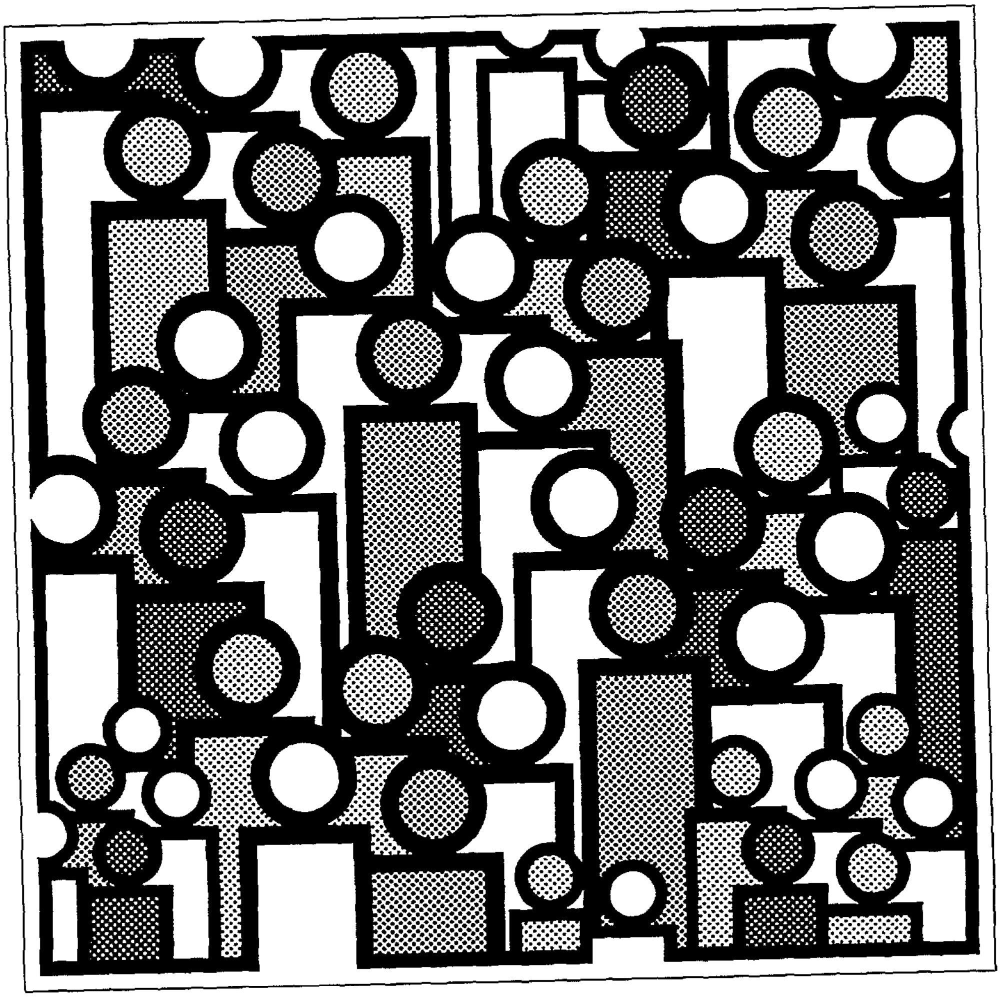
U.S. Decennial Life Tables for 1979-81

Volume I, Number 3 Methodology of the National and State Life Tables



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Methodology of the National and State Life Tables: 1979–81

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Introduction

This report describes the methodology employed in the preparation of the 1979–81 decennial life tables for the United States, the 50 individual States, and the District of Columbia. These tables are based on the 1980 U.S. census populations and the deaths of 1979–81. The methodology involved in the development of the 1979–81 life tables for the United States by causes of death is highly specialized and is described in the report containing those tables.[1] In addition, definitions of the usual life-table functions may be found in other reports of this series.

An ad hoc committee was formed in 1977 to consider the content and methodology of the 1979–81 decennial life tables. All of its recommendations were adopted, including use of the 1969–71 life-table methodology and first-time publication of standard errors.

The mortality rates at ages 85 years and over, based on the experience of the Social Security Medicare Program, were developed by Alice Wade of the Office of the Actuary, Social Security Administration, under the supervision of Francisco Bayo, Deputy Chief Actuary, who also devised the procedure for blending the mortality rates derived from census populations and registered deaths into those based on medicare experience. This same blending procedure was used for the 1969–71 tables.

The publications of decennial life tables based on the 1970 census included actuarial tables. Although the actuarial tables are no longer included in this set of reports, they will be produced by the Office of the Actuary, Social Security Administration.

Two aspects of these decennial life tables are new. First, standard errors of both the probability of dying and the average remaining lifetime are included in these reports for the first time. Second, where the numbers of deaths were large enough to justify it, State reports include life tables for the black population as well as the white and other-than-white populations.

Preliminary adjustment of data

The census populations used in the construction of the life tables are not exactly the "official" census data,[2] but are data adjusted by the U.S. Bureau of the Census. Misunderstanding by some respondents of certain items in the self-enumeration form used in the 1980 census is believed to have produced (1) an overstatement of the population of races other than white or black at the expense of the white population, and (2) an overstatement of the number of centenarians at the expense of the age groups from 85 to 99 years.[3] Thanks are due to Jeffrey Passel and Louisa Miller of the U.S. Bureau of the Census for furnishing magnetic tapes containing special population data adjusted for these errors.

A further relatively minor adjustment relates to the fact that the tabulations of deaths include a very small number (a total of 1,834 out of nearly 5.9 million deaths during the 3-year period) for which age is not reported. The assumption was made that these deaths were distributed among the various age groups in the same proportions as the deaths for which age was reported. To this end an adjustment factor was computed for each population category for which a life table was to be constructed. This factor was obtained by dividing the total number of deaths reported for the given category for the 3-year period 1979–81 by the total less the number for which age was not reported. The number of deaths reported in each age group for the given category was then multiplied by the adjustment factor.

In the preparation of these decennial life tables, no specific allowance was made for possible incompleteness in the enumeration of the population or in the registration of births or deaths. In the calculation of previous decennial life tables the use of birth statistics (rather than population data) in calculating the denominators of the mortality rates at ages under 2 years has been justified largely on the basis that the census populations under age 2 years were believed to be underenumerated. However, there are other advantages of using the methodology based on birth data, since it may be expected to produce a more accurate estimate of the average population during the 3-year period than is provided by the population enumerated on the census date. Accordingly, its use was continued with these 1979–81 life tables.

No adjustments have been made in the underlying census data for the misreporting of age. This is only the second time in several decades that such adjustments have not been considered necessary in the construction of the decennial life tables, the first being the 1969–71 life tables. For example, in the 1960 census there was substantial evidence of overreporting of 1900 as a year of birth in the population other than white, and, accordingly, the population was redistributed by age between ages 55 and 64 years, inclusive, before construction of the life tables was begun.

Data used for calculation of life-table values

The underlying data used in the preparation of each of the 1979–81 decennial life tables consisted of (1) reported deaths occurring in the 3-year period classified by age at death, (2) population data by age on the census date April 1, 1980, corrected as previously described for overstatement of the number of centenarians and overstatement of the population of races other than white and black, and (3) total registered births for each of the years 1977 to 1981, inclusive. These data were available separately by sex and race.

Populations and deaths were available by single years of age through age 5 years and by 5-year age groups from 5 to 99 years with the final age group being 100 years and over. There is a slight overlap because the population was needed at age 5 separately and as part of the age group 5–9 years. In each case the age referred to is the age in completed years—that is, the exact age on the individual's last birthday. In addition, deaths occurring at ages under 1 year were available for four subdivisions of the first year of life: Under 1 day, 1–6 days, 7–27 days, and 28–364 days. Life-table values were calculated for these subdivisions of the first year (but not published in the case of the State life tables) and for single years of age throughout the remainder of the life span.

With regard to the census data, actuarial theory would suggest that the populations to be used in the calculations should be those of the central date of the 3-year period, that is, July 1, 1980. However, the enumerated populations as of April 1, 1980 (adjusted as previously described) were used as if they were July 1 populations. This was done for two reasons. First, estimates of the latter are available for the Nation as a whole, but not for the individual States. Second, as shown in table A, the percent differences between the two sets of population figures are very small at the national level, and it was not considered necessary to produce new estimates for each of the States to reflect the lapse of time between April 1 and July 1, 1980. The national population estimates as of July 1, 1980, are contained in a report of the U.S. Bureau of the Census.[3]

Table A. Comparison of corrected enumerated populations, April 1, 1980, and comparable estimated populations, July 1, 1980, by race and sex: United States

Item	White		All other		Black	
	Male	*Female*	*Male*	*Female*	*Male*	*Female*
	Number in thousands					
Enumerated population	94,976	99,835	15,077	16,658	12,585	14,046
Estimated population	95,176	100,010	15,237	16,813	12,661	14,125
	Percent					
Increase	0.21	0.18	1.06	0.93	0.60	0.56

SOURCES: U.S. Bureau of the Census: Estimates of the population of the United States by age, sex, and race, 1980 to 1983, *Current Population Reports.* Series P–25, No. 949. Washington. U.S. Government Printing Office, 1984, table 2, p. 17.

U.S. Bureau of the Census: specially modified tape.

Numbers of survivors at ages 2 years and under

At ages under 2 years, the first life-table quantities to be calculated were the values of ${}_td_x$, the number of deaths occurring between exact ages x and $x+t$ in the life table cohort commencing with l_0 live births. This was calculated by the formula

$${}_td_x = \frac{l_0\,{}_tD_x}{{}_tE_x}$$

where ${}_tD_x$ denotes the number of deaths (adjusted as described earlier for nonreporting of age) occurring in 1979–81 between exact ages x and $x+t$, and ${}_tE_x$ denotes the appropriate denominator as indicated in table B. These denominators are based on the assumption of uniform distribution over the year of the births of 1977, 1978, 1979, 1980, and 1981. In each case l_0 is taken as 100,000. The appearance of overlapping in the designations of the age intervals occurs because exact ages are involved.

The unrounded values of ${}_td_x$ were then used to calculate values of l_x up to age 2 years by successive application of the formula

$$l_{x+t} = l_x - {}_td_x$$

Table B. Denominators ${}_tE_x$ used in calculating ${}_td_x$ for ages under 2 years

Age interval x to x + t	Denominator of ${}_td_x$
0–1 day	$\frac{1}{730}(B_{1978}+730B_{1979}+730B_{1980}+729B_{1981})$
1–7 days......	$\frac{1}{730}(8B_{1978}+730B_{1979}+730B_{1980}+722B_{1981})$
7–28 days	$\frac{1}{730}(35B_{1978}+730B_{1979}+730B_{1980}+695B_{1981})$
28–365 days...	$\frac{1}{730}(393B_{1978}+730B_{1979}+730B_{1980}+337B_{1981})$
1–2 years	$\frac{1}{2}(B_{1977}+2B_{1978}+2B_{1979}+B_{1980})$

NOTE: B_z denotes the reported number of births occurring during the calendar year z for the population category (by sex, race, and geographic area) involved.

Mortality rates at ages 2–94 years

The life-table mortality rate q_x is the fraction or proportion of a group of persons at exact age x who are expected to die before attaining age $x+1$. If m_x denotes the ratio d_x/L_x, commonly called the central death rate, then it is well known, on the assumption of uniform distribution of deaths over the year at age x, that

$$q_x = \frac{2m_x}{2+m_x} \qquad (1)$$

This approximation is sufficiently accurate when the life table is by single years of age. Formula (1) was the basis of the calculation of mortality rates at ages 2–94 years. Completion of the calculations depends, therefore, on the ability to calculate central death rates m_x at these ages. For this purpose different methods were used at ages 2–4 years and at ages 5–94 years, as will now be described.

Central death rates at ages 2–4 years

If D_x denotes the adjusted number of deaths in a population category at age x (in completed years) occurring in 1979–81, and P_x denotes the population at age x in the middle of the period, then,[4]

$$m_x = \frac{D_x}{3P_x} \qquad (2)$$

As previously noted, the populations actually used were those of April 1, 1980.

However, because the deaths occurring in a given single year of age during 1979–81 were drawn from three consecutive annual cohorts of the population, it was considered that the accuracy of these m_x values would be improved by replacing $3P_x$ in the denominator of equation (2) by the sum of the populations at age $x-1$, x, and $x+1$. Thus the formula becomes

$$m_x = \frac{D_x}{P_{x-1}+P_x+P_{x+1}} \qquad (3)$$

The combination of formulas (1) and (3) is equivalent to the single formula

$$q_x = \frac{D_x}{P_{x-1}+P_x+P_{x+1}+\tfrac{1}{2}D_x}$$

which was used for $x = 2$, 3, and 4.

Mortality rates at ages 5–94 years

The combination of formulas (1) and (2) is equivalent to

$$q_x = \frac{D_x}{3P_x+\tfrac{1}{2}D_x} \qquad (4)$$

which was used for ages 5–94 years, with values of D_x and P_x obtained by interpolation from data by 5-year age intervals. The procedure of interpolating populations and deaths separately has a long history. Apparently it was first used by Dr. John Tatham in the preparation of English Life Table No. 5 covering the period 1881–90 and published in 1895.[5] The Tatham method was also used in the construction of the U.S. life tables based on the censuses of 1900–30, inclusive. When life tables are being mass-produced, as is the case with the life tables for the United States, the 50 States, and the District of Columbia, the Tatham method of osculatory interpolation has an operational advantage in that the interpolated deaths at a given age for any class is exactly the sum of those for the subclasses of which it is composed, and a similar statement applies to populations. For example, at any age the interpolated deaths for total white persons is the sum of those for white males and white females, and the interpolated deaths for the United States is exactly the sum of those for the 50 States and the District of Columbia.

For the 1979–81 life tables, the interpolation of both deaths and populations was performed by means of the interpolation coefficients developed by H. S. Beers (table C). In certain age intervals the headings of this table are not to be taken precisely as stated. In interpolating at ages 90–94 years, the numbers (of deaths or populations) at ages 100 years and over were used as if they applied to ages 100–104 years. Moreover, in interpolating at ages 5–14 years, the value used for the "quinquennial sum starting at age 0" was not the actual number reported at ages 0–4 years but a fictitious value. Because of the mortality peak in infancy, the use of the actual numbers at ages 0–4 years probably would not yield plausible values. There is also no reason to expect that the interpolated values for ages 5–9 years would join smoothly with the numbers reported at ages 2–4 years if actual numbers for the age group 0–4 years were used.

As a result, a fictitious quinquennial sum was used for the age interval 0–4 years. The numbers were chosen so that the sum of the interpolated values at ages 2–4 years would be equal to the number reported in this 3-year age interval (adjusted for nonreporting of age in the case of deaths).

Table C. Beers' interpolation coefficients for subdividing quinquennial sums to obtain estimated numbers by single years of age (minimized fifth-difference formula with smoother ends)

Age	Quinquennial sum beginning at age—				
	0 years	*5 years*	*10 years*	*15 years*	*20 years*
	Coefficient for 2–4 years				
2 years	0.1924	0.0064	0.0184	−0.0256	0.0084
3 years	0.1329	0.0844	0.0054	−0.0356	0.0129
4 years	0.0819	0.1508	−0.0158	−0.0284	0.0115
	Coefficient for 5–9 years				
5 years	0.0404	0.2000	−0.0344	−0.0128	0.0068
6 years	0.0093	0.2268	−0.0402	0.0028	0.0013
7 years	−0.0108	0.2272	−0.0248	0.0112	−0.0028
8 years	−0.0198	0.1992	0.0172	0.0072	−0.0038
9 years	−0.0191	0.1468	0.0822	−0.0084	−0.0015

Age	Quinquennial sum beginning at age—				
	5m − 10 years	*5m − 5 years*	*5m years*	*5m + 5 years*	*5m + 10 years*
	Coefficient for 10–94 years				
5*m* years	−0.0117	0.0804	0.1570	−0.0284	0.0027
5*m* + 1 years	−0.0020	0.0160	0.2200	−0.0400	0.0060
5*m* + 2 years	0.0050	−0.0280	0.2460	−0.0280	0.0050
5*m* + 3 years	0.0060	−0.0400	0.2200	0.0160	−0.0020
5*m* + 4 years	0.0027	−0.0284	0.1570	0.0804	−0.0117

SOURCE: H. S. Beers: Reply to the discussion of his paper Six-term formulas for routine actuarial interpolation. *Rec. Amer. Inst. Act.* 34:60, 1945.

If W_x denotes the quinquennial sum commencing with age x and V denotes the sum of the interpolated numbers for ages 2–4 years (which shall be required to be equal to the reported number), these coefficients give

$$V = 0.4072\,W_0 + 0.2416\,W_5 + 0.0080\,W_{10} - 0.0896\,W_{15} + 0.0328\,W_{20}$$

Solving for W_0, the fictitious quinquennial sum, gives

$$W_0 = 2.45580\,V - 0.59332\,W_5 - 0.01965\,W_{10} + 0.22004\,W_{15} - 0.08055\,W_{20}$$

This formula was used to compute the fictitious value for the age interval 0–4 years. The interpolated populations from the census and deaths from the National Center for Health Statistics were used in formula (4) to calculate the mortality rates up to age 94 years.

At ages 85–94 years, the mortality rates obtained, as described here, were blended with those derived (as explained in the next section) from experience of the medicare program. Thus the rates actually used in the construction of the life tables were obtained by the formula

$$q_x = \frac{1}{11}[(95 - x)q_x^C + (x - 84)q_x^M]$$

where q_x = life-table mortality rate at age x

q_x^C = mortality rate calculated with formula (4)

q_x^M = corresponding rate based on medicare experience

Mortality rates at ages 95 years and over

As in the 1969–71 tables, mortality rates at ages 95 years and over were based solely on experience of the medicare program and were provided by the Office of the Actuary, Social Security Administration. Medicare data were used at ages 95 years and over because they were considered more accurate than conventional death rates, which have problems in the accuracy of the reporting of age among the extremely elderly. As mentioned in the preceding section, mortality rates at ages 85–94 years based on medicare data were blended with those based on census populations and registered deaths. Therefore mortality rates based on medicare experience were required at all ages 85 years and over.

These mortality rates were differentiated by sex and race but not by geographic area. Thus in the life-table mortality rates for the United States and for the States, there is no distinction by geographic area at ages 95 years and over, and the influence of geographic area on the rates diminishes with increasing age at ages 85–94 years. Life tables for the black population have been calculated in addition to those for the population other than white (which includes black persons) for the United States and each State. For this purpose the rates based on medicare experience for persons other than white have been regarded as applying to the black population, because separate data for black persons are not available from the medicare experience.

The mortality rates based on medicare experience used in the construction of the life tables are shown in table D. The procedure by which these were obtained consists of a series of steps. First, "crude" mortality rates for ages 66–105 years, inclusive, for white males, white females, males other than white, and females other than white were computed directly from the data on deaths and enrollments. The data used for this purpose were limited to the "HI-insured" group (that is, insured for Hospital Insurance).[6] In general terms, this excludes persons who were "blanketed into" the medicare program even though they had no covered employment or only a minimal amount of such employment under the Social Security Program or the Railroad Retirement Program.[7] It is believed that ages

Table D. Graduated mortality rates from medicare experience by sex, race, and age at last birthday: United States, 1979–81

Age	*Total*			*White*			*All other*		
	Both sexes	*Male*	*Female*	*Both sexes*	*Male*	*Female*	*Both sexes*	*Male*	*Female*
85 years	0.104708	0.132868	0.091564	0.104524	0.132792	0.091361	0.107674	0.133539	0.093982
86 years	0.114327	0.143244	0.101166	0.114298	0.143263	0.101098	0.115674	0.142695	0.101814
87 years	0.124568	0.154266	0.111466	0.124720	0.154422	0.111566	0.124008	0.152194	0.109994
88 years	0.135433	0.165881	0.122506	0.135807	0.166227	0.122815	0.132640	0.161945	0.118484
89 years	0.147023	0.178118	0.134368	0.147674	0.178723	0.134940	0.141535	0.171835	0.127254
90 years	0.159414	0.191064	0.147069	0.160417	0.192020	0.147965	0.150658	0.181720	0.136271
91 years	0.172609	0.204762	0.160532	0.174061	0.206183	0.161827	0.159948	0.191428	0.145492
92 years	0.186536	0.219090	0.174623	0.188550	0.221111	0.176408	0.169311	0.200748	0.154851
93 years	0.200975	0.233677	0.189132	0.203688	0.236461	0.191526	0.178607	0.209546	0.164257
94 years	0.215540	0.247924	0.203772	0.219104	0.251626	0.206916	0.187649	0.217809	0.173605
95 years	0.229763	0.261494	0.218225	0.234323	0.266169	0.222280	0.196255	0.225540	0.182789
96 years	0.243384	0.274375	0.232211	0.249004	0.280007	0.237293	0.204347	0.232744	0.191696
97 years	0.256371	0.286540	0.245596	0.263038	0.293109	0.251730	0.211931	0.239435	0.200215
98 years	0.268682	0.297973	0.258337	0.276380	0.305452	0.265510	0.219009	0.245630	0.208253
99 years	0.280295	0.308674	0.270399	0.288996	0.317029	0.278591	0.225593	0.251350	0.215773
100 years	0.291198	0.318650	0.281761	0.300870	0.327843	0.290944	0.231696	0.256617	0.222785
101 years	0.301392	0.327919	0.292417	0.311995	0.337908	0.302554	0.237337	0.261458	0.229301
102 years	0.310888	0.336504	0.302370	0.322378	0.347244	0.313421	0.242538	0.265896	0.235338
103 years	0.319704	0.344433	0.311633	0.332034	0.355879	0.323552	0.247322	0.269958	0.240913
104 years	0.327863	0.351737	0.320225	0.340984	0.363843	0.332965	0.251712	0.273670	0.246050
105 years	0.335393	0.358449	0.328170	0.349256	0.371172	0.341682	0.255733	0.277057	0.250772
106 years	0.342326	0.364606	0.335499	0.356882	0.377900	0.349734	0.259410	0.280143	0.255103
107 years	0.348695	0.370243	0.342242	0.363895	0.384066	0.357151	0.262766	0.282951	0.259068
108 years	0.354534	0.375394	0.348433	0.370331	0.389705	0.363968	0.265826	0.285504	0.262692
109 years	0.359877	0.380094	0.354105	0.376226	0.394855	0.370220	0.268612	0.287822	0.266000

have been more accurately determined for the HI-insured group. Table E shows by age and sex the percent of medicare beneficiaries who were HI-insured as of January 1, 1980. Also excluded from the calculations were persons of unknown race, estimated to have been roughly 3 percent.

Crude mortality rates for the remaining five categories of the population (total male, total female, total white, total other than white, and the total population) were obtained as weighted averages of the four previously determined categories, using as weights the proportions of the enrolled population in the categories involved on January 1, 1980. In calculating these proportions, the entire population enrolled for medicare (not merely the HI-insured) was used, except that, of course, persons of unknown race had to be excluded in obtaining proportions involving race. A comparison between the medicare proportions and the available age grouping proportions in the 1980 census showed little difference.

The crude rates for each of the nine population categories from age 66 through 105 years, inclusive, were then graduated (that is, smoothed) by a Whittaker-Henderson Type B formula.[8] Such a formula involves minimizing the quantity

$$\sum_{x=\alpha}^{\beta} W_x(q''_x - q_x)^2 + k\sum_{x=\alpha}^{\beta-z}(\Delta^z q_x)^2 \qquad (5)$$

where x = index for single years of age

α = minimum age

β = maximum age

W_x = designated positive weight

q''_x = crude rate

q_x = smoothed rate

Δ^z = zth finite difference, z being commonly chosen as 2 or 3

The first summation is a measure of the departure of the smoothed rates from the crude rates, and the second summation measures the roughness of the smoothed rates (that is, the smaller this quantity, the smoother these rates). The smoothing constant k indicates the degree of importance the user attaches to smoothness in relationship to closeness of fit to the observed data.

Minimization of expression (5) leads to a system of linear equations to be solved for the smoothed mortality rates q_x. It has been suggested[9,10] that on theoretical grounds the weights W_x should be taken as the reciprocals of the (binomial) variances of the smoothed mortality rates q_x, that is,

$$W_x = \frac{E_x}{q_x(1 - q_x)}$$

where E_x denotes the "exposed to risk." In fact, it may be pointed out that with this choice of weights the first term of expression (5) becomes the value of the chi-squared statistic that is used in the chi-squared goodness-of-fit test. However, the use of such weights would lead to a system of nonlinear equations in the variables q_x that could not easily be solved. This difficulty may be overcome by using the crude rate as an approximation to the smoothed rate in the computation of weights so that the approximate formula becomes

$$W_x = \frac{E_x}{q''_x(1 - q''_x)}$$

In graduating the mortality rates at ages 66–105 years based on medicare experience, the finite difference z was taken as 3 and the smoothing constant k as 140,000, so that the expression to be minimized becomes

$$\sum_{x=66}^{105} W_x(q''_x - q_x)^2 + 140{,}000\sum_{x=66}^{102}(\Delta^3 q_x)^2$$

It was considered desirable to have the graduated rates at the older ages increase smoothly, with the amount of increase declining somewhat with advancing age. Accordingly, the graduated rates at certain of the oldest ages were rejected and replaced by rates obtained by a method of extrapolation. The last rate accepted was that at the youngest age y such that

$$\frac{(q_{y+1}/q_y) - 1}{q_y/q_{y-1} - 1} < 0.9$$

For the different population categories, this occurred at various ages between 94 and 99 years, inclusive; therefore the mortality rates were extrapolated to age 111 years by the formula

$$\frac{q_{x+1}}{q_x} - 1 = 0.9\left(\frac{q_x}{q_{x-1}} - 1\right) \qquad (6)$$

This formula is the same one used in the construction of the 1969–71 tables.

As the underlying data were available in such a form that the mortality rate referred to as q_x was in reality $q_{x-½}$, interpolation was performed to produce rates for integral ages from 85 to 109 years, inclusive, using the approximation formula

$$q_x \cong -\frac{1}{16}q_{x-1½} + \frac{9}{16}q_{x-½} + \frac{9}{16}q_{x+½} - \frac{1}{16}q_{x+1½} \qquad (7)$$

This formula would be exact if the four consecutive rates in the right member were exactly fitted by a third-degree polynomial.

Table E. Percent of HI-insured medicare beneficiaries by age and sex: United States, January 1, 1980

Age	*Male*	*Female*
	Percent	
65–69 years	98.6	97.2
70–74 years	98.0	96.4
75–79 years	96.9	94.5
80–84 years	95.2	91.9
85–89 years	93.5	88.1
90–94 years	90.6	79.5
95–99 years	80.1	62.4
100 years and over	53.9	33.7

Calculation of the remaining life-table values

Mortality rates q_x were now available for all ages 2–109 years, inclusive. From these, using double precision, numbers of survivors l_x were calculated for ages 3–110 years, inclusive, by the formula

$$l_{x+1} = l_x - l_x q_x \qquad (8)$$

where x ranges from 2 to 109 years. Values of l_x were now available for all integral ages from 0 to 110, inclusive, as well as for certain ages between 0 and 1 year (1 day, 7 days, and 28 days).

Values of q_x were available for ages 2 years and above but not for ages under 2 years. Accordingly q_0 and q_1 were calculated by the formulas

$$q_0 = 1 - \frac{l_1}{l_0}$$

$$q_1 = 1 - \frac{l_2}{l_1}$$

Moreover, for the United States, mortality rates ${}_tq_x$ for subdivisions of the first year of life were calculated by the formula

$${}_tq_x = 1 - \frac{l_{x+t}}{l_x}$$

Such values of ${}_tq_x$ were not published for States.

Values of $\mathring{e}_{110}$, the expectation of life at age 110 years, for each of the nine race-sex categories were furnished by the Office of the Actuary, Social Security Administration. These were calculated by extrapolating $q_{x-½}$ values by means of formula (6) to age 132 years, and obtaining values of q_x up to age 130 years by interpolation using formula (7). Values of l_x for $x = 111$ to 130 years, inclusive, were calculated by formula (8). Values of L_x, the number of person years lived between age x and $x + 1$, for age 110–130 years, inclusive, were obtained from the general formula

$$L_x = \int_0^1 l_{x+t}\, dt$$

$$= f_x l_{x+1} + (1 - f_x) l_x$$

where f_x was determined by assuming that the force of mortality is constant between ages x and $x + 1$; that is,

$$f_x = \frac{1}{q_x} + \frac{1}{\ln(1 - q_x)}$$

T_x, the number of person years lived after age x, was computed sequentially from age 130 years back to age 110 years by

$$T_x = T_{x+1} + L_x$$

with T_{131} taken equal to zero. The final step was to compute

$$\mathring{e}_{110} = \frac{T_{110}}{l_{110}}$$

These values are shown in table F.

With the values of $\mathring{e}_{110}$ available, T_x for the ages included in the life tables was computed by the formulas

$$T_{110} = l_{110}\mathring{e}_{110}$$

and

$$T_x = T_{x+1} + \frac{1}{2}(l_x + l_{x+1})$$

Table F. Values of $\mathring{e}_{110}$ extrapolated from medicare experience by race and sex: United States, 1979–81

Race and sex	$\mathring{e}_{110}$
Total	
Both sexes	2.155395
Male	2.025408
Female	2.192263
White	
Both sexes	2.034531
Male	1.925943
Female	2.069592
All other	
Both sexes	3.094117
Male	2.875030
Female	3.109930

proceeding from age 109 years back to age 0. For the subdivisions of the first year of life, the corresponding formula is

$$T_x = T_{x+t} + \frac{t}{2}(l_x + l_{x+t})$$

where t is the appropriate fraction of a year corresponding to the interval involved. The values of t used are as follows:

Age	t
0	1/365
1 day	6/365
7 days	21/365
28 days	337/365

The average remaining lifetime (or "expectation of life") $\mathring{e}_x$ was calculated at all ages from 0 to 109 years, including ages between 0 and 1 year, by the formula

$$\mathring{e}_x = \frac{T_x}{l_x}$$

The values of q_x, l_x, T_x, and $\mathring{e}_x$ were then rounded to the number of decimal places shown in the published life tables. In other words, l_x and T_x were rounded to the nearest integer, q_x to five decimal places, and $\mathring{e}_x$ to two decimal places. Because l_x and q_x were independently rounded, calculation of q_x from the published (rounded) values of l_x by

$$q_x = \frac{l_{x+1}}{l_x}$$

may not always agree with the published value in the fifth decimal place. Finally, d_x and L_x were obtained by differencing the rounded values of l_x and T_x, respectively:

$$d_x = l_x - l_{x+1}$$

and

$$L_x = T_x - T_{x+1}$$

Calculation of standard errors of the life-table functions

The 1979–81 U.S. decennial life tables are the first set of decennial life tables to show standard errors for certain life-table functions. Specifically, the standard errors for the probabilities of dying and for the life expectancies are shown. It is important to consider that these standard errors reflect only stochastic variation and are based on an assumption that the age-specific deaths follow a binomial distribution. Stochastic variation is not the only source of error for life-table functions; measurement error, such as age misstatements on death certificates or on census reports, also affects the accuracy of the life-table functions. While the extent of measurement error on life-table functions has not been quantified, it is generally thought that measurement errors could be larger than stochastic errors. Because the life tables for the United States and for the published States are based on relatively large numbers of deaths, the standard errors presented are rather small.

For ages less than 85 years, a binomial distribution assumption yields the following estimate for the variance of q_x:

$$S^2(q_x) = \frac{q_x^2(1 - q_x)}{D_x^*} \qquad (9)$$

where D_x^* is the age-specific number of deaths, smoothed by interpolation and adjusted for the number of deaths with age not stated.

For ages 85–109 years, medicare data were used to estimate the probabilities of dying; equation (9) cannot be used. An empirical investigation, described elsewhere,[11] led to estimates of $S^2(q_x)$ for these ages as well as for $S^2(\mathring{e}_{110})$. For the variances of the life expectancies at ages 0–109 years, an equation from Chiang,[12] with a slight modification, was used, namely:

$$S^2(\mathring{e}_x) = \frac{l_{110}^2 S^2(\mathring{e}_{110}) + \sum_{y=x}^{109} l_y^2 \left(\mathring{e}_{y+1} + \frac{1}{2} \right)^2 S^2(q_y)}{l_x^2} \qquad (10)$$

Special adjustments in the U.S. and State life tables

For each of the 50 States and the District of Columbia, life tables were calculated for each of the 12 race-sex groups shown in the U.S. report. However, in some States not all the 12 tables were published, because it was considered that the amount of data for one race was too small to produce reliable results. If for any racial group fewer than 700 male or 700 female deaths at all ages were registered in the given State for the 3-year period 1979–81, the tables for that racial group were not published. The number 700 was chosen after experimenting with data from the last decennial period to determine the minimum number of deaths that could be smoothed into a reliable life table. As a result of applying this criterion, life tables for persons other than white were not published for 16 States: Alaska, Idaho, Iowa, Maine, Minnesota, Montana, Nebraska, Nevada, New Hampshire, North Dakota, Oregon, Rhode Island, South Dakota, Utah, Vermont, and Wyoming. Life tables were not published for the black population for these same States plus the four States of Arizona, Hawaii, New Mexico, and Washington. In 8 of the above 20 States (Alaska, Arizona, Minnesota, Nebraska, Nevada, Oregon, South Dakota, and Washington) the number of deaths of black persons or of persons other than white was fewer than 700 for one sex (females in all cases) and not for the other. Life tables for white persons were published for every State.

In most of the State life tables special adjustments were made at certain ages to correct or mitigate anomalous behavior of the life-table values that may be attributed to the small numbers involved. After each life table to be subjected to such adjustment had been calculated and printed out, the q_x values for individual years of age were examined and certain tests of consistency applied. The other life-table functions are completely determined by the q_x values, so no tests needed to be applied to them.

It was considered that, in each life table, the q_x values should decrease from age 0 to about age 10 or 11 years and then increase to the early twenties. They should increase again from about age 30 years to the end of the table. Strict increase in mortality rates with increasing age was not required between 20 and 30 years, because a slight decrease in the mortality curve in this age range (due to violent and accidental deaths) is a feature of many of the life tables. Abrupt age-to-age changes in q_x values (indicated by relatively large second differences) were also examined.

Such adjustments were made directly only to the life tables for white males, white females, males other than white, females other than white, black males, and black females. After the data underlying these six tables had been adjusted to remove anomalies from the life tables, the adjusted data were combined in various ways to produce the remaining six tables. It was assumed that if the six basic components were free from anomalies, this would also be true of the various combinations. For some States the data for population groups for which life tables were not published (for example, males and females other than white in 16 States and black males and females in an additional 4 States) were not adjusted. It was assumed that if the life tables for white persons were free from anomalies and the deaths of persons other than white were too few to warrant publication of the life tables, there would be no anomalies in the life tables for total males and total females. For 11 of the 16 States (Alaska, Iowa, Maine, Montana, Nebraska, Nevada, New Hampshire, North Dakota, Oregon, Rhode Island, and South Dakota) this assumption was not justified, and adjustments were found to be necessary in the data for persons other than white, even though the corresponding life tables were not published, to avoid anomalies in the life tables for total persons, total males, and total females. These adjustments were most commonly made at ages 1–4 years.

For each pair of published life tables for males and females of a given race in a given State it was considered that the q_x value for females at each age should be less than the corresponding value for males. When life tables for the population other than white were published, the q_x value for white persons at each age should be less than the corresponding value for persons other than white of the same sex up to about age 70 years. If the values for persons other than white do become lower at about age 70 years or later, they should remain lower. In other words, corresponding mortality curves for white persons and others should not be permitted to cross and recross a number of times. This criterion was not applied, however, to the State of Hawaii, where the population other than white is composed predominantly of ethnic groups having mortality rates closely comparable to those of the white population. Similarly, when life tables for the black population were published, the q_x value for persons other than white at each age should be less than the corresponding value for black persons of the same sex. This should be true at all ages.

In every instance in which an adjustment was considered necessary, it was effected by redistributing by age the numbers of deaths in two or more usually adjacent age groups, so that the total number of deaths at all ages remained unchanged. In using this type of adjustment, the intention was to change the local shape of the mortality curve while preserving the overall

Table G. Number of published State and District of Columbia life tables with special adjustments by race, sex, and selected age intervals: United States, 1979–81

Age interval between exact ages	White		All other[1]		Black	
	Male	Female	Male	Female	Male	Female
	Number[2]					
0–2 years	35	35	29	29	25	23
2–5 years	45	45	32	32	28	28
5–30 years	41	48	33	34	30	30
30–50 years	34	28	24	16	20	18
50–100 years	7	4	16	11	14	15
	Number of life tables published					
Total	51	51	35	35	31	31

[1]Excludes Alaska, Iowa, Maine, Montana, Nebraska, Nevada, New Hampshire, North Dakota, Oregon, Rhode Island, and South Dakota, where minor adjustments were made, even though the life tables were not published.
[2]In some instances the age interval involved in a single redistribution of deaths by age included parts of 2 or more of the age intervals shown in the table. Thus the sum of the entries in any column, in general, exceeds the total number of separate redistributions made.

mortality level. In some cases, the numbers of deaths in the age groups involved were redistributed by age in proportion to the corresponding numbers for the same sex and race for the United States. This procedure was used most frequently at ages 1–4 years. When this process failed to remove the observed anomalies, deaths were redistributed by age in a more arbitrary manner.

Sometimes several trial runs had to be made for a given State before satisfactory q_x values were obtained. For each trial run small changes were made in the numbers of deaths; then q_x values were recalculated. When redistributions by age of the numbers of deaths resulted in appropriate q_x values, the process of computation of the various life-table functions, as previously described, was carried out with the q_x values based on the redistributed deaths.

The redistributed deaths for the six basic demographic categories (white males, white females, males other than white, females other than white, black males, and black females) were then combined to produce the redistributed deaths by age for the six remaining categories (total population, total males, total females, total white, total other than white, and total black), and computation of life tables for the latter categories was completed. If these life tables contained anomalies, some additional redistribution of deaths was done in the individual race-sex groups. The life tables for the United States were not corrected to reflect redistributions by age of deaths in the States.

While the U.S. life tables that were prepared 10 years ago did not require any smoothing, some minor smoothing was required in the 1979–81 tables. The unsmoothed national data for both sexes showed higher death rates for white persons than for persons other than white at ages 15–19. This was smoothed by moving deaths of white males and white females out of the age group 15–19 into adjacent age groups. The reverse was done for the population other than white; this necessitated a smiliar shifting of deaths by age in the black population.

Table G gives some idea of the number of special adjustments made in the State life tables. Further details are given in the appendix, which shows, for each sex and race in each State, the ages at which these adjustments were made.

By far, more adjustments were required in the age interval 1–44 years than in any other. There were two reasons for this. First, at ages 1–4 years deaths by single years of age were used, and these numbers are small and subject to severe statistical fluctuations. Second, the U.S. life tables were themselves smoothed at ages 15–44 years for white males and white females; at ages 10–44 years for females other than white, black females, and black males; and at ages 5–44 years for males other than white. This indicated that most State life tables would have to be smoothed at these same ages. Of all the published life tables, only three did not require special adjustments at any age. These were the life tables for white males for Iowa, Michigan, and New York.

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Appendix
Ages at which special adjustments were made in the 1979–81 State life tables by sex and race

State	Male			Female		
	White	*All other*	*Black*	*White*	*All other*	*Black*
	Years					
Alabama	1–4, 10–24	10–24	10–24	1–4, 15–29	0, 1, 3–24	1, 3–9, 15–24
Alaska	0, 2–44, 75–89			0–34, 75–89		
Arizona	2–4, 20–34	1–4, 20–49, 65–89		1–9, 15–34	0, 2–24	
Arkansas	1–39	1, 3–49	1, 3–49	1–29	1–29, 90–94	1–24
California	4–19, 30–39	4–19, 25–29, 40–49	2–4, 10–19	1, 4–34	1, 5–44	10–34
Colorado	4–44	1–3, 5–29, 55–79, 90–94	1–94	5–29	1, 3–9, 15–19, 50–89	3–9, 15–29, 45–84, 90–94
Connecticut	2–24, 30–44	0–29, 45–64	0–29, 45–64	2–34	0–19, 40–59	0–9, 15–24, 40–59
Delaware	0–4, 10–49	0–1, 3–29, 40–69	0–29, 40–69, 80–84	1–34	0–9, 25–44, 55–64	0–9, 25–44, 55–64, 90–94
District of Columbia	0–34, 55–64	0–29, 90–94	0–29	0–9, 15–34, 45–59	0–19, 25–29, 35–39	0–19, 25–29, 35–39
Florida	5–19	1–4, 10–24, 65–74	1–4, 10–29, 65–74	3–34	1, 3, 10–19, 70–89	1, 3, 10–19, 70–89
Georgia	2–9, 15–39	10–44	10–44, 90–94	2–4, 15–34	1–4, 15–39	1–4, 15–39, 90–94
Hawaii	1–3, 5–14, 20–59	1–4, 30–44		1–14, 30–49	1–3, 5–9	
Idaho	3–29			1, 2, 10–14		
Illinois	2–4	3, 5–9, 25–34	3, 5–9, 25–34	15–24	3, 4	3, 4
Indiana	2–4	2–4, 10–34	2–4, 10–34	1, 3, 5–14, 20–29	1–9	2–14, 50–59
Iowa				2–4, 15–24		
Kansas	1–4, 10–19, 25–39	0–4, 10–19, 25–34, 45–64	0, 2–19, 25–34, 45–49, 55–69, 90–99	1–4, 30–39	1–3, 5–14, 20–39	0–3, 5–14, 30–54, 60–69
Kentucky	1–39	0–34, 50–69	0–34, 50–69	1, 4–24	15–24, 50–54	15–24, 50–54
Louisiana	1–3, 5–39	1–4, 10–29	1–34	3–29	1–9, 15–34	1–34
Maine	1–4, 10–29			2, 5–19		
Maryland	1, 2, 4, 10–19	1–3, 5–29	1–29	3, 4, 15–24	1–19	1, 3–19
Massachusetts	1–4, 15–39	1–9, 15–34, 65–89	1–29, 60–89	1–4	1–29, 65–79	0–24, 65–84
Michigan		1, 2, 4–29	1–29, 90–104	1–14	1–19, 25–29	1, 2, 4–19, 25–34
Minnesota	1–3, 30–34			1–34		
Mississippi	1, 2, 4, 10–34	1–29, 45–54	0–29, 35–39, 45–54, 65–69, 80–84	1–44	1–34	1–34, 50–54
Missouri	1–4, 10–39	1–3, 10–24	1–3, 10–24	1–9, 15–24	1–14, 80–89	1–14, 80–89
Montana	1–4, 25–29, 35–54			1–4, 15–29		
Nebraska	1–3, 10–24, 30–39			1–3, 5–34		
Nevada	0–4			0–4, 15–39		
New Hampshire	1–3, 10–19			1, 2, 4, 15–29		
New Jersey	1–4, 10–19	1–3, 10–24	1–3, 10–24	1–4, 15–29	1–4, 15–24	1–3, 15–24
New Mexico	1, 4–19, 35–44, 65–74	1–69, 85–94		1–34, 60–79	1–3, 5–19, 50–54, 60–79	
New York		1, 3, 4, 75–89	1, 3, 4, 60–69, 80–89	1, 3	3, 4, 10–19	3, 4, 10–19

State	Male			Female		
	White	All other	Black	White	All other	Black
			Years			
North Carolina	3, 4, 10–19, 25–34	1–3, 15–19, 30–39	1–3, 5–44	1, 3–39	2, 4–39	0–44, 90–94
North Dakota	1–49			2, 3, 10–39		
Ohio	1–4, 10–39	1–4, 10–39	1–39	1, 3, 4, 15–29	1, 3–24	3–29
Oklahoma	1–19, 25–39, 60–84	1–4, 10–24, 30–34, 65–74	1, 3–24, 60–69	1–34	1, 2, 4–24, 45–49	1–34, 45–54, 90–94
Oregon	1, 2			1–3, 15–24		
Pennsylvania	1–4, 15–29	1–29	1–34	1–3, 10–34	1, 2, 4–9, 15–19, 25–34	1, 2, 4–9, 15–19, 25–34
Rhode Island	1, 2, 4–9			2–9		
South Carolina	1, 4, 10–34	1–34	1–34	1, 3–39	1, 3, 5–19, 25–34	1, 3, 5–39
South Dakota	1–3, 35–44			1–3, 15–44		
Tennessee	1–4, 10–44	1–4, 10–34	1–34	2–4, 10–34	1–29, 35–49	1–29, 35–49
Texas	10–39	5–39	5–39	10–34	5–24	5–24
Utah	1–4			5–24, 30–39		
Vermont	1–4, 10–34			1, 2, 4–29		
Virginia	3, 4, 10–39	1–24, 30–39	1–39	10–34	1–44	1–44
Washington	15–39, 60–79	1–3, 25–34, 40–49, 55–59, 65–69		5–9, 15–34, 65–79	1–3, 5–14, 20–24, 65–89	
West Virginia	1–4, 15–19, 30–39	0–29, 40–44 60–64, 85–94	0–29, 40–44, 50–54, 60–69, 75–94	1, 3, 5–39	0, 1, 3–24, 30–44	0, 1, 3–24, 30–44, 60–64, 70–74
Wisconsin	1, 2, 4, 15–19, 30–39	1, 3–39, 80–89	0–94	1–4, 15–24	2–4, 15–39, 50–54, 60–79	0, 2–19, 25–39, 45–89
Wyoming	1–3, 5–14, 25–34			1, 3–24, 30–39		

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