-24, 35-84 | 1-29, 35-74 | 1-69 | 1-44 | 1-44 |
| Florida | 1-14 | 10-19 | 3, 4, 10-19 | 1, 3-24 | 3-14 | 2-14, 75-84 |
| Georgia | 1-4, 10-24 | 0-4, 70-89 | 0-4, 70-89 | 1-29 | 1-4, 15-24 | 1-4, 15-24 |
| Hawaii | 1-14, 30-34, 70-89 | 1-3, 25-34 |  | 1-64 | 1-44 |  |
| Idaho . . . . | 1-14, 25-44 |  |  | 1-3, 5-29, 35-44 |  |  |
| Illinois . . . . |  | 1-4, 10-19 | 1, 2, 4, 10-19 | 1-4, 15-24 | 1-14 | 1-14 |
| Indiana . . . |  | $\begin{aligned} & 1-3,5-24,65-74 \\ & 80-89 \end{aligned}$ | $\begin{aligned} & 1-3,5-24,65-74 \\ & 80-89 \end{aligned}$ | 1-3, 15-29 | 1-29, 65-74 | 1-29, 65-74 |
| Iowa . . . . |  |  |  | 1-3, 10-24, 30-34 |  |  |
| Kansas . . . | 1-4, 15-19, 30-44 | $\begin{aligned} & 1-29,35-44,60-69 \\ & 80-89 \end{aligned}$ | 1-29, 35-44, 60-69 | 1-34 | $\begin{aligned} & 2-19,25-29,35-44, \\ & 50-69,80-89 \end{aligned}$ | 1-14, 25-34, 50-69, 80-89 |
| Kentucky | 1-29 | 1-44, 65-84, 90-94 | 1-44, 65-84, 90-94 | 1-3, 5-29 | $\begin{aligned} & 1-24,35-39,50-59 \\ & 65-79 \end{aligned}$ | 1-19, 50-59, 65-94 |
| Louisiana | 0-29 | $\begin{aligned} & 1-3,10-19,35-44 \\ & 55-74 \end{aligned}$ | 1-3, 10-19, 55-74 | 2-19, 25-29 | 1-19, 65-84 | 2, 3, 5-14, 25-34, 65-84 |
| Maine . . . . |  |  |  | 1-34 |  |  |
| Maryland | 1-14, 30-39 | 1, 2, 5-19, 65-84 | 1-19, 65-84 | 1, 2, 4-24 | 1-24, 65-84 | 1-24, 65-84 |
| Massachusetts . | 1, 2, 5-19 | 1-24, 30-34, 55-89 | 1-89 | 1-24 | 1-19, 35-49, 60-79 | 1-19, 35-49, 60-79 |
| Michigan . | 1-3, 10-19 | 1-29, 80-89 | 1-29, 55-59, 70-89 | 1-29 | 1-34 | 1-34, 65-84 |
| Minnesota . . |  | 1-9, 35-79, 85-94 |  | 1-34 | 1-24, 30-94 |  |
| Mississippi . | 0-29, 70-79 | 1-39, 70-84, 90-94 | $\begin{aligned} & 1-3,5-39,60-84 \\ & 90-94 \end{aligned}$ | 1-34 | $\begin{aligned} & 1-34,40-49,55-59 \\ & 70-84,95-99 \end{aligned}$ | 1-24, 30-39, 55-59, 65-94 |
| Missouri . . . | 1-4, 15-29 | 1-4, 50-89 | 1-4, 55-89 | 1-4, 15-34 | 1-39, 55-84 | 1-39, 55-84 |
| Montana . . . . |  |  |  | 1-34 |  |  |
| Nebraska . . | 1-9, 25-29 | 1-54, 65-74 |  | 1-34 | 1-24, 30-74, 80-94 |  |
| Nevada . . . . | 1-4, 15-24, 65-79 | 0-44, 65-99 |  | 1-24, 55-69, 75-89 | $\begin{aligned} & 0-14,20-29,35-69, \\ & 75-89 \end{aligned}$ |  |
| New Hampshire | 1-14, 20-34 |  |  | 1-34 |  |  |
| New Jersey | 1-14 | 1-14, 35-59, 70-84 | 1-4, 35-59, 70-84 | 2-24 | 1-14, 35-59, 70-79 | 1-19, 35-59, 70-84 |
| New Mexico . . | 2, 3, 5-14, 25-29 | 1-14, 20-24, 30-79 |  | 1-34 | 1-19, 25-74 |  |

Ages at which special adjustments were made in the 1989-91 State life tables by sex and race-Con.

| State | Male |  |  | Female |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | White | All other | Black | White | All other | Black |
| New York | 1-4 | 2-19, 35-49 | 2-19, 35-49 | 2, 4-14, 20-24 | 1-19, 35-44 | 1, 3-24, 35-44 |
| North Carolina | 1, 3-29 | $\begin{aligned} & 2-4,10-14,20-29, \\ & 70-84 \end{aligned}$ | 1-4, 10-29, 70-84 | 1, 3-39 | 2-24, 30-39, 70-84 | $\begin{aligned} & 1,3-24,30-39,50-59, \\ & 70-84 \end{aligned}$ |
| North Dakota | 1-39 |  |  | 1-24, 30-44 |  |  |
| Ohio |  | 1-14, 65-79 | 1-14, 65-79 | 1, 4, 10-34 | 1-34, 65-84 | 1-34, 65-84 |
| Oklahoma . | 1-34 | 1-24, 65-74, 80-94 | 1-24, 60-84 | 2-39 | 1-54, 60-84, 90-94 | 1, 2, 4-29, 45-54, 65-94 |
| Oregon . . . | 1-4, 10-14, 20-39 | 3-64 |  | 1-29, 45-74 | 1-19, 25-94 |  |
| Pennsylvania | 1,2, 4 | 1, 2, 4, 65-89 | 1-4, 65-89 | 1-4, 10-14, 20-29 | 5-19, 65-89 | 5-19, 65-89 |
| Rhode Island | 1-14, 25-44 |  |  | 1-29 |  |  |
| South Carolina | 1-29 | 1-24, 55-64, 75-84 | 1-24, 55-64, 75-84 | 1-3, 5-39 | 1-29, 70-94 | 1-29, 70-84 |
| South Dakota | 1-39 |  |  | 1-34, 45-54 |  |  |
| Tennessee . | 1-19, 25-29 | 2-4, 55-64 | 2-4, 55-64 | 1-34 | 2-4, 55-64 | 2-4, 55-64 |
| Texas. | 2-4, 10-19 | 1-4, 10-29, 65-84 | 1-24, 65-84 | 1-29 | 1-4, 10-24, 75-84 | 1-24, 75-84 |
| Utah . . . . |  |  |  | 1-3, 5-39 |  |  |
| Vermont . . . |  |  |  | 1-14, 20-34 |  |  |
| Virginia . . . | 1-29 | 1-24, 70-84 | 1-24, 65-94 | 5-29 | 3-39, 75-84 | 1-39, 75-84 |
| Washington | 1-14, 25-29 | 1-49, 60-84 | $\begin{aligned} & 1-9,15-34,40-54 \\ & 60-99 \end{aligned}$ | 1-3, 5-9, 15-29 | 1-49, 55-94 | 1-94 |
| West Virginia | $\begin{aligned} & 1-4,10-14,20-24, \\ & 30-39 \end{aligned}$ | $\begin{aligned} & 1-3,5-9,15-24 \\ & 30-44,65-74 \end{aligned}$ | $\begin{aligned} & 1-3,5-9,15-19 \\ & 25-54,65-74 \end{aligned}$ | 1-29 | 1-14, 20-59, 65-74 | 1-14, 20-39, 50-59, 65-89 |
| Wisconsin . . | 1-24 | 1-29, 60-94 | 1-24, 60-69 | 1-29 | $\begin{aligned} & 1-4,10-14,25-29 \\ & 45-54,60-99 \end{aligned}$ | $\begin{aligned} & 1-9,15-19,25-29,45-54, \\ & 60-99 \end{aligned}$ |
| Wyoming . . . | 1-44 |  |  | 1-3, 5-44 |  |  |

## U.S. Decennial Life Tables, 1989-91

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#### Abstract

\section*{VOLUME I}

Number 1 United States Life Tables. This first report contains life tables by single years of age from birth to age 110 for the United States. Tables are included for the total population, the white population, the population other than white, and the black population. Within these large populations are tables showing the race-sex categories of male, female, and both sexes combined. Standard error tables for the probability of dying and of the average remaining lifetime are included. Number 2 Methodology of the National and State Life Tables. This report describes in detail the methods of construction of the national and State life tables. Number 3 Some Trends and Comparisons of United States Life Table Data: 1900-1991. This report deals with trends and interpretations related to life expectancy and survivorship. Number 4 United States Life Tables Eliminating Certain Causes of Death. This report provides life tables analyzed by major groups of causes of death.


## VOLUME II

Numbers
1 through 51 Alabama through Wyoming, State Life Tables. Each of these 51 reports contains life tables for a particular State and a table that ranks each State in the order of life expectancy. All States have tables for the total population and the white population by sex. In addition, 40 States have tables for the other than white population and 33 have tables for the black population. Standard error tables for the probability of dying and of the average remaining lifetime are included.

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