FEDERAL SECURITY AGENCY
UNITED STATES PUBLIC HEALTH SERVICE
NATIONAL OFFICE OF VITAL STATISTICS

UNITED STATES LIFE TABLES
and
ACTUARIAL TABLES
1939–1941

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[1] As this volume was going to press, it was learned that Mr. John S. Thompson has become President of the Mutual Benefit Life Insurance Co.

CONTENTS

PART I.—INTRODUCTION

PART II.—LIFE TABLES

PART III.—ACTUARIAL TABLES

PART IV.—MATHEMATICAL THEORY AND USE OF THE ACTUARIAL TABLES

PART V.—METHOD OF CONSTRUCTION AND GRADUATION OF THE LIFE TABLES

APPENDIX

PRINCIPAL TABLES

FIGURES

UNITED STATES LIFE TABLES AND ACTUARIAL TABLES, 1939–1941

PART I

INTRODUCTION

Plan and scope of this volume

The life tables in this volume are based on the 1940 census of population and the deaths of the 3-year period 1939–1941. Separate life tables have been prepared for each sex for each of three racial groups: white, Negro, and other races. This is the first time official life tables have been prepared for races other than whites and Negroes in the United States. Life tables are also included for the total population of each sex, for the total population of each racial group without distinction by sex, and for the entire population without distinction by race or sex. Each of the 12 life tables is based on data for the entire continental United States. Also included are certain actuarial tables derived from the life tables for white males, white females, and total whites, to be used in calculating premiums and values for life annuities, life assurances, and other monetary benefits contingent on death or survival. Other sections give a brief synopsis of the elementary mathematical theory of life contingencies, including those involving more than one life; instructions for using the actuarial tables, with numerical examples; and a complete account of the methods and processes used in constructing the life tables. Because of the increasing interest in the preparation of life tables on the part of demographers, public health workers, and other groups, an effort has been made to render this statement of methods and processes intelligible to readers having a reasonable knowledge of mathematics and statistics, but without specific actuarial training. For this reason, some of the explanations will doubtless seem to the actuary unnecessarily full, and even somewhat tedious. An appendix, intended primarily for actuaries, explains the special processes used in the construction of the actuarial tables, and certain other technical matters.

Accuracy of the tables

It is well known that the statistics on which these life tables are based are subject to various errors, the magnitude of which is, in most cases, difficult to estimate with precision. These errors, whether found in statistics of populations, deaths, or births, fall into two general classes: (1) incompleteness or underenumeration, and (2) incorrect reporting of some of the pertinent information, such as age, race, or sex. Very little specific information is available as to the extent of incompleteness of reporting, except in the case of birth statistics.[1] However, it is believed that the unreported cases constitute, in general, a small percentage of the totals involved, except in the case of data for very young children (including births). In the latter case, a serious attempt has been made to introduce a suitable correction in the process of constructing the life tables.[2] It should be mentioned also that when death statistics are related to the corresponding population data, as in the computation of rates of mortality, any incompleteness in the enumeration of the population tends to offset whatever deficiency may exist in the reporting of deaths. It is believed, therefore, that errors of incomplete reporting are not likely, in general, to be of sufficient magnitude to seriously affect the life table values for white persons. However, there is some indication that in the rural areas of the South the reporting of Negro deaths may be appreciably less complete than the enumeration of Negroes in the census.[3] Since 49 percent of the total Negro population is found in the rural parts of the South, it is possible that mortality rates for Negroes may be somewhat understated. There is a more serious possibility of error in the case of the group of "other races" which includes Indians living on reservations, a class which presents real difficulty from the standpoint of complete reporting and enumeration.

Among the errors due to incorrect reporting, those arising from incorrect statements of age are by far the most important class, as regards the construction of life tables. These errors in age fall into two general types: (1) systematic errors, which arise from a preference for ages ending with certain digits, such as 0, 5, and the even numbers generally, and (2) errors characteristic of particular ages or periods of life. The systematic errors are believed to have been largely eliminated in the graduation of the data described in part V. A typical example of an age error of the second type would be that described by Wolfenden[4] as "a natural inclination to overstate the age until the attainment of majority, and then to understate at adult ages,

[1] See p. 102.
[2] See pp. 106–108.
[3] U. S. Bureau of the Census, *United States Abridged Life Tables, 1939, Urban and Rural, by Regions, Color, and Sex*, p. 5, June 1943.
[4] Wolfenden, Hugh H., *Population Statistics and Their Compilation* (*Actuarial Studies, No. 3*), p. 27, Actuarial Society of America, New York, 1925

with some overstatement in advanced years." Errors of this sort are not easy to detect, especially if the same type of error occurs in both population and death statistics. Only in one instance, in which the effect was particularly noticeable, has any adjustment been made for such errors in the construction of the life tables in this volume. This point is fully discussed in part V.

Errors in the reporting of race probably are relatively infrequent, except in the case of persons of mixed white and Indian blood. There is no general agreement as to what proportion of Indian blood entitles one to be called an Indian, and it is likely that the information furnished on death certificates may often fail to be consistent in this respect with the definition adopted in the population census. Any error arising from this source could scarcely be of sufficient magnitude to have any appreciable influence on mortality rates for the white population, but could easily have a disturbing effect on those for "other races." It is believed that any errors in the reporting of sex would not be sufficiently numerous to seriously affect any of the life tables.

In addition to errors resulting from actual inaccuracies in the data, there are errors due to chance fluctuation in the number of deaths: that is, what is known as sampling error. This is of importance only in fairly small classes, in which a small variation in the absolute number of deaths in a given age group may make a considerable difference in the rate of mortality. Table A, showing the total enumerated population and the total deaths in the 3-year period in each of the six subdivisions of the population for which separate life tables were prepared, indicates the size of the exposure underlying each life table. Sampling errors tend to be largely corrected by the graduation process, in which the mortality rates in each age group are adjusted so as to bring them into line with those in the neighboring age groups. In any case, it is believed that the effect of sampling error is negligible in the life tables for white persons, and of minor importance in those for Negroes, except at the very old ages. However, it may have significantly affected the results for "other races."[5]

If allowance is made for all the possible sources of error discussed above, the life tables for whites and Negroes are believed to be sufficiently accurate and reliable for all ordinary purposes. However, those for "other races" can be regarded only as reasonable approximations. For reasons explained in part V, this is also true of the life table values for subdivisions of the first year of life in all the tables.[6]

In connection with the accuracy of the tables, it should be clearly understood that the values cannot be considered reliable, in most cases, to anything like the number of decimal places or significant figures shown in the tables. The chief purpose of retaining additional figures beyond those which can be regarded as dependable is to secure a reasonable degree of smoothness in the results. This is always desirable, and in many of the uses to which life tables are put excessive roughness is a serious inconvenience. A further reason exists in the case of the actuarial tables, because of the mathematical relationships which hold between different actuarial functions, such as the values of life annuities and assurances. The actuary wishing to make use of the tables is inconvenienced if, because of excessive rounding, these relationships do not hold with a fair degree of precision.

TABLE A.—1940 ENUMERATED POPULATIONS, AND TOTAL DEATHS REPORTED IN 1939–1941, BY RACE AND SEX: UNITED STATES

RACE AND SEX	1940 population	1939–1941 deaths
White:		
Male	59, 448, 548	2, 048, 620
Female	58, 766, 322	1, 603, 192
Negro:		
Male	6, 269, 038	282, 490
Female	6, 596, 480	246, 492
Other races:		
Male	344, 006	13, 803
Female	244, 881	8, 211

Comparisons based on the life tables

Variation by race and sex.—The most usual measure of the comparative longevity of different populations is the average duration of life, also called the expectation of life at birth. This is the average number of years lived by the members of a specified cohort, or closed group of persons, assumed to be subject throughout life to the life table rates of mortality. A comparison on this basis is given in table B. This table indicates that females live, on the average, longer than males, white persons longer than Negroes, and Negroes not quite so long as those of "other races." There is, however, some objection to the use of the average duration of life as a standard of comparison because the method of calculating it gives great weight to the relatively large number of deaths occurring in the first year of life. This influence may be entirely eliminated by considering instead the average lifetime remaining to those members of the cohort who survive to age 1. This comparison is presented in table C, which shows, in general, about the same relationships as table B. However, the differences between the corresponding values for Negroes and "other races" are slightly increased now that the effect of the high infant mortality among "other races" is no longer reflected in the figures.

TABLE B.—AVERAGE DURATION OF LIFE IN YEARS, BY RACE AND SEX: UNITED STATES, 1939–1941

RACE	Both sexes	Male	Female
All races	63. 62	61. 60	65. 89
White	64. 92	62. 81	67. 29
Negro	53. 85	52. 26	55. 56
Other races	54. 35	53. 56	55. 84

[5] In connection with the distribution of "other races" deaths by subdivisions of the first year of life, a correction was applied for sampling error. See p. 109.

[6] See p. 108.

TABLE C.—AVERAGE FUTURE LIFETIME IN YEARS AT AGE 1, BY RACE AND SEX: UNITED STATES, 1939–1941

RACE	Both sexes	Male	Female
All races	65.76	64.00	67.73
White	66.84	64.98	68.93
Negro	57.15	55.93	58.46
Other races	58.90	58.40	60.14

Another possible standard for comparing the longevity of different populations is provided by the median length of life, or "probable lifetime," which is the age at which exactly half the original members of the cohort have died, and half are still alive. In other words, it is the age to which an infant born alive has just an even chance of surviving. The values of the median length of life (shown in table D) are greater in every case than those of the average length of life,[7] the difference ranging from 3.81 years in the case of Negro females to 8.70 years in the case of females of "other races." The use of the probable lifetime as a measure of longevity results in a somewhat more favorable showing for "other races," as compared with Negroes, than when the average duration of life was used. In fact, the probable lifetime of males of "other races" slightly exceeds that of Negro females. The reverse was true of the corresponding average durations of life.

TABLE D.—MEDIAN LENGTH OF LIFE IN YEARS, BY RACE AND SEX: UNITED STATES, 1939–1941

RACE	Both sexes	Male	Female
All races	69.85	67.68	72.22
White	70.86	68.67	73.19
Negro	57.86	56.42	59.37
Other races	62.67	61.89	64.54

Still another measure of comparative longevity is the number of persons surviving to stated ages in a cohort of, say, 100,000 live births. Such a comparison is presented in table E for survivors to age 21, and in table F for survivors to age 65. These ages have been chosen as representing, respectively, the attainment of manhood or womanhood, and the retirement age prescribed by the Social Security Act. Table E shows that relatively more Negroes reach age 21 than persons of "other races." This reflects higher rates of mortality in the "other races" group over almost the entire age period in question. However, between ages 21 and 65 the relationship is reversed, and the proportion surviving to the latter age is greater among "other races" than among Negroes.

TABLE E.—SURVIVORS TO AGE 21 OUT OF 100,000 LIVE BIRTHS, BY RACE AND SEX: UNITED STATES, 1939–1941

RACE	Both sexes	Male	Female
All races	92,234	91,392	93,116
White	92,951	92,098	93,848
Negro	87,367	86,494	88,264
Other races	82,853	82,412	83,302

[7] The explanation of this fact and a discussion of the relative merits of different measures of longevity are given on p. 23.

TABLE F.—SURVIVORS TO AGE 65 OUT OF 100,000 LIVE BIRTHS, BY RACE AND SEX: UNITED STATES, 1939–1941

RACE	Both sexes	Male	Female
All races	60,366	55,776	65,523
White	63,201	58,305	68,701
Negro	37,838	35,371	40,504
Other races	46,130	44,689	49,303

In considering the mortality and longevity of the group of "other races," it should be kept in mind that this is a heterogeneous class made up of elements which differ widely both in the general level of mortality and in its incidence by age. The racial composition of the group is shown in table G, and age-specific death rates for the principal races separately appear in table H, together with comparable figures for whites and Negroes.

TABLE G.—POPULATION OF OTHER RACES,[1] BY SPECIFIED RACE AND SEX: UNITED STATES, 1940

RACE	POPULATION			PERCENT BY RACE		
	Total	Male	Female	Total	Male	Female
Total other races	588,887	344,006	244,881	100.0	100.0	100.0
Indian	333,969	171,427	162,542	56.7	49.8	66.4
Chinese	77,504	57,389	20,115	13.2	16.7	8.2
Japanese	126,947	71,967	54,980	21.6	20.9	22.4
Filipino	45,563	39,723	5,840	7.7	11.6	2.4
All other	4,904	3,500	1,404	0.8	1.0	0.6

[1] All except white and Negro

TABLE H.—DEATH RATES PER 1,000 ENUMERATED POPULATION, BY AGE, RACE, AND SEX: UNITED STATES, 1939–1941

SEX AND AGE	White	Negro	Indian	Chinese	Japanese	Other
MALE						
0–4	13.2	22.8	35.8	13.7	12.1	12.7
5–9	1.2	1.6	3.3	[1] 1.0	1.1	[1] 1.6
10–14	1.1	1.7	2.8	1.6	1.3	[1] 2.1
15–19	1.7	3.7	5.7	3.5	1.8	[1] 1.4
20–24	2.3	6.4	7.5	4.7	2.6	4.9
25–29	2.5	7.8	6.6	5.0	2.9	5.1
30–34	3.1	9.7	8.3	6.9	4.8	4.7
35–39	4.2	11.4	8.2	9.5	4.5	7.4
40–44	6.1	15.7	9.6	12.8	6.0	7.6
45–49	9.1	20.8	13.0	17.1	9.4	12.9
50–54	13.7	29.4	16.3	23.8	11.4	19.4
55–59	20.7	36.1	24.1	38.3	17.6	27.5
60–64	30.0	43.8	30.1	47.7	27.4	59.6
65–74	53.1	54.5	48.4	80.2	45.7	92.2
75 and over	135.0	119.8	109.9	192.1	110.0	103.7
FEMALE						
0–4	10.4	18.1	32.1	13.7	9.4	10.7
5–9	.9	1.3	2.8	[1] .9	1.0	3.0
10–14	.7	1.5	3.0	1.5	.8	[1] 1.4
15–19	1.2	4.2	6.4	2.8	1.5	[1] 2.7
20–24	1.6	5.8	9.5	3.6	1.9	[1] 4.5
25–29	2.0	6.6	9.1	4.3	3.3	[1] 2.5
30–34	2.4	8.2	8.4	2.5	2.5	[1] 5.5
35–39	3.1	9.9	9.4	4.6	3.3	[1] 4.4
40–44	4.3	14.0	9.6	5.6	3.9	18.5
45–49	6.1	17.6	11.2	9.8	6.7	23.2
50–54	9.0	25.7	16.0	15.1	7.9	45.3
55–59	13.5	32.4	21.9	17.0	13.9	57.5
60–64	20.7	40.0	28.0	28.2	17.3	142.9
65–74	40.8	44.9	43.0	42.5	37.0	91.7
75 and over	120.8	96.5	103.7	93.8	49.0	[1] 388.9

[1] Rate based on less than 10 deaths.

A more detailed comparison of life table values by race and sex is offered by figures 1 to 6, in which are plotted graphically the values at all ages of the rate of mortality, the number of survivors out of 100,000 live births, and the average future lifetime for each of the 12 life tables. These graphs bring out certain

FIGURE 1.—ANNUAL RATE OF MORTALITY PER 1,000, FOR EACH RACE BY SEX: UNITED STATES, 1939–1941

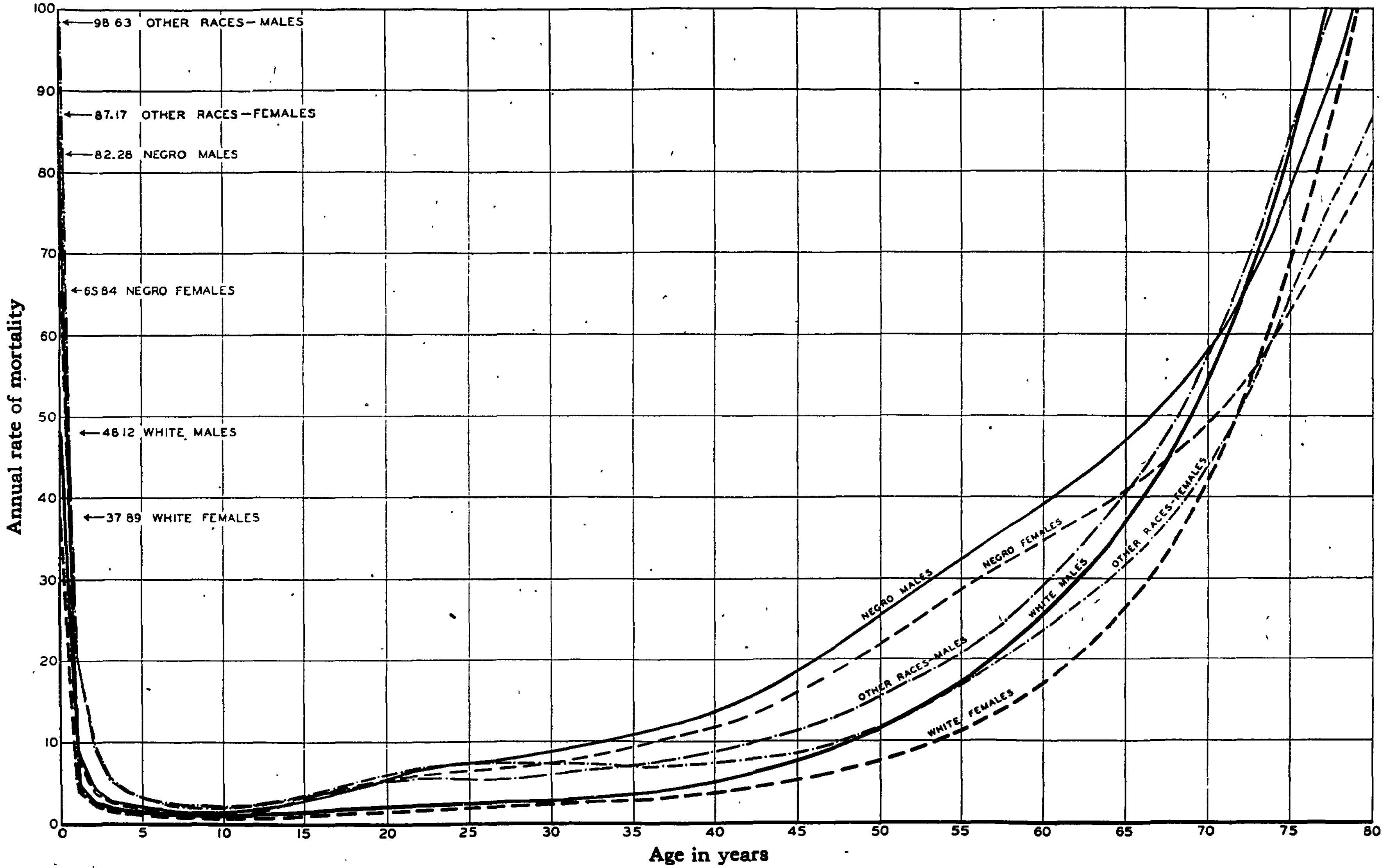


FIGURE 2.—ANNUAL RATE OF MORTALITY PER 1,000, BY RACE AND BY SEX: UNITED STATES, 1939–1941

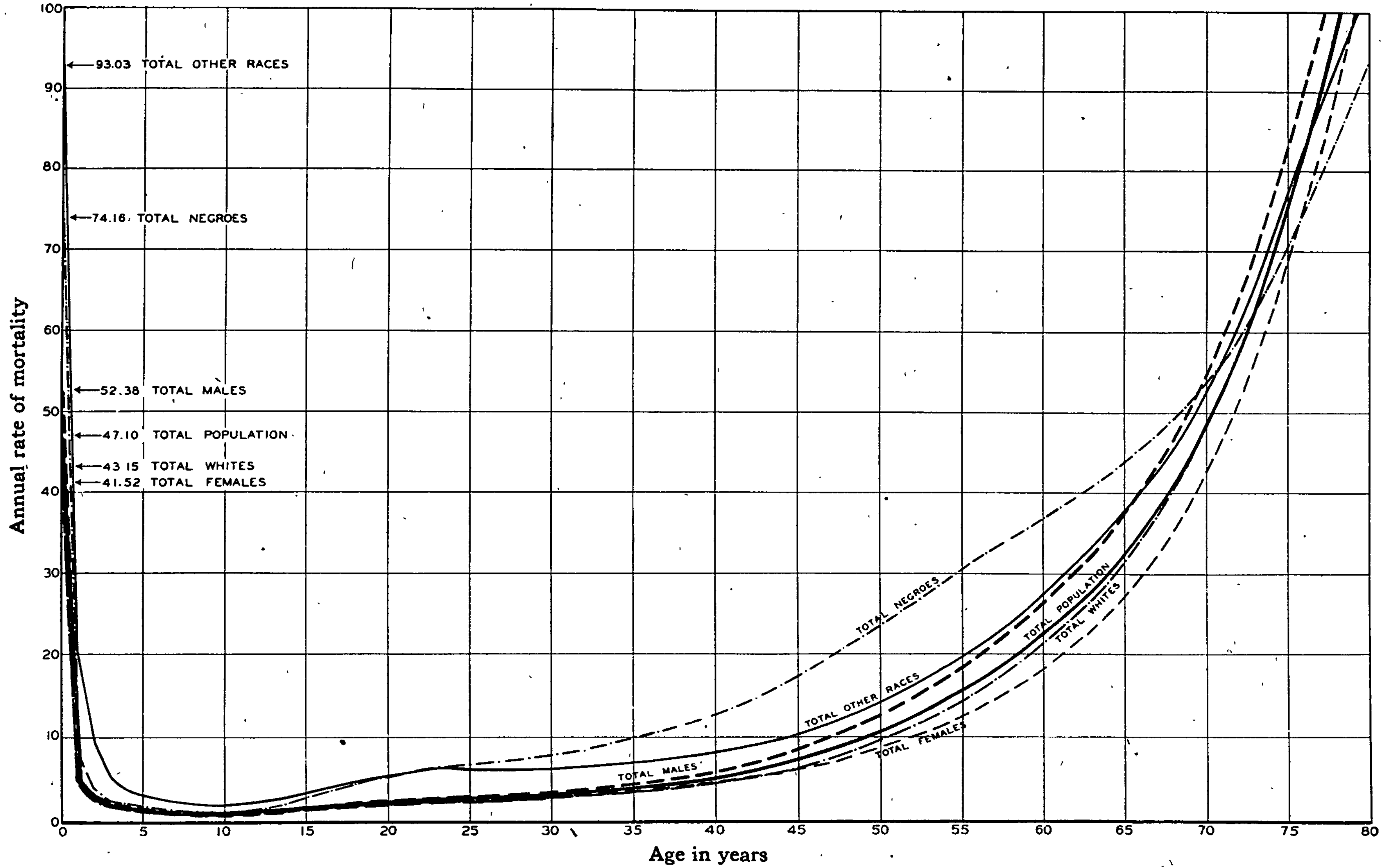


FIGURE 3.—NUMBER OF SURVIVORS OUT OF 100,000 BORN ALIVE, FOR EACH RACE BY SEX: UNITED STATES, 1939–1941

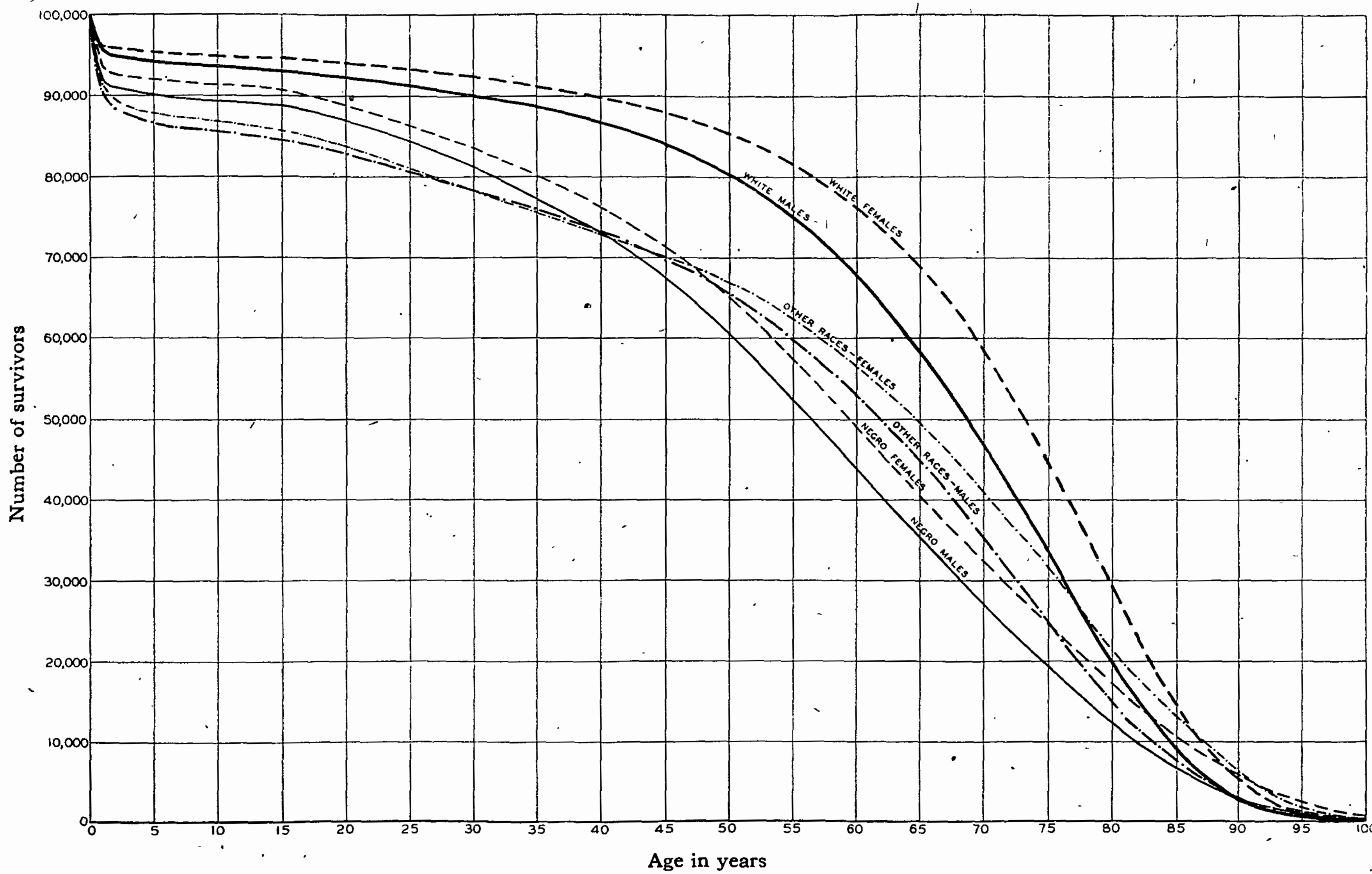


FIGURE 4.—NUMBER OF SURVIVORS OUT OF 100,000 BORN ALIVE, BY RACE AND BY SEX: UNITED STATES, 1939–1941

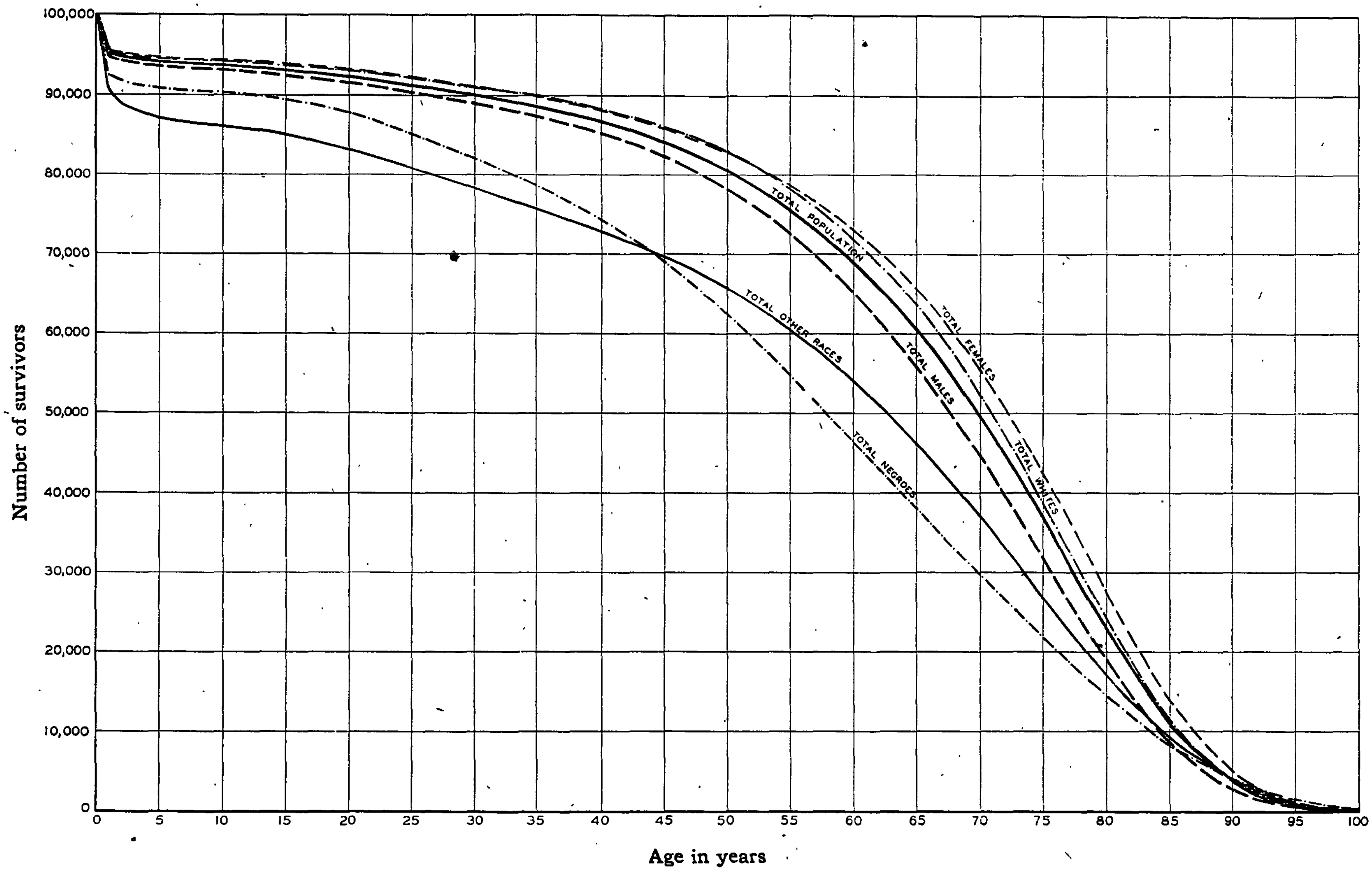


FIGURE 5.—AVERAGE FUTURE LIFETIME, FOR EACH RACE BY SEX: UNITED STATES, 1939–1941

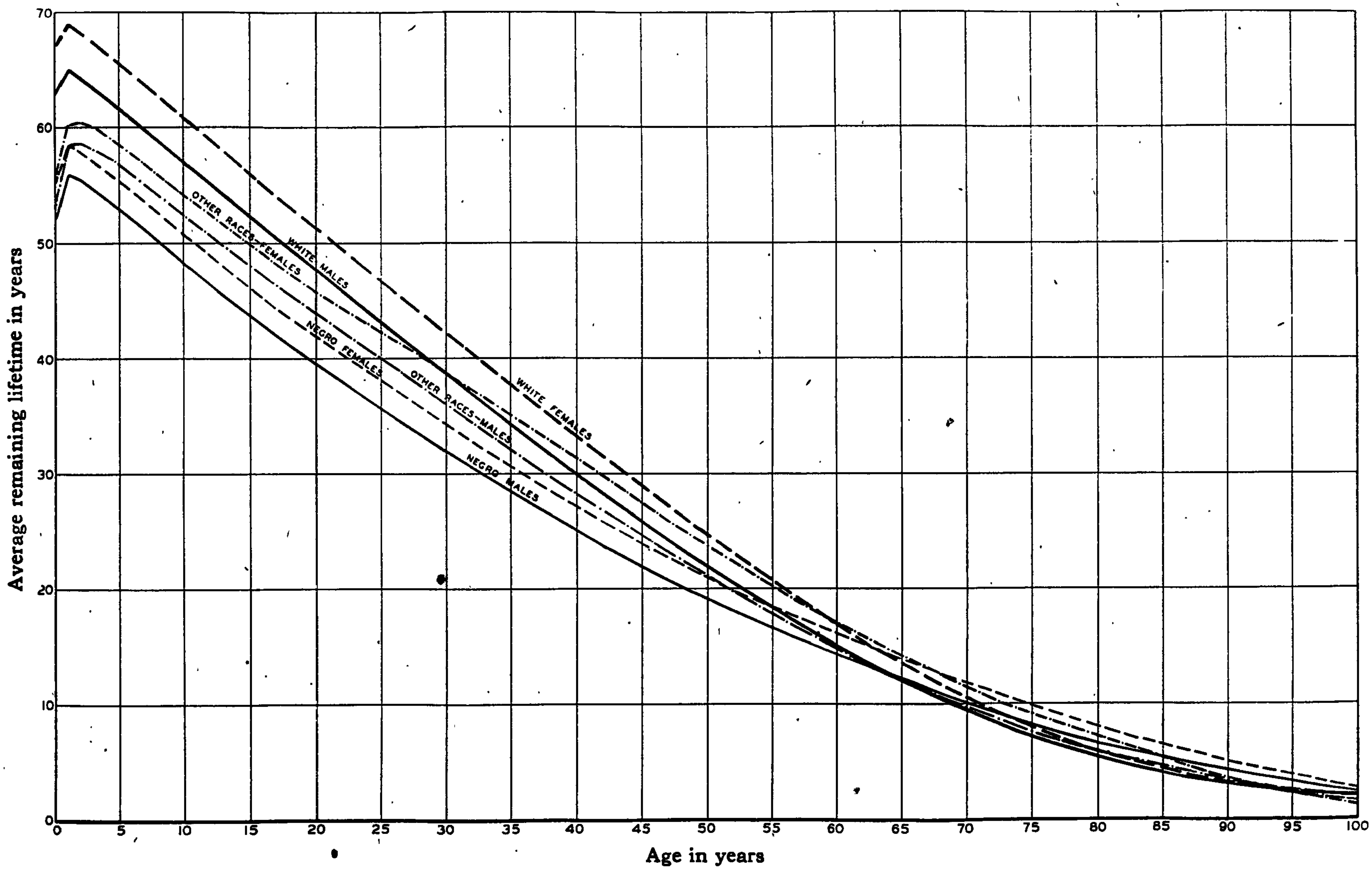
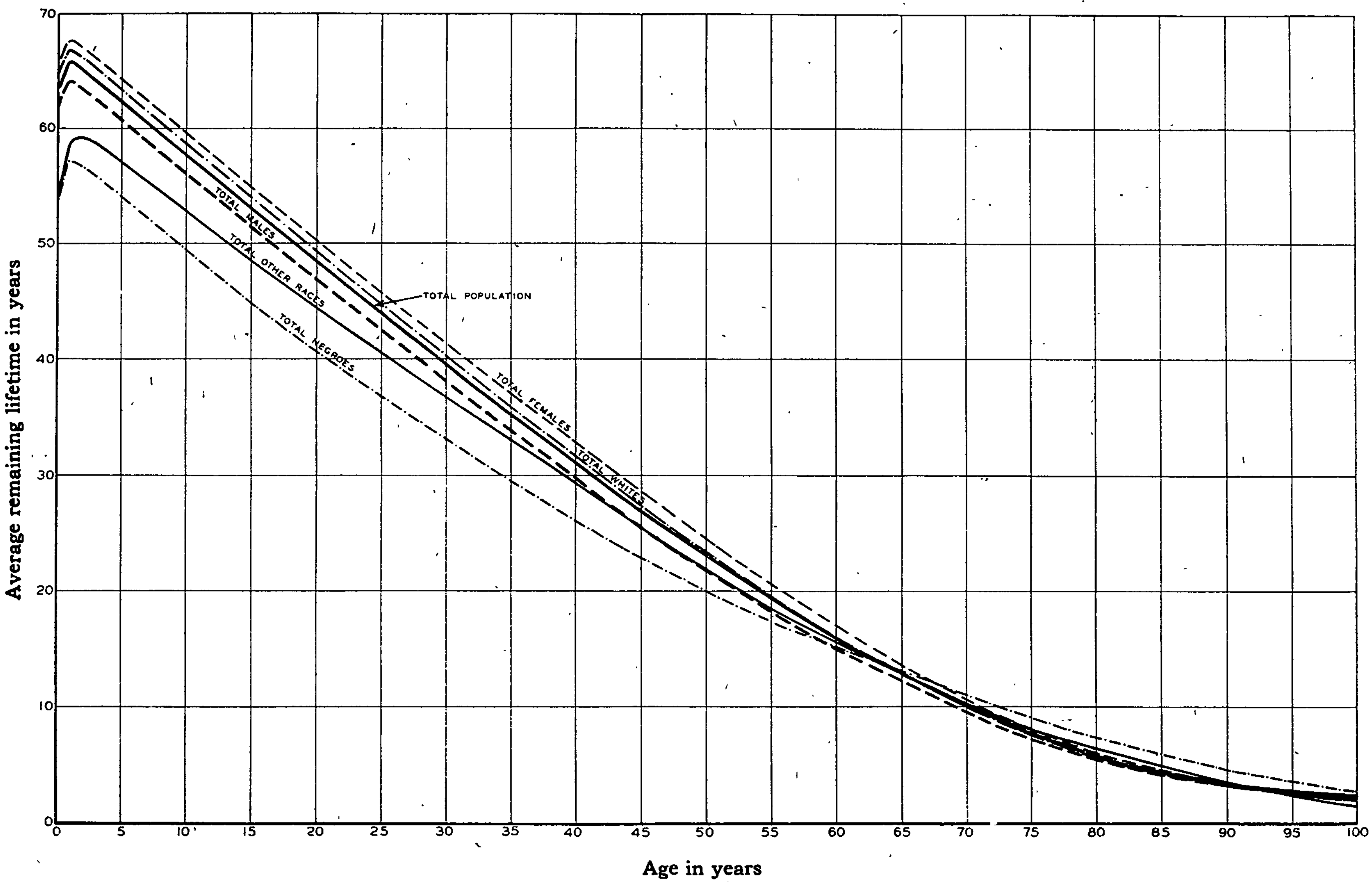


FIGURE 6.—AVERAGE FUTURE LIFETIME, BY RACE AND BY SEX: UNITED STATES, 1939–1941



relationships which may, at first sight, appear somewhat surprising. For example, figure 2 shows that the mortality rates for total females are lower than those for total whites at all ages above 42, notwithstanding the fact that Negro females show a much higher mortality than white males during a large part of this age interval. This seeming inconsistency is due to the fact that Negro and "other races" females form a much smaller group than white males, so that combining them with white females produces less change in the mortality rate of the entire group (as compared with that for white females before the addition) than if the white males had been added.

Because mortality rates for females are so consistently lower, age by age, than the corresponding rates for males, particular interest attaches to the few instances in which exceptions to this general rule occur. The exceptions are found among Negroes at ages 14 to 19, inclusive, and among "other races" at ages 12 to 34. In the case of Negroes, further analysis by particular causes of death shows that the phenomenon is primarily due to the effect of higher mortality from tuberculosis among females at younger ages. Deaths from puerperal causes appear to be a negligible factor at the ages in question. In the case of "other races," examination of data showing a more detailed racial classification makes it clear that the Indians are almost entirely responsible for the higher female mortality, as the Chinese show heavier mortality for males in all age groups and the Japanese show a very slightly higher mortality among females in the late twenties only. In 1940, Indians constituted 66.4 percent of the total female population excluding whites and Negroes, as indicated in table G. Here again, an excess of deaths from tuberculosis among females is the chief factor involved, but in this case deaths from puerperal causes exert a significant, although secondary influence.

The rates of mortality for "other races" show a further peculiarity in that they decrease with increasing age at ages 24 to 26 for males and at ages 29 to 36 for females. Both of the peculiarities mentioned—the higher mortality of females at certain ages, and interruptions in the steady increase of the mortality rate—are usually prominent features of life tables for depressed countries where the general level of mortality is high, such as British India, Japan, and Bulgaria, and they are commonly associated with a high tuberculosis death rate, combined perhaps with a higher mortality from puerperal causes. It is curious to observe, however, that both peculiarities have been consistently noted in the life tables of Canada, a relatively prosperous country having a level of mortality lower, in general, than that of the United States. For example, in the Canadian life table for 1930–1932 the mortality rate of females exceeds that of males at ages 23 to 42, and the mortality rate of males decreases with advancing age at ages 24 to 26. Analysis of deaths by cause indicates that the higher mortality of Canadian females at certain ages has been primarily due, as in other countries where this occurs, to deaths from tuberculosis. While the over-all death rate for tuberculosis was, in the period under consideration, only slightly higher in Canada than in the United States, the deaths from this cause in Canada are found to be more heavily concentrated among females and at the younger ages.

Comparison with earlier United States life tables.—Table J presents a comparison of values based on the life tables in this volume with those of earlier United States life tables. In addition, rates of mortality for white males and white females are plotted in figures 7 and 8 for five life tables covering the period 1900 to 1941. Although the life tables for periods prior to 1930 do not cover the entire United States, any possible geographic variation in the mortality of white persons could account for only a small part of the spectacular improvement which the comparison shows. In the 40 years between 1900 and 1940 the average duration of life increased by more than 14 years for white males and more than 16 years for white females. The proportion of persons surviving to age 65 has increased by one-half, and the rate of infant mortality has declined to little more than one-third of its value in 1900. Similar improvement is shown throughout childhood and young adulthood and, to a lesser degree, in middle age. The mortality rate at age 40 has diminished to less than half its former value. At older ages the improvement becomes in proportion progressively less, but the recent figures are slightly lower even at the oldest ages shown. The improvement is more marked in the case of females, and remains substantial in amount to a later age than for males.

In the case of Negroes, only the 1929–1931 and 1939–1941 life table values are shown, since the life tables for Negroes show a considerable geographic variation (perhaps due as much to geographic differences in the completeness of registration of Negro deaths as to actual differences in mortality) which makes it inadvisable to present any comparisons not involving identical areas. However, even in the 10-year period between 1930 and 1940, the improvement is striking, the average duration of life of Negroes having risen during the decade nearly 5 years for males and more than 6 years for females.

Figure 7 calls attention to one rather curious feature which calls for special comment. This is the low level of mortality above age 45 in the life table for white males in 1919–1921 in the death-registration States of 1920. From age 52 to 69, the rates of mortality in this life table are actually lower than those of the 1939–1941 table for white males in the United States. The years 1919 to 1921, coming immediately after the influenza epidemic of 1918, were years of unusually low mortality, probably because many persons who, under ordinary circumstances, would have died in these 3 years actually died in 1918. These conditions, of course, affected both sexes and a much broader range

TABLE J.—LIFE TABLE VALUES FOR SELECTED SPECIFIC AGES, BY SEX: DEATH-REGISTRATION STATES OF 1900 AND 1920, AND THE UNITED STATES, AT 10-YEAR INTERVALS, 1900–1941

[The abbreviation D. R. S. stands for death-registration States]

SEX AND AGE	WHITE									
	Annual rate of mortality per 1,000 (1,000q_x)					Number of survivors out of 100,000 live births (l_x)				
	1939–1941 (U. S.)	1929–1931 (U. S.)	1919–1921 (D. R. S. of 1920)	1909–1911 (D. R. S. of 1900)	1900–1902 (D. R. S. of 1900)	1939–1941 (U. S.)	1929–1931 (U. S.)	1919–1921 (D. R. S. of 1920)	1909–1911 (D. R. S. of 1900)	1900–1902 (D. R. S. of 1900)
MALE										
0	48.12	62.32	80.25	123.26	133.45	100,000	100,000	100,000	100,000	100,000
1	4.87	9.93	16.19	28.21	34.47	95,188	93,768	91,975	87,674	86,655
5	1.38	2.66	3.95	4.71	6.06	94,150	91,738	88,842	82,972	80,864
10	1.00	1.47	2.11	2.38	2.74	93,601	90,810	87,530	81,519	79,109
15	1.43	2.13	2.91	2.83	3.34	93,089	90,074	86,546	80,549	78,037
20	2.12	3.18	4.27	4.89	5.94	92,293	88,904	84,997	79,116	76,376
25	2.43	3.71	5.04	5.54	7.04	91,241	87,371	83,061	77,047	73,907
30	2.79	4.13	5.73	6.60	7.99	90,092	85,707	80,888	74,810	71,219
35	3.63	5.10	6.69	8.52	9.32	88,713	83,812	78,441	72,108	68,245
40	5.13	6.79	7.50	10.22	10.60	86,880	81,457	75,733	68,848	64,954
45	7.66	9.29	9.26	12.64	12.63	84,285	78,345	72,696	65,115	61,369
50	11.55	12.78	11.74	15.53	15.37	80,521	74,288	69,107	60,741	57,274
55	17.37	18.19	16.53	21.50	21.18	75,156	68,981	64,574	55,622	52,491
60	25.48	26.44	24.62	30.75	28.59	67,787	61,933	58,498	48,987	46,452
65	36.85	38.65	34.99	43.79	41.66	58,305	52,964	50,663	40,862	39,245
70	54.54	57.96	54.63	62.14	58.94	46,739	41,880	40,873	31,527	30,640
75	83.13	85.26	81.91	92.53	88.43	33,404	29,471	29,205	21,585	21,387
80	124.71	129.97	119.73	135.75	133.53	19,860	17,221	17,655	12,160	12,266
85	181.04	184.68	182.32	191.11	191.76	9,013	7,572	8,154	5,145	5,252
90	248.94	245.50	238.19	255.17	262.78	2,812	2,356	2,568	1,523	1,523
FEMALE										
0	37.89	49.63	63.92	102.26	110.61	100,000	100,000	100,000	100,000	100,000
1	4.32	8.79	14.59	25.83	31.15	96,211	95,037	93,608	89,774	88,939
5	1.10	2.20	3.49	4.47	5.89	95,309	93,216	90,721	85,349	83,426
10	.70	1.13	1.79	2.06	2.46	94,890	92,466	89,564	83,979	81,723
15	.96	1.64	2.49	2.65	3.39	94,534	91,894	88,712	83,093	80,680
20	1.45	2.77	4.33	4.20	5.54	93,984	90,939	87,281	81,750	78,978
25	1.82	3.39	5.52	5.22	6.79	93,228	89,524	85,163	79,865	76,588
30	2.20	3.74	6.03	6.03	7.72	92,320	87,972	82,740	77,676	73,887
35	2.78	4.33	6.42	7.13	8.39	91,211	86,248	80,206	75,200	70,971
40	3.68	5.32	6.76	8.03	9.31	89,805	84,256	77,624	72,425	67,935
45	5.23	7.02	8.14	9.91	10.63	87,920	81,780	74,871	69,341	64,677
50	7.62	9.59	10.67	12.59	13.37	85,267	78,572	71,547	65,629	61,005
55	11.28	13.75	14.63	17.93	18.69	81,520	74,321	67,323	61,053	56,509
60	17.14	20.63	21.73	25.83	25.06	76,200	68,462	61,704	54,900	50,752
65	26.43	31.25	31.68	37.86	36.41	68,701	60,499	54,299	47,086	43,806
70	42.33	48.66	50.23	56.63	53.69	58,363	49,932	44,638	37,482	35,206
75	68.89	74.60	75.97	82.52	80.39	44,685	37,024	32,777	26,569	25,362
80	108.19	117.42	113.41	125.79	121.15	28,882	23,053	20,492	15,929	15,349
85	162.94	170.86	170.44	178.32	174.60	14,487	10,937	9,909	7,152	7,149
90	231.41	231.51	230.61	247.59	245.32	5,061	3,719	3,372	2,291	2,322

SEX AND AGE	WHITE					NEGRO					
	Average future lifetime in years ($\mathring{e}_x$)					Annual rate of mortality per 1,000 (1,000 q_x)		Number of survivors out of 100,000 live births (l_x)		Average future lifetime in years ($\mathring{e}_x$)	
	1939–1941 (U. S.)	1929–1931 (U. S.)	1919–1921 (D. R. S. of 1920)	1909–1911 (D. R. S. of 1900)	1900–1902 (D. R. S. of 1900)	1939–1941 (U. S.)	1929–1931 (U. S.)	1939–1941 (U. S.)	1929–1931 (U. S.)	1939–1941 (U. S.)	1929–1931 (U. S.)
MALE											
0	62.81	59.12	56.34	50.23	48.23	82.28	87.32	100,000	100,000	52.26	47.55
1	64.98	62.04	60.24	56.26	54.61	9.37	16.57	91,772	91,268	55.93	51.08
5	61.68	59.38	58.31	55.37	54.43	1.86	2.95	90,082	88,412	52.95	48.69
10	57.03	54.96	54.15	51.32	50.59	1.38	2.11	89,303	87,311	48.34	44.27
15	52.33	50.39	49.74	46.91	46.25	2.74	4.33	88,610	86,152	43.74	39.83
20	47.76	46.02	45.60	42.71	42.19	5.44	8.58	86,968	83,621	39.52	35.95
25	43.28	41.78	41.60	38.79	38.52	7.33	10.96	84,227	79,516	35.72	32.67
30	38.80	37.54	37.65	34.87	34.88	8.72	12.75	80,979	75,083	32.05	29.45
35	34.36	33.33	33.74	31.08	31.29	10.71	14.84	77,221	70,049	28.48	26.39
40	30.03	29.32	29.86	27.43	27.74	13.62	18.13	72,780	64,710	25.06	23.36
45	25.87	25.28	26.00	23.86	24.21	18.59	22.40	67,346	58,432	21.88	20.59
50	21.96	21.51	22.22	20.39	20.76	25.36	27.50	60,495	51,748	19.06	17.92
55	18.34	17.97	18.59	17.03	17.42	32.48	33.92	52,426	44,436	16.60	15.46
60	15.05	14.72	15.25	13.98	14.35	39.10	41.40	43,833	36,790	14.37	13.15
65	12.07	11.77	12.21	11.25	11.51	46.85	50.72	35,371	29,314	12.21	10.87
70	9.42	9.20	9.51	8.83	9.03	57.99	70.18	27,236	21,741	10.11	8.78
75	7.17	7.02	7.30	6.75	6.84	78.03	92.82	19,456	14,419	8.17	6.99
80	5.38	5.26	5.47	5.09	5.10	107.30	129.91	12,186	8,239	6.58	5.42
85	4.02	3.99	4.06	3.88	3.81	137.83	177.61	6,444	3,660	5.34	4.30
90	3.06	3.03	3.18	2.99	2.85	174.17	220.32	2,836	1,246	4.23	3.42
FEMALE											
0	67.29	62.67	58.53	53.62	51.08	65.84	72.04	100,000	100,000	55.56	49.51
1	68.93	64.93	61.51	58.69	56.39	7.96	14.37	93,416	92,796	58.46	52.33
5	65.57	62.17	59.43	57.67	56.03	1.75	2.84	91,906	90,185	55.40	49.81
10	60.85	57.65	55.17	53.57	52.15	1.04	1.61	91,308	89,201	50.75	45.33
15	56.07	53.00	50.67	49.12	47.79	3.07	5.12	90,594	88,088	46.13	40.87
20	51.38	48.52	46.46	44.88	43.77	5.32	8.82	88,736	85,078	42.04	37.22
25	46.78	44.25	42.55	40.88	40.05	6.27	10.34	86,198	81,067	38.20	33.93
30	42.21	39.99	38.72	36.96	36.42	7.33	11.59	83,384	76,816	34.40	30.67
35	37.70	35.73	34.86	33.09	32.82	9.24	13.22	80,092	72,192	30.71	27.47
40	33.25	31.52	30.94	29.26	29.17	11.81	16.25	76,084	67,271	27.19	24.30
45	28.90	27.39	26.98	25.45	25.51	16.02	20.18	71,157	61,365	23.89	21.39
50	24.72	23.41	23.12	21.74	21.89	21.87	26.65	64,885	54,920	20.95	18.60
55	20.73	19.60	19.40	18.18	18.43	28.58	34.99	57,314	47,074	18.38	16.27
60	17.00	16.05	15.93	14.92	15.23	34.72	42.20	48,928	38,761	16.10	14.22
65	13.56	12.81	12.75	11.97	12.23	40.90	49.35	40,504	30,852	13.93	12.24
70	10.50	9.98	9.94	9.38	9.59	49.12	61.74	32,354	23,341	11.82	10.38
75	7.92	7.56	7.62	7.20	7.33	62.94	73.41	24,502	16,576	9.81	8.62
80	5.88	5.63	5.70	5.35	5.50	81.27	97.84	17,039	10,822	8.02	6.90
85	4.34	4.24	4.24	4.06	4.10	105.29	128.34	10,622	6,033	6.41	5.48
90	3.24	3.17	3.16	3.00	3.02	141.32	172.03	5,652	2,774	4.96	4.20

FIGURE 7.—ANNUAL RATE OF MORTALITY PER 1,000 FOR WHITE MALES: DEATH-REGISTRATION STATES OF 1900 AND 1920, AND THE UNITED STATES, AT 10-YEAR INTERVALS, 1900–1941

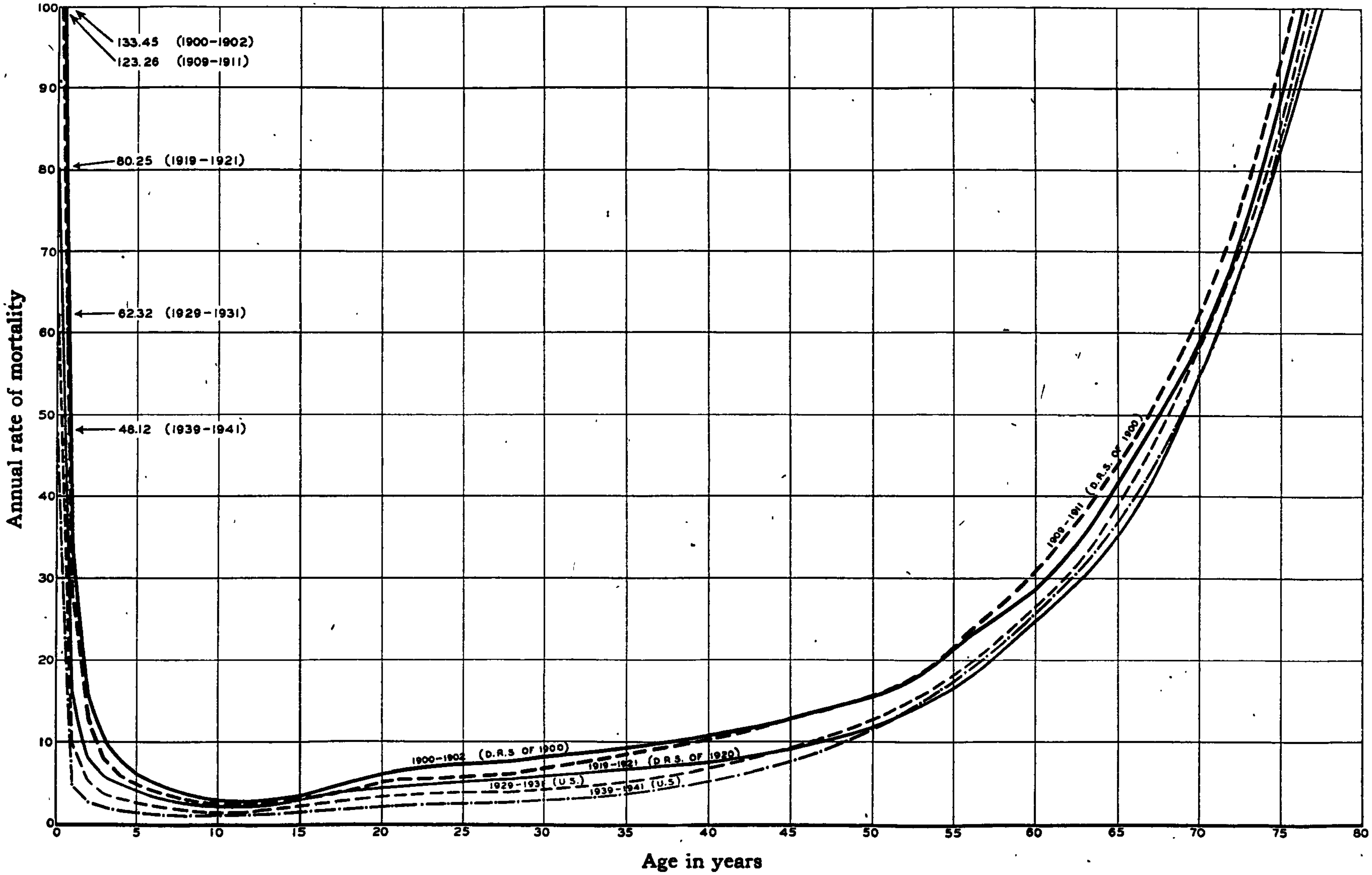


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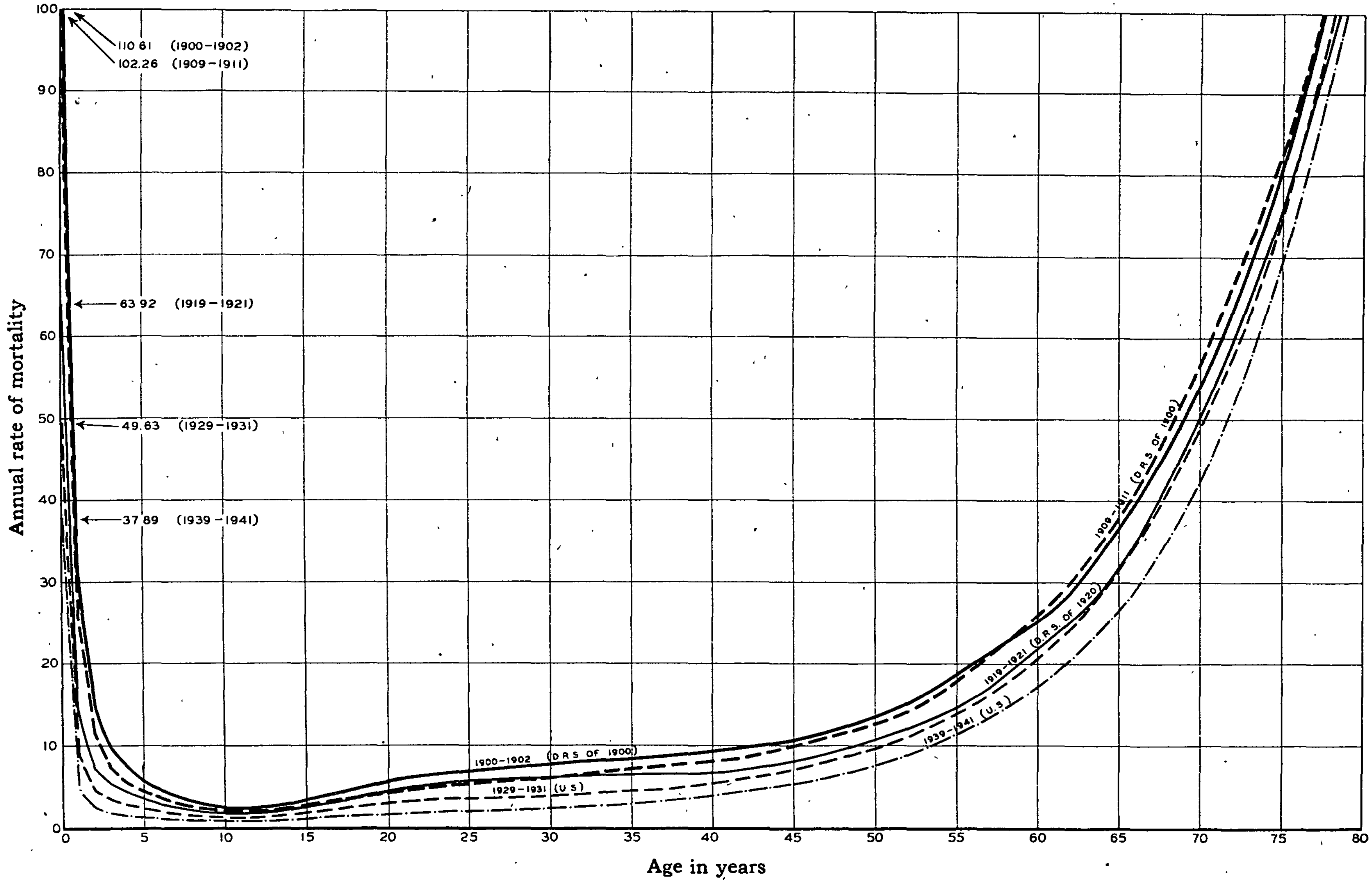
FIGURE 8.—ANNUAL RATE OF MORTALITY PER 1,000 FOR WHITE FEMALES: DEATH-REGISTRATION STATES OF 1900 AND 1920, AND THE UNITED STATES, AT 10-YEAR INTERVALS, 1900–1941



of ages than that indicated above. However, among males at younger ages and among females at most ages, there has been a steady improvement in mortality since 1921, so that the low mark set in the post-epidemic years has since been surpassed. However, in the case of males above age 55 there has been only a slight decline in the mortality rate during the last 20 years, so that the low level of 1919–1921 has not yet been equaled.

Comparison with recent life tables for other countries.—In tables K, L, and M, life table values for the United States in 1929–1931 and 1939–1941 are compared with the corresponding values for a number of other countries. The selection of these countries has been influenced, to a considerable extent, by the availability of reliable life tables covering recent periods. (Only those for periods ending in or after 1930 have been used.) It is natural, therefore, that the majority of the countries selected are European. Figures 9 and 10 exhibit graphically the number of survivors to successive ages for white males and white females in the United States in 1929–1931 and 1939–1941, in comparison with similar curves for England and Wales, British India, Italy, Japan, Mexico, and New Zealand. These six countries were selected mainly in order to secure a wide range in the general levels of mortality represented. New Zealand and British India, in particular, have long been regarded as representing, respectively, the lowest and the highest general level of mortality among those countries for which reliable life tables are available. Values for the United States death-registration States of 1900 in 1900-1902 have also been plotted, so that the amount of improvement over the 40-year period can be compared with the variation in present conditions as between different parts of the world. These charts show that survival rates in Mexico and British India were still in 1930 far below the level which characterized the United States in 1900, and those of Japan had not quite reached that level. The English life tables of 1930–1932 exhibit lower survival rates up to about age 45 than the United States tables covering approximately the same period. However, at subsequent ages the cumulative effect of the lower adult mortality of England and Wales (which began at age 20) results in a larger number of survivors than in the United States. Ten years later, in 1939–1941, the United States had not attained the low mortality found in New Zealand about 4 years earlier. This indicates the possibility of still further improvement; and the 1934–1938 life table for New Zealand is by no means to be regarded as reflecting the ultimate low level of mortality which is never to be surpassed.

All the values employed in the preparation of tables K, L, and M and figures 9 and 10 have been obtained from official government life tables except those for Austria and Mexico and some of the values for Canada. The Austrian life table was published by an association of insurance companies, while the Mexican table appeared in a signed article in a journal published by the Department of Public Health. The official Canadian life table commences not at birth but at age 5, so that the numbers of survivors in the life table cohort are not comparable with similar figures for other countries. Accordingly, all these values, as well as the values of the rate of mortality and the average future lifetime at ages 0 and 1, have been taken from an unofficial life table[8] covering the same period of years. For four countries, it was not possible to secure the original publications, and the figures were obtained from secondary sources. In the case of Denmark and Sweden, the available sources were official yearbooks, while the figures for Austria and Czechoslovakia were obtained from a German Government publication.

The Mexican life table was not graduated, and the rates of mortality were generally overstated at the ages which are multiples of 5 (these being the ages for which values are shown in table K), because of a decided preference for such ages in the reporting of ages at death. For this reason, it would have been unfair to Mexico to use the published mortality rates in the comparison. Accordingly, the values shown in table K have been corrected for this error.[9]

[8] See list of sources of foreign life table values on p. 20.

[9] The correction was made by referring to a graduated life table for Mexico, that of J. B. Solórzano as adjusted by Giorgio Mortara (published in *Estadística*, Journal of the Inter American Statistical Institute, vol. 11, No. 5, pp. 78-80, March 1944). For each age x (except ages 0, 1, 5, and 10) for which the mortality rate is shown in table K, a corrected value of the number of deaths at age x in the life table cohort was obtained by accepting as correct the total number of deaths at ages x to $x+4$, and assuming the number of deaths at age x to be the same fraction of the total deaths at ages x to $x+4$ as in the Solórzano-Mortara table. The latter table is for both sexes combined and covers the period 1929–1933.

TABLE K.—ANNUAL RATE OF MORTALITY PER 1,000, FROM RECENT LIFE TABLES FOR SELECTED COUNTRIES, BY SEX FOR SELECTED SPECIFIC AGES

SEX AND AGE	Australia, 1932–1934	Austria, 1930–1933	Belgium, 1928–1932	British India, 1921–1930	Canada, 1930–1932	Czechoslovakia, 1929–1932	Denmark, 1931–1935	England and Wales, 1930–1932	France, 1928–1933	Germany, 1932–1934	Italy, 1930–1932
MALE											
0	45.43	115.40	100.75	248.7	99.97	148.69	81.47	71.86	90.18	85.35	115.32
1	7.75	14.00	17.11	91.8	12.82	19.32	9.01	15.30	16.90	9.26	38.97
5	1.84	3.41	3.12	19.3	2.62	3.80	1.34	3.43	2.85	2.32	3.65
10	1.19	1.86	1.54	7.9	1.60	1.99	1.13	1.46	1.63	1.33	1.99
15	1.49	2.03	2.30	9.8	2.07	2.39	1.47	1.97	2.49	1.57	2.38
20	2.19	3.74	4.34	12.7	3.08	4.29	2.56	3.16	5.18	2.83	4.14
25	2.49	4.26	3.98	15.3	3.40	4.55	2.66	3.30	5.23	2.97	4.27
30	2.71	4.38	4.44	19.3	3.41	4.64	2.68	3.40	5.88	3.24	4.66
35	3.46	5.62	5.19	24.1	3.98	5.52	3.24	4.21	7.07	3.94	5.30
40	4.60	7.03	6.40	29.4	4.94	7.07	4.01	5.62	8.90	4.82	6.36
45	6.59	9.51	8.35	34.9	6.30	9.23	5.84	7.99	11.64	6.58	7.94
50	9.66	12.99	11.51	41.0	9.03	12.85	8.32	11.28	15.33	9.39	10.63
55	14.93	18.93	16.59	48.1	13.29	17.97	12.44	16.14	20.71	14.18	14.68
60	22.16	26.72	24.77	57.9	19.38	25.79	18.66	24.15	29.18	21.72	21.92
65	33.11	40.77	37.87	72.7	29.75	38.96	30.97	37.91	42.33	34.04	33.19
70	50.82	60.33	58.71	97.6	46.34	59.74	48.25	60.35	64.28	54.01	53.23
75	78.08	95.56	91.52	142.7	74.03	93.93	78.37	95.19	101.60	87.40	87.79
80	126.59	147.73	142.20	218.0	115.27	143.87	121.81	145.00	152.56	136.68	137.99
85	188.64	222.52	218.41	360.8	171.67	212.11	189.55	210.48	234.42	207.69	206.64
90	249.86	312.73	327.51	577.0	247.11	289.36	284.52	286.14	303.40	287.73	290.32
FEMALE											
0	36.42	92.45	78.55	232.3	83.58	124.57	63.08	54.55	71.62	68.39	102.25
1	6.45	13.07	14.78	86.5	13.79	18.57	7.18	13.45	15.13	8.23	39.05
5	1.58	3.43	2.68	16.5	2.32	3.76	1.32	2.98	2.79	2.15	3.66
10	.87	1.75	1.50	8.1	1.40	2.10	.78	1.34	1.60	1.14	1.79
15	1.13	1.94	2.40	11.5	1.95	2.50	1.23	1.91	3.04	1.30	2.64
20	1.83	3.26	3.79	17.6	2.95	3.85	2.24	2.68	4.82	2.27	3.88
25	2.43	3.62	3.81	21.6	3.67	4.37	2.78	2.98	5.00	2.70	4.46
30	2.79	3.96	4.06	25.1	3.98	4.48	3.05	3.19	4.78	3.01	4.39
35	3.41	4.42	4.48	29.3	4.48	5.02	3.56	3.64	5.14	3.48	4.81
40	4.02	5.14	5.22	34.5	5.12	5.69	4.56	4.40	6.08	4.22	5.43
45	5.23	7.06	6.49	39.0	6.15	6.89	5.74	5.84	7.50	5.46	6.20
50	7.44	9.40	8.69	43.1	8.04	9.50	7.82	8.16	9.77	7.91	8.20
55	10.19	13.15	12.42	47.5	11.62	13.49	11.82	11.74	13.38	11.53	11.36
60	14.66	19.91	18.81	54.3	17.14	20.51	17.59	17.70	19.26	17.46	17.47
65	23.65	32.39	29.69	66.6	26.03	33.08	27.70	27.55	29.86	28.53	28.40
70	38.02	51.22	48.12	88.8	40.57	54.30	44.22	44.51	48.13	47.61	46.53
75	62.29	85.97	78.97	130.1	67.35	84.42	75.87	74.14	78.75	80.33	79.61
80	101.06	131.56	129.67	206.6	107.69	131.28	119.78	118.58	127.93	126.51	127.02
85	158.37	202.12	210.38	347.6	160.86	192.59	187.34	179.42	200.02	193.66	191.19
90	233.91	279.42	332.04	566.7	228.60	263.29	255.64	250.61	284.63	273.64	267.86

SEX AND AGE	Japan, 1926–1930	Mexico, 1930	New Zealand, 1934–1938	Scotland, 1930–1932	Sweden, 1931–1935	Switzerland, 1933–1937	UNION OF SOUTH AFRICA		UNITED STATES		
							Whites, 1935–1937	Nonwhites, 1935–1937	Whites, 1939–1941	Whites, 1929–1931	Negroes, 1939–1941
MALE											
0	140.10	223.69	36.53	93.46	54.86	52.42	66.41	183.65	48.12	62.32	82.28
1	43.12	105.89	4.71	22.31	7.65	7.77	14.64	70.78	4.87	9.93	9.37
5	6.44	16.86	1.72	3.36	1.60	2.15	2.38	7.16	1.38	2.66	1.86
10	2.63	6.47	1.00	1.80	1.33	1.23	1.54	3.64	1.00	1.47	1.38
15	5.02	5.60	1.47	1.98	1.87	1.73	1.85	5.41	1.43	2.13	2.74
20	9.82	10.00	2.18	3.26	3.90	3.26	3.46	8.39	2.12	3.18	5.44
25	8.61	10.96	2.34	3.52	3.55	3.47	3.50	9.36	2.43	3.71	7.33
30	7.39	12.17	2.32	3.83	3.66	3.43	3.52	10.43	2.79	4.13	8.72
35	7.70	13.72	3.03	5.04	4.07	4.33	4.72	12.70	3.63	5.10	10.71
40	9.58	16.14	4.41	6.76	4.55	5.50	6.00	15.34	5.13	6.79	13.62
45	12.69	18.97	5.84	8.96	6.43	7.86	9.30	17.88	7.66	9.29	18.59
50	17.50	22.32	8.43	11.51	8.46	11.67	13.08	21.36	11.55	12.78	25.36
55	24.95	27.16	12.56	16.56	11.87	17.47	18.64	25.14	17.37	18.19	32.48
60	36.71	36.64	19.51	25.18	17.68	26.11	25.56	35.76	25.48	26.44	39.10
65	54.86	46.14	30.17	39.89	27.65	39.48	37.29	46.30	36.85	38.65	46.85
70	80.35	71.91	47.89	62.95	44.89	59.35	53.87	60.30	54.54	57.96	57.99
75	117.53	91.69	75.60	98.97	70.13	93.10	84.88	79.55	83.13	85.26	78.03
80	170.20	138.53	120.01	150.98	116.63	144.80	120.95	114.10	124.71	129.97	107.30
85	243.20	185.59	183.86	211.54	186.33	210.90	192.20	175.88	181.04	184.68	137.83
90	341.41	218.64	263.84	293.88	286.31	288.00	300.71	276.70	248.94	245.50	174.17
FEMALE											
0	124.14	196.75	28.70	73.04	41.82	40.83	53.48	163.00	37.89	49.63	65.84
1	42.10	108.70	4.25	20.65	6.74	6.35	14.02	70.62	4.32	8.79	7.96
5	7.09	17.12	1.40	3.16	1.54	1.82	2.19	7.71	1.10	2.20	1.75
10	3.00	6.46	.80	1.53	1.18	1.03	1.47	4.07	.70	1.13	1.04
15	7.32	5.24	.97	2.18	2.04	1.49	1.43	6.90	.96	1.64	3.07
20	10.59	9.24	1.62	2.93	3.37	2.56	2.33	10.50	1.45	2.77	5.32
25	9.64	10.12	2.10	3.33	3.47	3.08	2.87	12.13	1.82	3.39	6.27
30	8.94	11.34	2.44	3.92	3.34	3.16	3.17	11.98	2.20	3.74	7.33
35	9.26	12.40	2.73	4.64	3.87	3.45	4.18	12.59	2.78	4.33	9.24
40	10.05	13.06	3.58	5.51	4.14	4.14	4.98	14.80	3.68	5.32	11.81
45	10.17	15.28	4.94	7.06	5.60	5.49	6.71	14.12	5.23	7.02	16.02
50	12.62	19.72	6.69	9.45	7.41	7.86	9.24	18.62	7.62	9.59	21.87
55	16.86	24.27	9.62	13.58	10.23	11.60	12.49	22.35	11.28	13.75	28.58
60	24.16	37.49	14.86	20.21	15.48	18.04	17.98	32.16	17.14	20.63	34.72
65	37.08	50.20	23.94	30.15	24.30	29.78	28.52	40.54	26.43	31.25	40.90
70	57.67	79.05	39.33	48.66	39.14	48.79	42.41	55.37	42.33	48.66	49.12
75	88.92	97.00	64.55	79.71	66.92	79.43	67.25	70.51	68.89	74.60	62.94
80	138.54	144.72	107.95	125.82	107.88	126.30	108.29	99.20	108.19	117.42	81.27
85	213.72	194.25	160.60	190.63	173.66	193.70	167.58	153.49	162.94	170.86	105.29
90	322.69	256.61	234.75	265.00	260.74	268.00	248.43	243.86	231.41	231.51	141.32

Table L.—Number of Survivors Out of 100,000 Live Births, From Recent Life Tables for Selected Countries, by Sex for Selected Specific Ages

Sex and age	Australia, 1932–1934	Austria, 1930–1933	Belgium, 1928–1932	British India, 1921–1930	Canada, 1930–1932	Czechoslovakia, 1929–1932	Denmark, 1931–1935	England and Wales, 1930–1932	France, 1928–1933	Germany, 1932–1934	Italy, 1930–1932
MALE											
0	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000
1	95,457	88,460	89,925	75,126	90,003	85,131	91,853	92,814	90,982	91,465	88,468
5	93,887	85,933	87,094	60,161	87,681	81,934	90,322	90,069	88,164	89,654	82,846
10	93,193	84,732	86,090	56,467	86,931	80,706	89,758	89,023	87,200	88,793	81,738
15	92,609	84,039	85,395	54,112	86,253	79,965	89,280	88,360	86,447	88,244	80,936
20	91,797	82,847	84,077	51,203	85,144	78,662	88,423	87,245	84,900	87,298	79,669
25	90,711	81,209	82,378	47,787	83,713	76,897	87,272	85,824	82,691	86,032	78,014
30	89,566	79,507	80,682	43,931	82,308	75,203	86,119	84,416	80,470	84,715	76,317
35	88,248	77,555	78,797	39,461	80,899	73,360	84,910	82,885	77,963	83,234	74,486
40	86,539	75,247	76,004	34,563	79,212	71,158	83,472	80,935	74,988	81,481	72,396
45	84,276	72,273	73,920	29,439	77,071	68,400	81,613	78,357	71,348	79,285	69,944
50	81,061	68,454	70,477	24,348	74,229	64,877	78,999	74,794	66,861	76,322	66,884
55	76,504	62,337	65,896	19,476	70,221	60,217	75,191	70,041	61,291	72,147	62,942
60	69,950	56,757	59,699	14,933	64,772	54,227	69,804	63,620	54,391	66,293	57,683
65	61,292	48,275	51,390	10,773	57,564	46,418	62,177	54,899	45,800	58,106	50,606
70	50,086	37,834	40,724	7,036	47,662	36,605	51,610	43,361	35,436	47,059	41,175
75	36,588	25,909	28,224	3,848	35,125	25,148	38,135	29,605	23,768	33,479	29,299
80	22,223	14,103	15,745	1,514	21,512	13,847	23,064	16,199	12,496	19,122	16,707
85	9,752	5,406	6,181	316	9,865	5,444	10,346	6,377	4,527	7,732	6,813
90	2,935	1,227	1,374	17	2,833	1,356	2,966	1,609	965	1,966	1,732
FEMALE											
0	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000
1	96,358	90,755	92,145	76,766	91,642	87,543	93,692	94,545	92,838	93,161	89,775
5	94,993	88,321	89,616	62,817	89,017	84,400	92,419	92,024	90,205	91,535	84,107
10	94,424	87,148	88,690	59,369	88,220	83,126	91,913	91,082	89,245	90,753	83,019
15	93,991	86,452	87,966	56,757	87,590	82,316	91,523	90,420	88,416	90,270	82,227
20	93,341	85,375	86,635	52,833	86,564	81,031	90,741	89,383	86,727	89,490	80,908
25	92,364	83,918	84,991	47,932	85,154	79,383	89,705	88,133	84,585	88,390	79,223
30	91,174	82,381	83,347	42,675	83,542	77,669	88,405	86,792	82,545	87,139	77,478
35	89,823	80,738	81,606	37,266	81,840	75,873	87,003	85,353	80,563	85,754	75,754
40	88,175	78,841	79,684	31,778	79,919	73,886	85,293	83,690	78,381	84,135	73,860
45	86,256	76,620	77,445	26,409	77,756	71,668	83,220	81,660	75,851	82,211	71,777
50	83,680	73,054	74,667	21,464	75,127	68,907	80,537	78,958	72,728	79,620	69,332
55	80,172	69,796	71,001	17,065	71,685	65,214	76,908	75,290	68,809	76,038	66,164
60	75,565	64,471	65,933	13,210	66,840	60,174	71,806	70,204	63,687	70,984	61,803
65	69,089	57,053	58,780	9,761	60,304	53,023	64,609	63,046	56,747	63,712	55,510
70	59,629	46,620	48,857	6,627	51,382	43,050	54,401	53,144	47,194	53,184	46,455
75	46,977	33,399	36,002	3,841	39,697	30,671	40,594	40,040	34,821	39,132	34,323
80	31,539	19,416	21,570	1,631	25,748	17,947	24,876	24,869	20,962	23,500	20,517
85	16,425	8,114	9,055	367	12,865	7,721	11,283	11,594	9,086	10,323	9,017
90	5,808	2,164	2,063	22	4,256	2,225	3,484	3,611	2,430	2,868	2,579

Sex and age	Japan, 1926–1930	Mexico, 1930	New Zealand, 1934–1938	Scotland, 1930–1932	Sweden, 1931–1935	Switzerland, 1933–1937	Union of South Africa		United States		
							Whites, 1935–1937	Nonwhites, 1935–1937	Whites, 1939–1941	Whites, 1929–1931	Negroes, 1939–1941
MALE											
0	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000
1	85,990	77,631	96,347	90,654	94,514	94,758	93,359	81,635	95,188	93,768	91,772
5	78,457	61,485	95,212	87,038	93,055	93,112	90,765	72,210	94,150	91,738	90,082
10	76,786	58,131	94,576	85,885	92,324	92,314	89,879	70,378	93,601	90,810	89,393
15	75,703	56,471	94,069	85,102	91,660	91,725	89,180	68,942	93,089	90,074	88,610
20	72,845	54,413	93,217	84,069	90,477	90,627	88,106	66,702	92,293	88,904	86,968
25	69,466	51,660	92,156	82,641	88,840	89,082	86,515	63,764	91,241	87,371	84,227
30	66,721	48,774	91,084	81,165	87,278	87,586	85,029	60,723	90,092	85,707	80,979
35	64,284	45,724	89,954	79,468	85,718	85,948	83,382	57,387	88,713	83,812	77,221
40	61,693	42,472	88,365	77,216	83,936	83,936	81,223	53,549	86,880	81,457	72,780
45	58,460	38,895	86,174	74,339	81,802	81,292	78,369	49,309	84,285	78,345	67,346
50	54,349	35,031	83,328	70,698	78,956	77,614	74,226	44,759	80,521	74,288	60,495
55	49,051	30,940	79,243	66,165	75,179	72,390	68,780	39,881	75,156	68,981	52,426
60	42,283	26,573	73,472	59,877	70,044	65,213	61,763	34,471	67,787	61,933	43,833
65	33,814	21,590	65,232	51,322	62,975	55,710	53,099	28,086	58,305	52,964	35,371
70	24,306	16,615	54,184	40,035	53,076	43,811	42,516	21,564	46,739	41,880	27,236
75	14,813	10,879	40,151	26,966	40,346	30,258	30,402	15,205	33,404	29,471	19,456
80	7,080	6,352	24,845	14,343	25,536	16,666	18,043	9,396	19,800	17,221	12,186
85	2,352	2,660	11,393	5,467	11,946	6,548	8,216	4,518	9,013	7,572	6,444
90	454	800	3,395	1,366	3,477	1,647	2,177	1,365	2,812	2,356	2,836
FEMALE											
0	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000
1	87,586	80,325	97,130	92,696	95,818	95,917	94,652	83,700	96,211	95,037	93,416
5	79,866	63,131	96,227	89,249	94,505	94,512	92,210	73,801	95,309	93,216	91,906
10	78,053	59,672	95,700	88,217	93,892	93,828	91,355	71,798	94,890	92,466	91,308
15	76,523	58,062	95,311	87,560	93,265	93,309	90,722	70,040	94,534	91,894	90,594
20	73,069	56,081	94,740	86,380	92,069	92,419	89,939	67,153	93,984	90,939	88,736
25	69,366	53,459	93,865	85,054	90,497	91,097	88,778	63,430	93,228	89,524	86,198
30	66,215	50,702	92,825	83,561	88,944	89,708	87,456	59,661	92,320	87,972	83,384
35	63,287	47,748	91,637	81,814	87,404	88,255	85,934	56,133	91,211	86,248	80,092
40	60,312	44,679	90,281	79,802	85,715	86,640	84,005	52,479	89,805	84,256	76,084
45	57,345	41,634	88,437	77,418	83,702	84,657	81,690	48,714	87,920	81,780	71,157
50	54,285	38,303	85,991	74,384	81,106	82,005	78,605	45,083	85,267	78,572	64,885
55	50,534	34,351	82,718	70,418	77,706	78,283	74,577	40,691	81,520	74,321	57,314
60	45,819	30,019	78,073	64,951	73,117	72,991	69,342	35,734	76,200	68,462	48,928
65	39,593	24,260	71,253	57,553	66,647	65,237	62,127	29,783	68,701	60,499	40,504
70	31,544	18,178	61,352	47,782	57,326	54,086	52,314	23,565	58,363	49,932	32,354
75	22,099	11,280	47,851	35,058	44,532	39,708	40,224	17,156	44,685	37,024	24,502
80	12,538	6,314	31,560	21,066	28,923	23,861	26,193	11,281	28,882	23,053	17,039
85	5,044	2,480	15,754	9,336	14,099	10,495	13,015	6,005	14,487	10,937	10,622
90	1,159	665	5,605	2,684	4,375	2,945	4,311	2,139	5,061	3,719	5,652

TABLE M.—AVERAGE FUTURE LIFETIME IN YEARS, FROM RECENT LIFE TABLES FOR SELECTED COUNTRIES, BY SEX FOR SELECTED SPECIFIC AGES

SEX AND AGE	Australia, 1932–1934	Austria, 1930–1933	Belgium, 1928–1932	British India, 1921–1930	Canada, 1930–1932	Czechoslovakia, 1929–1932	Denmark, 1931–1935	England and Wales, 1930–1932	France, 1928–1933	Germany, 1932–1934	Italy, 1930–1932
MALE											
0	63.48	54.5	56.02	26.91	58.46	51.92	62.0	58.74	54.30	59.86	53.76
1	65.49	60.5	61.25	34.68	63.96	59.90	66.5	62.25	58.63	64.43	59.71
5	62.57	58.3	59.21	38.96	62.30	58.19	63.6	60.11	56.47	61.70	59.68
10	58.02	54.1	54.88	36.38	57.96	54.04	59.0	55.79	52.06	57.28	55.46
15	53.36	49.5	50.29	32.85	53.41	49.52	54.3	51.19	47.50	52.62	50.98
20	48.81	45.2	46.04	29.57	49.05	45.29	49.8	46.81	43.30	48.16	46.75
25	44.37	41.0	41.95	26.50	44.83	41.27	45.4	42.54	39.40	43.83	42.69
30	39.90	36.9	37.78	23.60	40.55	37.15	41.0	38.21	35.42	39.47	38.58
35	35.46	32.7	33.61	20.99	36.23	33.02	36.5	33.87	31.47	35.13	34.47
40	31.11	28.7	29.48	18.60	31.98	28.96	32.1	29.62	27.62	30.83	30.39
45	26.87	24.7	25.47	16.40	27.79	25.02	27.8	25.51	23.90	26.61	26.37
50	22.83	21.0	21.61	14.31	23.72	21.24	23.6	21.60	20.33	22.54	22.45
55	19.03	17.4	17.04	12.27	19.88	17.68	19.7	17.89	16.93	18.69	18.70
60	15.57	14.2	14.53	10.25	16.29	14.35	16.0	14.43	13.76	15.11	15.16
65	12.40	11.2	11.43	8.26	12.98	11.32	12.6	11.30	10.86	11.87	11.92
70	9.60	8.6	8.69	6.35	10.06	8.67	9.7	8.62	8.29	9.05	9.05
75	7.19	6.3	6.41	4.61	7.57	6.46	7.2	6.43	6.11	6.68	6.68
80	5.22	4.6	4.65	3.13	5.61	4.73	5.2	4.74	4.44	4.84	4.85
85	3.90	3.3	3.35	1.95	4.10	3.48	3.8	3.50	3.23	3.52	3.52
90	2.99	2.4	2.43	1.12	2.97	2.61	2.8	2.63	2.65	2.63	2.59
FEMALE											
0	67.14	58.5	59.79	26.56	60.23	55.18	63.8	62.88	59.02	62.81	56.00
1	68.67	63.5	63.84	33.48	64.72	61.96	67.1	65.48	62.53	66.41	61.32
5	65.64	61.2	61.63	36.61	63.17	60.21	64.0	63.24	60.32	63.56	61.37
10	61.02	57.0	57.25	33.61	58.72	56.10	59.4	58.87	55.95	59.09	57.15
15	56.29	52.4	52.68	30.04	54.15	51.63	54.6	54.28	51.45	54.39	52.67
20	51.67	48.0	48.43	27.08	49.76	47.40	50.0	49.88	47.40	49.84	48.49
25	47.19	43.8	44.33	24.58	45.54	43.33	45.6	45.55	43.52	45.43	44.47
30	42.77	39.6	40.17	22.30	41.38	39.24	41.2	41.22	39.54	41.05	40.41
35	38.37	35.3	35.97	20.18	37.19	35.10	36.8	36.87	35.45	36.67	36.27
40	34.04	31.1	31.77	18.23	33.02	30.98	32.5	32.55	31.37	32.33	32.14
45	29.74	27.0	27.62	16.43	28.87	26.86	28.3	28.30	27.33	28.02	28.00
50	25.58	22.9	23.55	14.65	24.79	22.83	24.1	24.18	23.39	23.85	23.89
55	21.58	19.1	19.64	12.79	20.84	18.98	20.1	20.23	19.57	19.85	19.91
60	17.74	15.4	15.93	10.81	17.15	15.35	16.4	16.50	15.94	16.07	16.13
65	14.15	12.1	12.57	8.76	13.72	12.06	12.9	13.07	12.57	12.60	12.66
70	10.98	9.2	9.60	6.74	10.63	9.24	9.9	10.02	9.58	9.58	9.61
75	8.23	6.8	7.12	4.86	7.98	6.95	7.3	7.45	7.07	7.09	7.09
80	6.01	5.0	5.20	3.25	5.92	5.12	5.4	5.46	5.09	5.15	5.18
85	4.30	3.6	3.76	2.00	4.38	3.80	4.0	4.00	3.64	3.70	3.78
90	3.05	2.7	2.86	1.18	3.24	2.87	3.0	2.98	2.75	2.72	2.82

SEX AND AGE	Japan, 1926–1930	Mexico, 1930	New Zealand, 1934–1938	Scotland, 1930–1932	Sweden, 1931–1935	Switzerland, 1933–1937	UNION OF SOUTH AFRICA		UNITED STATES		
							Whites, 1935–1937	Nonwhites, 1935–1937	Whites, 1939–1941	Whites, 1929–1931	Negroes, 1939–1941
MALE											
0	44.82	32.44	65.46	56.0	63.22	60.7	58.95	40.18	62.81	59.12	52.26
1	51.07	40.64	66.92	60.7	65.88	63.0	62.12	48.14	64.98	62.04	55.93
5	51.85	46.97	63.70	59.2	62.89	60.1	59.86	50.27	61.68	59.38	52.95
10	47.93	44.57	59.11	54.9	58.37	55.6	55.43	46.53	57.03	54.96	48.34
15	43.58	40.80	54.42	50.4	53.77	50.9	50.84	42.44	52.33	50.39	43.74
20	40.18	37.25	49.89	46.0	49.44	46.5	46.43	38.78	47.76	46.02	39.52
25	37.01	34.10	45.43	41.7	45.31	42.3	42.24	35.45	43.28	41.78	35.72
30	33.43	30.97	40.94	37.4	41.07	38.0	37.93	32.10	38.80	37.54	32.05
35	29.61	27.88	36.42	33.2	36.78	33.6	33.63	28.81	34.36	33.33	28.48
40	25.74	24.84	32.03	29.1	32.50	29.4	29.45	25.69	30.03	29.22	25.06
45	22.02	21.89	27.78	25.1	28.28	25.2	25.43	22.69	25.87	25.28	21.88
50	18.49	19.05	23.64	21.3	24.21	21.3	21.70	19.74	21.96	21.51	19.06
55	15.21	16.23	19.72	17.5	20.29	17.7	18.21	16.84	18.34	17.97	16.60
60	12.23	13.50	16.06	14.1	16.59	14.3	14.97	14.08	15.05	14.72	14.37
65	9.64	11.05	12.76	11.0	13.15	11.3	11.99	11.71	12.07	11.77	12.21
70	7.43	8.66	9.82	8.4	10.12	8.7	9.34	9.49	9.42	9.20	10.11
75	5.61	6.90	7.36	6.3	7.49	6.5	7.05	7.42	7.17	7.02	8.17
80	4.15	5.17	5.35	4.6	5.37	4.7	5.20	5.50	5.38	5.26	6.58
85	3.02	3.98	3.86	3.5	3.76	3.5	3.57	3.84	4.02	3.99	5.34
90	2.17	3.69	2.79	2.5	2.60	2.6	2.40	2.56	3.06	3.03	4.23
FEMALE											
0	46.54	34.07	68.45	59.5	65.33	64.6	63.06	40.86	67.29	62.67	55.56
1	52.10	41.30	69.46	63.1	67.17	66.4	65.60	47.74	68.93	64.93	58.46
5	53.00	48.19	66.10	61.5	64.09	63.3	63.30	49.99	65.57	62.17	55.40
10	49.18	45.87	61.45	57.2	59.49	58.8	58.87	46.33	60.85	57.65	50.75
15	45.11	42.07	56.69	52.7	54.87	54.1	54.27	42.42	56.07	53.00	46.13
20	42.12	38.46	52.02	48.3	50.55	49.6	49.72	39.13	51.38	48.52	42.04
25	39.23	35.23	47.48	44.0	46.38	45.3	45.33	36.28	46.78	44.25	38.20
30	35.98	32.01	42.98	39.8	42.15	40.9	40.98	33.41	42.21	39.99	34.40
35	32.53	28.84	38.51	35.6	37.85	36.6	36.66	30.36	37.70	35.73	30.71
40	29.01	25.66	34.05	31.4	33.54	32.2	32.44	27.29	33.25	31.52	27.19
45	25.39	22.35	29.70	27.3	29.29	27.9	28.29	24.21	28.90	27.39	23.89
50	21.67	19.09	25.47	23.3	25.14	23.7	24.30	20.96	24.72	23.41	20.95
55	18.09	15.98	21.38	19.4	21.13	19.7	20.47	17.95	20.73	19.60	18.38
60	14.68	12.92	17.49	15.9	17.29	16.0	16.82	15.07	17.00	16.05	16.10
65	11.58	10.38	13.91	12.6	13.71	12.5	13.46	12.58	13.56	12.81	13.93
70	8.88	8.06	10.73	9.6	10.51	9.6	10.50	10.23	10.50	9.98	11.82
75	6.59	6.46	8.02	7.1	7.78	7.1	7.88	8.13	7.92	7.56	9.81
80	4.73	4.76	5.85	5.2	5.62	5.2	5.75	6.08	5.88	5.63	8.02
85	3.30	3.43	4.30	3.8	3.97	3.8	4.13	4.29	4.34	4.24	6.41
90	2.24	2.81	3.00	2.8	2.84	2.9	2.93	2.88	3.24	3.17	4.96

FIGURE 9.—NUMBER OF SURVIVORS OUT OF 100,000 LIVE BIRTHS, FROM RECENT LIFE TABLES FOR SELECTED COUNTRIES

I. MALES

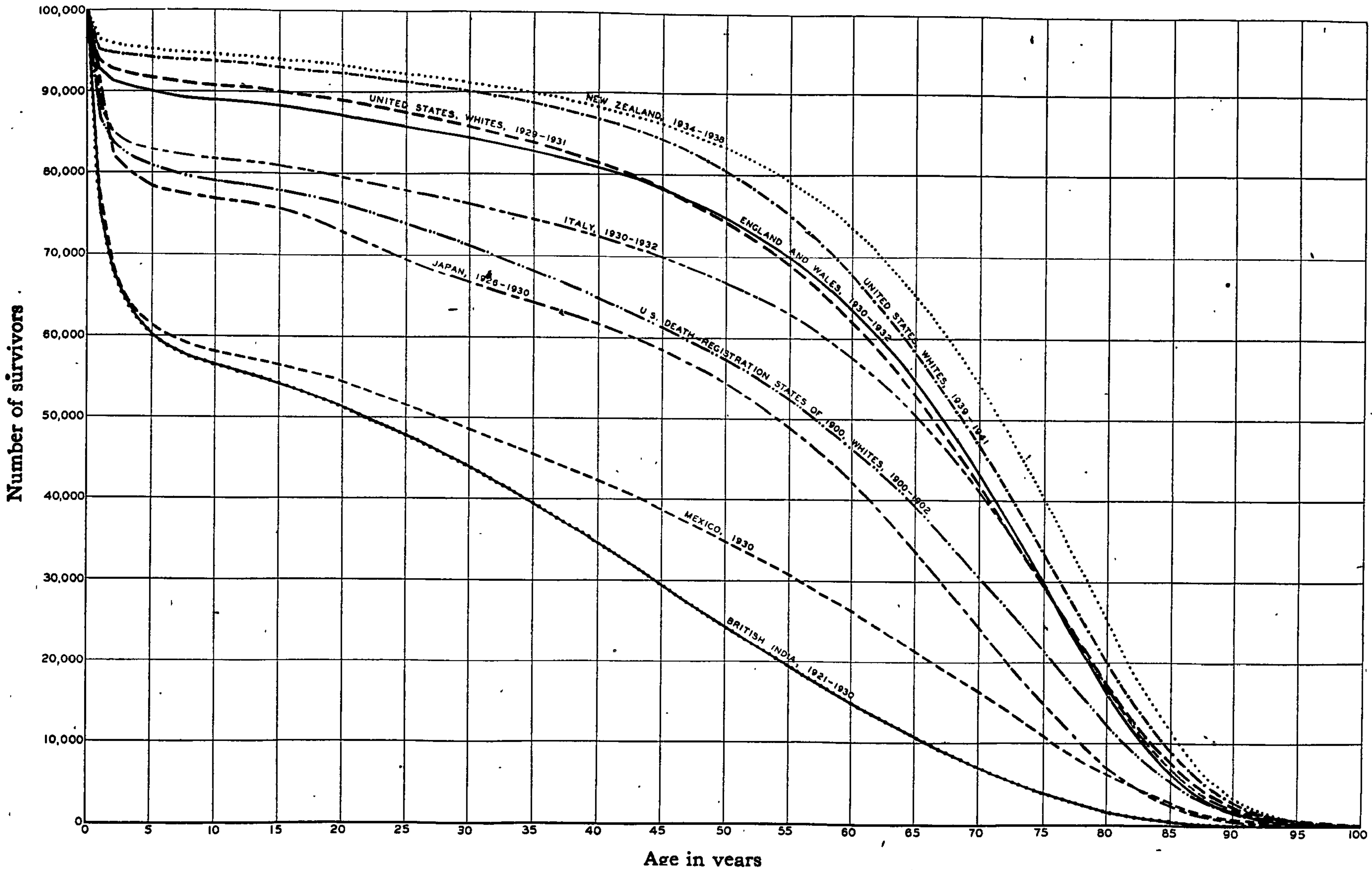
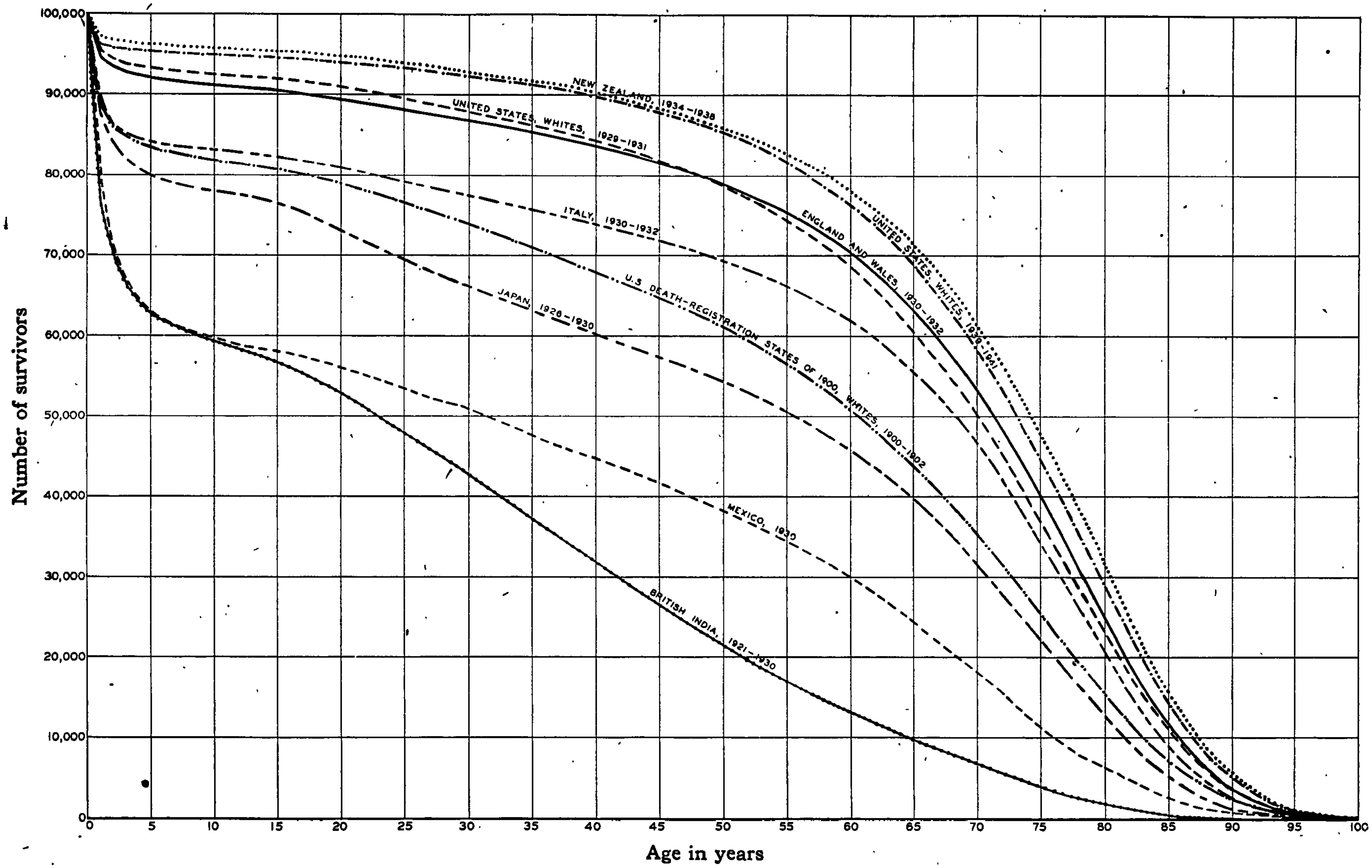


FIGURE 10.—NUMBER OF SURVIVORS OUT OF 100,000 LIVE BIRTHS, FROM RECENT LIFE TABLES FOR SELECTED COUNTRIES

II. FEMALES



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PART II

LIFE TABLES

This part contains the principal life tables presented in this volume. Life tables are given for whites, Negroes, and other races, separately by sex, and for both sexes combined, and also for the total population and for total males and total females. This makes altogether 12 life tables. In addition, table 13 gives, for the same 12 classes and combinations of classes, life table values for certain subdivisions of the first year of life. All these tables are based on the 1940 census of population and the deaths of the 3-year period 1939–1941.

Explanation of the columns of the life table

Both the descriptive titles and the conventional actuarial symbols appear at the head of the columns in each of the tables. The description which follows gives a more detailed explanation of each column of the life table, and may be helpful to some readers.

Column 1—Year of age (x *to* $x+1$).—The year of age, shown in column 1, is the interval between two successive birthdays. For instance, "4–5" indicates the interval between the fourth birthday and the fifth, in other words, the fifth year of life.

Column 2—Mortality rate (1,000 q_x).—This column shows the number of deaths within 1 year after the birthday indicated, among 1,000 persons alive on that birthday. For example, the rate of mortality at age 45 for white males (table 5) is 7.66 per 1,000. In other words, during 1939–1941, 7.66 out of every 1,000 white males who were alive on their forty-fifth birthday died before reaching age 46. The rates of mortality form the basis of the life table, all the other columns being derived from them.

Column 3—Number living (l_x).—This column shows the number of persons who would survive to each age out of a cohort of 100,000 live births, subject throughout life to the rates of mortality shown in column 2. Thus, table 5 shows that out of 100,000 white male babies born alive, 95,188 will complete the first year of life and enter the second; 94,724 will begin the third year; 92,098 will reach age 21; and 33,404 will live to age 75.

Column 4—Number dying (d_x).—This column shows the number dying in each successive year of age out of 100,000 live births. Out of 100,000 white males born alive (table 5), 4,812 die in the first year of life, 464 in the second year, 195 in the twenty-first year, and 2,762 in the seventy-fifth year. Each figure in column 4 is the difference between two successive figures in column 3.

Columns 5 and 6—Stationary population (L_x *and* T_x).—Suppose that a group of 100,000 individuals like that assumed in columns 3 and 4 is born every year, each such group being subject throughout life to the rates of mortality shown in column 2. If there were no migration and if the births were evenly distributed over the calendar year, the survivors of these births would make up what is called a stationary population because in such a population the number of persons living in any given age group would never change. When an individual left the group, either by death or by growing older and entering the next higher age group, his place would immediately be taken by someone entering from the next lower age group. Thus, a census taken at any time in such a stationary community would always show the same total population and the same numerical distribution of that population among the various ages. In such a stationary population, column 3 shows the number of persons who, each year, reach the birthday indicated in column 1 while column 4 shows the number who die each year in the indicated age interval.

Column 5 shows the number of persons in the stationary population in the indicated age interval. For example, the figure given for white males in the year of life 45–46 is 83,962. This means that in a stationary population of white males supported by 100,000 annual births and subject always to the rates of mortality shown in column 2, a census taken on any date would show 83,962 persons between 45 and 46 years old.

Column 6 shows the total number of persons in the stationary population (column 5) in the indicated age interval and all subsequent age intervals. For example, in the stationary population of white males referred to in the last illustration, column 6 shows that there would be at any given moment a total of 2,180,567 persons who have passed their forty-fifth birthday. The population at all ages 0 and above (in other words, the total population of the stationary community) would be 6,281,188.

Column 7—Average future lifetime ($\mathring{e}_x$).—The average future lifetime (also called the complete expectation of life) at any age is the average number of years remaining to be lived by those surviving to that age, on the basis of a given set of mortality rates. The values in column 5 can also be interpreted in terms of a single life table cohort, without introducing the concept of the stationary population. From this point of view, each figure in column 5 represents the total time (in years) lived between the indicated birthdays by those reaching the earlier birthday among the survivors of a cohort of 100,000 live births. Thus, the figure 83,962 for white males in the year of life 45–46 is the total number of years that will be lived between the forty-fifth and forty-sixth birthdays by the 84,285 (column 3) who reach their forty-fifth birthday out of 100,000 white males born alive. The corresponding figure in column 6 (2,180,567) is the total number of years that will be lived after attaining age 45 by the 84,285 reaching that age. This number of years divided by the number of persons (2,180,567 divided by 84,285) gives 25.87 as the average future lifetime of white males at age 45.

Care must be exercised in drawing conclusions from the figures in column 7. Thus, observing that the "expectation of life" at birth is always greater for white persons than for Negroes, one should not conclude that the oldest ages reached by white persons necessarily exceed those attained by the most long-lived Negroes. The difference in the average length of life is due to the fact that a greater proportion of Negroes die before reaching old age. For example, the number surviving to age 65 out of 100,000 born alive is far greater among whites than among Negroes; yet the average length of life remaining at age 65 is practically the same for both races.

Table 13—Subdivisions of the first year of life.—What has been said about the various columns of the life table applies also, with certain obvious modifications, to the life table values for subdivisions of the first year of life, given in table 13. The figures corresponding to age "2–3 weeks" for white males may be taken as an illustration. The age interval (column 1) is the period beginning with the exact age 2 weeks and extending up to the exact age 3 weeks: in other words, the third week of life. The mortality rate of 1.64 in column 2 means that out of every 1,000 white male infants alive exactly 2 weeks after birth during 1939–1941 this number, on the average, died during the following week. The number living (97,194 in column 3) signifies that this many would still be alive exactly 2 weeks after birth out of the life table cohort of 100,000 live births, on the assumption that the mortality rates shown in column 2 have prevailed during the first 2 weeks of life. The number dying (159 in column 4) means that out of the 97,194 alive exactly 2 weeks after birth this number would die during the following week. The figure 1,861 in column 5 indicates that during the third week of life the survivors of the life table cohort of 100,000 white male births have lived a total of 1,861 person-years of life. Or, alternatively, this figure is the number of infants aged 2–3 weeks in a stationary population of white males supported by 100,000 annual births and subject always to the mortality rates shown for white males in column 2 of this table and of table 5. The figure 6,277,446 in column 6 represents the total number of person-years of life lived beyond the first 2 weeks of life by all the 97,194 survivors to the age of exactly 2 weeks in the life table cohort which started with 100,000 white male births. Alternatively, it is the entire population at all ages beyond 2 weeks in the stationary population already referred to. Finally, the average future lifetime of 64.59 shown in column 7 is the average number of years lived beyond the first 2 weeks of life by the 97,194 survivors to the age of exactly 2 weeks in the life table cohort.

Use of life tables in estimating and forecasting populations

One of the most important applications of life tables in demographic research is their use in estimating the age distribution of a population on a given postcensal date. In particular cases, this may be either a past, present, or future date. While an exhaustive discussion of the subject would be beyond the scope of this volume,[1] an outline of the general procedure will be given. Basically this consists, in the usual method of population projection, in multiplying the number enumerated at each age in the census by a survival rate derived from a life table, in order to obtain the estimated number of survivors on the given date. It is usually most appropriate to obtain the survival rates from the L_x column of the life table (column 5 of the tables on pp. 26 to 49). For example, suppose that in a certain group of white males there were enumerated, in the 1940 census, 32,000 at age 47 on the last birthday, and that it is desired to estimate the number of survivors just 6 years later (that is, on April 1, 1946), on the supposition that the mortality during the 6-year period will be approximately the same as that indicated at the ages in question by the 1939–1941 life table for white males. Now the original group of 32,000 presumably included persons at all ages between exact age 47 and exact age 48, and was, therefore, similar in its age composition to the group at age 47 on the last birthday in the stationary life table population, which numbered 82,568. Now, since the hypothetical life table population does not change with the passage of time either in its total number or in its age composition, the survivors 6 years later of this group of 82,568 would be merely the number at age 53 in the life table population, which is 76,953. Therefore, the survival rate to be applied to the group of 32,000 is 76,953 divided by 82,568, which is .93200; and the estimated number of survivors is 32,000 multiplied by .93200, which gives 29,824. In algebraic terms, the L_{x+6} persons aged $x+6$ in 1946 are the survivors of the L_x persons aged x in 1940. Therefore, the survival rate to be applied to the population at age x is L_{x+6}/L_x.

If migration during the 6 years is thought to have been a significant factor, it is of course necessary to

[1] For a detailed discussion of the subject, see *Estimates of Future Population of the United States, 1940–2000* (prepared by Warren S. Thompson and P. K. Whelpton, and issued by the National Resources Planning Board), Government Printing Office, Washington, D. C., 1943.

obtain some information or to make some assumption as to the number and age composition of the net migrants each year, and to adjust the number of survivors accordingly.

Estimation of the populations at ages under 6 on April 1, 1946, would require a knowledge of the number of births during each of the 6 years. For example, suppose 51,000 white males entered the group through birth during the year April 1, 1943, to April 1, 1944. On April 1, 1946, the survivors of these births would be between exact ages 2 and 3. Now, in the life table population, the number of births during any year is the radix [2] of the life table—in this case, 100,000—while the number of survivors on a date just 2 years after the end of the year in which the births occurred would be merely the number at age 2 in the life table population (or 94,592 in column 5). Therefore, the survival rate to be applied to the 51,000 births is 94,592 divided by 100,000 which is .94592; and the estimated number of survivors is 51,000 multiplied by .94592, which gives 48,242. In algebraic terms, the survival rate to be applied to the births of the nth year preceding the date of the estimate is L_{n-1}/l_o.

In the original example of the 32,000 enumerated at age 47, suppose it had been desired to estimate the number of survivors 6 months later, on October 1, 1946. These individuals would then be at ages ranging from exact age 53½ to exact age 54½. Now the number of persons between these ages in the life table population is approximately l_{54} (column 3): that is, the number of survivors to age 54 out of the life table cohort of 100,000 live births, as indicated in column 3. In this particular case, the figure is 76,380. Therefore, the survival rate to be applied is l_{54}/L_{47}, or 76,380 divided by 82,568, which is .92506; and the estimated number of survivors is 32,000 multiplied by .92506, which gives 29,602.

If the population data are given in 5-year age groups, or can be combined into such groups, it is possible to shorten the arithmetic with very little loss of accuracy by using an average survival rate for each 5-year age group as a whole. Thus, the survival rate over a 6-year period for the age group x to $x+4$ would be $(T_{x+6}-T_{x+11})\div(T_x-T_{x+5})$. Other situations which may arise can be dealt with along similar lines.

The life table as a frequency distribution

The ages at death in the hypothetical life table cohort (as shown in column 4 of the life tables on pp. 26 to 49) constitute a frequency distribution. In the following discussion, the case of the life table for white males (table 5) will be taken as an illustration, but the remarks to be made will apply equally to all the life tables, except for some difference in the ages and numerical values quoted. The frequency distribution based on the white males life table is exhibited graphically in figure 11. Perhaps the most obvious characteristic of this distribution is that it is bimodal: that is, it has two modes or maxima, one in the year of age 0–1 and another in the year of age 75–76. The mode at age 0–1 is the higher, more deaths occurring in this than in any other single year of age. It is also clear that the frequency distribution is decidedly skewed toward the left: that is, the frequencies rise very gradually from the "trough" at age 10 to the "peak" at age 75, and then drop off sharply above age 75. The arithmetic mean of the distribution is the average age at death in the hypothetical cohort, or in other words, the average duration of life. Its value in this case is 62.81 years (column 7 of the life table). It is clear that the value of the arithmetic mean is very much influenced by the large number of deaths in the first year of life. If the deaths occurring in the first year were excluded from the distribution, the average age at death of the remaining 95,188 individuals would be one plus the average future lifetime at age 1: that is, 65.98 years. This represents a difference of more than 3 years in the value.

The median of the distribution (that is, the value which has the same number of elements on either side of it) is the median length of life, or probable lifetime, another possible measure of longevity to which reference was made in part I.[3] Since the distribution of ages at death in a life table cohort is always characterized by a greater dispersion below the median value than above it, the median always exceeds the arithmetic mean. In the particular case under consideration, the median is 68.67 years, which exceeds the mean value by 2.69 years.

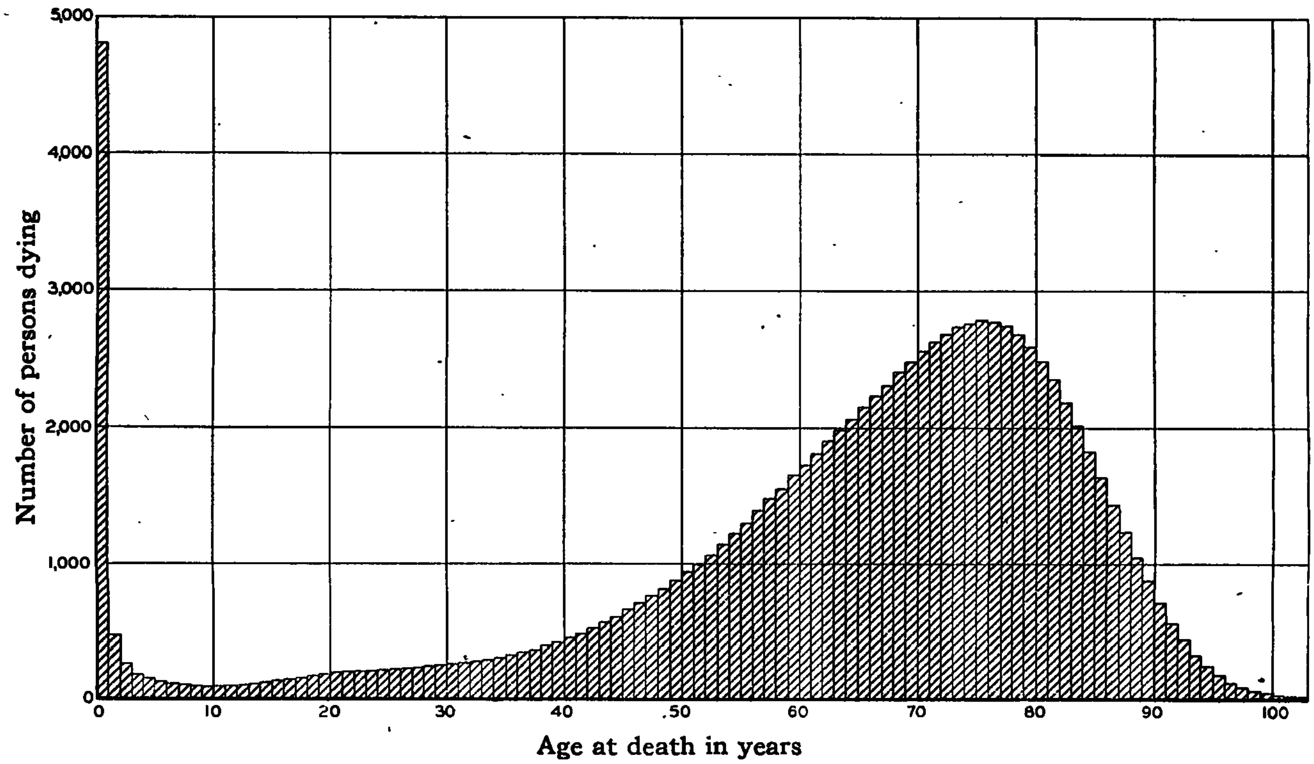
In part I the longevity of different subdivisions of the population was compared also by means of a third criterion, the number of persons surviving to specified ages in the hypothetical cohort. Which of these is the better measure of longevity is a question that cannot be answered categorically. The answer perhaps depends primarily on the purpose such a measure is intended to serve. Certainly no one figure can contain within itself all the information which is provided by the complete frequency distribution.

In view of the pronounced skewness of the distribution, it may be felt that the arithmetic average is not sufficiently representative. The layman, in inquiring what is the "life expectancy" of a newborn infant, probably has in the back of his mind the idea of an age to which the infant has a reasonably good chance of surviving. If he is told that the infant's "expectation of life" is 62.81 years, he may be surprised to be told later that more than 62 percent of white male infants alive at birth outlive their expectation of life while less than 38 percent die before reaching that age. The alternative statement that 68.67 years is the probable lifetime, the age to which the infant has a fifty-fifty chance of surviving, is probably a more satisfactory answer to the layman's question.

[2] The *radix* of a life table is the number of births with which the life table cohort begins, or, in algebraic terms, the value of l_o. In the tables on pp. 26 to 49, this is shown in column 3 opposite the year of age 0–1, and is always 100,000.

[3] See p. 3.

FIGURE 11.—FREQUENCY DISTRIBUTION OF AGES AT DEATH IN A COHORT STARTING WITH 100,000 LIVE BIRTHS, BASED ON THE MORTALITY OF WHITE MALES: UNITED STATES, 1939–1941



On the other hand, the objection may be made that the probable lifetime is not sufficiently sensitive to changes in the ages at death of the members of the life table cohort. In fact, its value is not affected by any change in which the age at death of an individual is not actually shifted from one side to the other of the probable lifetime itself. If, for example, the deaths of the 4,812 dying before age 1 in the white males life table were equally spread over all the years of age between birth and age 68, many of these individuals would live much longer; yet the value of the probable lifetime would be unchanged. However, the effect of transferring deaths from one age to another in the hypothetical life table cohort is not entirely relevant, since the mortality rates in the life table were not obtained by observing a single cohort over a period of time, but rather by observing many cohorts simultaneously, a different one at each age. Therefore, the important thing is the effect of a specified change in the rate of mortality at a particular age, without reference to any offsetting change elsewhere. Any change in the mortality rate at any age less than the probable lifetime (unaccompanied by other changes) will alter the value of the probable lifetime. However, changes in mortality rates at ages greater than the probable lifetime will have no effect whatever on its value. Similar remarks apply to the third criterion suggested, the number of survivors to a designated age. The value of the average duration of life, on the contrary, is affected in some measure by any change in the rate of mortality at any age, or in the ages at death in the life table cohort.

Use of the life table in studying there productive capacity of populations

Another important application of life tables in demographic research is their use in conjunction with fertility rates in investigating the inherent capacity of a population to reproduce itself. This is studied, for the most part, by means of certain specific measures devised for that purpose, the most important of which are the gross and net reproduction rates [4] and the true rate of natural increase.[5] While life table survival rates are an important component in the calculation of these measures, they involve other considerations of a highly technical nature, which are outside the scope of this volume.

Mathematical notation employed

One of the mathematical symbols used in the headings of table 13 represents a departure from the standard notation in use by actuaries. This is the symbol

[4] See Robert R. Kuczynski, *The Balance of Births and Deaths*, 2 vols., The Macmillan Co., New York, 1928; *Fertility and Reproduction*, Falcon Press, New York, 1932; *The Measurement of Population Growth*, Oxford University Press, New York, 1936; D. V. Glass, *Population Policies and Movements in Europe* (Appendix), Oxford University Press, London, 1940.

[5] See Louis I. Dublin and Alfred J. Lotka, *Length of Life*, The Ronald Press Co., New York, 1936; *On the True Rate of Natural Increase*, Journal of the American Statistical Association, vol. 20, No. 151, pp. 305–339, September 1925; Alfred J. Lotka, *The Geographic Distribution of Intrinsic Natural Increase in the United States, and an Examination of the Relation Between Several Measures of Net Reproductivity*, *ibid.*, vol. 31, No. 194, pp. 273–294, June 1936; *Some Recent Results in Population Analysis*, *ibid.*, vol. 33, No. 201, pp. 164–178, March 1938. See also Glass' book cited in the preceding footnote.

$_tq_x$, which appears in the heading of column 2 and which is used here to denote the probability that an individual alive at exact age x will die within time t thereafter, both x and t being measured in years. The standard actuarial symbol for this probability is $_{|t}q_x$ when t is 1 year or less and $_{|t}Q_x$ when t is greater than 1 year. The latter notation has been conceded by actuaries to be awkward and unnecessary.[6] Moreover, a subcommittee designated by the Permanent Committee of the International Congresses of Actuaries to study the revision of the international actuarial notation has gone on record recommending the replacement of the two symbols just mentioned by the one employed here.[7] The latter symbol has also been widely used, even by actuaries, on the continent of Europe,[8] and has also appeared in several publications in this country.[9]

Consistency of the tables

Consistency requires that the rates of mortality in the life tables for combinations of classes shall always be intermediate between the rates at the same ages for the component classes. This is true in every case, notwithstanding the fact that the interpolation [10] of the rates of mortality for the combination tables was carried out entirely independently of the corresponding interpolation for the separate classes, except above age 92, where the rates of mortality for separate classes were extrapolated from the data for earlier ages, and those for the various combinations were obtained by a special process in order to insure consistency.

Such consistency as regards the rates of mortality does not, however, guarantee the same kind of consistency in the values of the other life table functions. This would follow if the rates of mortality were obtained by observing a fixed cohort of persons from birth until death, but does not hold when the persons under observation at different ages belong to distinct cohorts, sometimes differing greatly in their race and sex composition. Under these conditions, in fact, such apparent inconsistencies are to be expected, and are not properly regarded as inconsistencies at all. In the life tables in this volume, such situations are few in number and are largely concentrated at the old ages and in the life tables for "other races," and in all these cases the numerical magnitude of the differences involved is small. It may be remarked that such situations have arisen in earlier life tables. For example, in Glover's life table for total males in 1910, the mortality rate is, at every age, intermediate between the corresponding rates for white males and Negro males.[11] Nevertheless, the values of l_x at ages 96–98 and d_x at age 55 for total males exceed the corresponding values for both white males and Negro males.[12]

[6] See *Notation Internationale*, pamphlet issued by the Comité Permanent des Congrès Internationaux d'Actuaires, p. 5, Bruxelles, Février 1939.

[7] *Op. cit.*, p. 91.

[8] *Op. cit.*, p. 62.

[9] See, for example, American Journal of Hygiene, vol. 30, No. 2, p. 35 et seq., September 1939; Record, American Institute of Actuaries, vol. 32, Part I, No. 65, p. 29 et seq., June 1943.

[10] For a detailed technical description of the process of interpolation, see pp. 122-126.

[11] U. S. Bureau of the Census, *United States Life Tables, 1890, 1901, 1910, and 1901–1910*, pp. 58–59, 68–69, 80–81, Government Printing Office, Washington, D. C., 1921.

[12] While it is true that the total males include a small number of males of "other races," this group constituted only 0.16 of 1 percent of the deaths of 1909–1911 at all ages and only 0.17 of 1 percent of the total estimated population, so that this is not likely to be the explanation of the peculiarity noted.

TABLE 1.—LIFE TABLE FOR THE TOTAL POPULATION OF THE UNITED STATES: 1939–1941

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+1$	$1,000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
0–1	47.10	100,000	4,710	96,058	6,362,494	63.62
1–2	5.21	95,290	496	94,997	6,266,436	65.76
2–3	2.67	94,794	254	94,660	6,171,439	65.10
3–4	1.88	94,540	177	94,448	6,076,779	64.28
4–5	1.51	94,363	143	94,288	5,982,331	63.40
5–6	1.32	94,220	125	94,157	5,888,043	62.49
6–7	1.17	94,095	110	94,041	5,793,886	61.57
7–8	1.05	93,985	98	93,936	5,699,845	60.65
8–9	.96	93,887	91	93,841	5,605,909	59.71
9–10	.91	93,796	86	93,754	5,512,068	58.77
10–11	.90	93,710	84	93,668	5,418,314	57.82
11–12	.92	93,626	86	93,583	5,324,646	56.87
12–13	.97	93,540	91	93,495	5,231,063	55.92
13–14	1.07	93,449	100	93,399	5,137,568	54.98
14–15	1.22	93,349	114	93,292	5,044,169	54.04
15–16	1.39	93,235	130	93,170	4,950,877	53.10
16–17	1.57	93,105	146	93,031	4,857,707	52.17
17–18	1.73	92,959	162	92,878	4,764,676	51.26
18–19	1.88	92,797	174	92,711	4,671,798	50.34
19–20	2.03	92,623	188	92,529	4,579,087	49.44
20–21	2.17	92,435	201	92,334	4,486,558	48.54
21–22	2.30	92,234	212	92,128	4,394,224	47.64
22–23	2.42	92,022	223	91,911	4,302,096	46.75
23–24	2.50	91,799	229	91,684	4,210,185	45.86
24–25	2.56	91,570	235	91,452	4,118,501	44.98
25–26	2.62	91,335	239	91,216	4,027,049	44.09
26–27	2.67	91,096	243	90,974	3,935,833	43.21
27–28	2.75	90,853	250	90,728	3,844,859	42.32
28–29	2.85	90,603	258	90,473	3,754,131	41.44
29–30	2.95	90,345	267	90,212	3,663,658	40.55
30–31	3.07	90,078	276	89,939	3,573,446	39.67
31–32	3.20	89,802	288	89,658	3,483,507	38.79
32–33	3.35	89,514	299	89,365	3,393,849	37.91
33–34	3.51	89,215	313	89,058	3,304,484	37.04
34–35	3.69	88,902	329	88,737	3,215,426	36.17
35–36	3.90	88,573	345	88,401	3,126,689	35.30
36–37	4.12	88,228	363	88,047	3,038,288	34.44
37–38	4.36	87,865	383	87,674	2,950,241	33.58
38–39	4.62	87,482	404	87,279	2,862,567	32.72
39–40	4.91	87,078	428	86,864	2,775,288	31.87
40–41	5.24	86,650	454	86,423	2,688,424	31.03
41–42	5.59	86,196	482	85,955	2,602,001	30.19
42–43	5.99	85,714	513	85,458	2,516,046	29.35
43–44	6.43	85,201	548	84,927	2,430,588	28.53
44–45	6.91	84,653	584	84,361	2,345,661	27.71
45–46	7.44	84,069	626	83,756	2,261,300	26.90
46–47	8.01	83,443	668	83,109	2,177,544	26.10
47–48	8.62	82,775	714	82,418	2,094,435	25.30
48–49	9.28	82,061	761	81,680	2,012,017	24.52
49–50	9.99	81,300	813	80,894	1,930,337	23.74
50–51	10.76	80,487	866	80,054	1,849,443	22.98
51–52	11.59	79,621	923	79,160	1,769,389	22.22
52–53	12.49	78,698	982	78,206	1,690,229	21.48
53–54	13.46	77,716	1,047	77,193	1,612,023	20.74
54–55	14.51	76,669	1,112	76,113	1,534,830	20.02

TABLE 1.—LIFE TABLE FOR THE TOTAL POPULATION OF THE UNITED STATES: 1939–1941—Continued

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+1$	$1{,}000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
55–56	15. 64	75, 557	1, 182	74, 966	1, 458, 717	19. 31
56–57	16. 84	74, 375	1, 252	73, 750	1, 383, 751	18. 60
57–58	18. 12	73, 123	1, 325	72, 460	1, 310, 001	17. 92
58–59	19. 49	71, 798	1, 400	71, 098	1, 237, 541	17. 24
59–60	20. 95	70, 398	1, 474	69, 661	1, 166, 443	16. 57
60–61	22. 51	68, 924	1, 552	68, 148	1, 096, 782	15. 91
61–62	24. 19	67, 372	1, 630	66, 557	1, 028, 634	15. 27
62–63	26. 01	65, 742	1, 710	64, 887	962, 077	14. 63
63–64	27. 97	64, 032	1, 791	63, 137	897, 190	14. 01
64–65	30. 12	62, 241	1, 875	61, 304	834, 053	13. 40
65–66	32. 48	60, 366	1, 960	59, 386	772, 749	12. 80
66–67	35. 09	58, 406	2, 050	57, 381	713, 363	12. 21
67–68	37. 98	56, 356	2, 140	55, 286	655, 982	11. 64
68–69	41. 20	54, 216	2, 234	53, 099	600, 696	11. 08
69–70	44. 77	51, 982	2, 327	50, 818	547, 597	10. 53
70–71	48. 73	49, 655	2, 420	48, 445	496, 779	10. 00
71–72	53. 12	47, 235	2, 509	45, 981	448, 334	9. 49
72–73	57. 98	44, 726	2, 593	43, 430	402, 353	9. 00
73–74	63. 33	42, 133	2, 668	40, 799	358, 923	8. 52
74–75	69. 18	39, 465	2, 730	38, 100	318, 124	8. 06
75–76	75. 54	36, 735	2, 775	35, 347	280, 024	7. 62
76–77	82. 39	33, 960	2, 798	32, 561	244, 677	7. 20
77–78	89. 75	31, 162	2, 797	29, 763	212, 116	6. 81
78–79	97. 61	28, 365	2, 769	26, 981	182, 353	6. 43
79–80	105. 99	25, 596	2, 713	24, 240	155, 372	6. 07
80–81	114. 91	22, 883	2, 629	21, 568	131, 132	5. 73
81–82	124. 38	20, 254	2, 519	18, 995	109, 564	5. 41
82–83	134. 44	17, 735	2, 385	16, 542	90, 569	5. 11
83–84	145. 08	15, 350	2, 226	14, 237	74, 027	4. 82
84–85	156. 25	13, 124	2, 051	12, 099	59, 790	4. 56
85–86	167. 88	11, 073	1, 859	10, 143	47, 691	4. 31
86–87	179. 92	9, 214	1, 658	8, 385	37, 548	4. 08
87–88	192. 29	7, 556	1, 453	6, 830	29, 163	3. 86
88–89	204. 93	6, 103	1, 250	5, 478	22, 333	3. 66
89–90	217. 79	4, 853	1, 057	4, 324	16, 855	3. 47
90–91	230. 81	3, 796	876	3, 358	12, 531	3. 30
91–92	243. 94	2, 920	713	2, 563	9, 173	3. 14
92–93	257. 11	2, 207	567	1, 924	6, 610	2. 99
93–94	270. 31	1, 640	443	1, 418	4, 686	2. 86
94–95	283. 44	1, 197	340	1, 027	3, 268	2. 73
95–96	296. 46	857	254	730	2, 241	2. 61
96–97	309. 35	603	186	510	1, 511	2. 50
97–98	322. 10	417	135	350	1, 001	2. 40
98–99	334. 75	282	94	235	651	2. 31
99–100	347. 36	188	65	155	416	2. 21
100–101	360. 05	123	45	101	261	2. 13
101–102	372. 98	78	29	64	160	2. 04
102–103	386. 34	49	19	39	96	1. 96
103–104	400. 36	30	12	24	57	1. 88
104–105	415. 25	18	7	15	33	1. 80
105–106	431. 17	11	5	8	18	1. 72
106–107	448. 20	6	3	5	10	1. 64
107–108	466. 33	3	1	2	5	1. 56
108–109	485. 39	2	1	2	3	1. 48
109–110	505. 10	1	1	1	1	1. 41

NOTE.—Rates of mortality at ages above 87 are not based on actual statistics at these ages, but have been obtained by mathematical extrapolation from mortality rates at younger ages. Other life table functions at these ages are based on the extrapolated rates of mortality, and may not necessarily represent actual conditions.

TABLE 2.—LIFE TABLE FOR TOTAL MALES IN THE UNITED STATES: 1939–1941

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+1$	$1,000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
0–1	52. 38	100, 000	5, 238	95, 591	6, 160, 087	61. 60
1–2	5. 53	94, 762	524	94, 453	6, 064, 496	64. 00
2–3	2. 89	94, 238	273	94, 093	5, 970, 043	63. 35
3–4	2. 01	93, 965	189	93, 867	5, 875, 950	62. 53
4–5	1. 62	93, 776	152	93, 697	5, 782, 083	61. 66
5–6	1. 45	93, 624	136	93, 556	5, 688, 386	60. 76
6–7	1. 30	93, 488	121	93, 428	5, 594, 830	59. 85
7–8	1. 19	93, 367	111	93, 312	5, 501, 402	58. 92
8–9	1. 11	93, 256	103	93, 204	5, 408, 090	57. 99
9–10	1. 06	93, 153	99	93, 103	5, 314, 886	57. 06
10–11	1. 05	93, 054	98	93, 005	5, 221, 783	56. 12
11–12	1. 07	92, 956	100	92, 906	5, 128, 778	55. 17
12–13	1. 13	92, 856	105	92, 804	5, 035, 872	54. 23
13–14	1. 24	92, 751	114	92, 694	4, 943, 068	53. 29
14–15	1. 39	92, 637	129	92, 572	4, 850, 374	52. 36
15–16	1. 57	92, 508	146	92, 435	4, 757, 802	51. 43
16–17	1. 76	92, 362	163	92, 281	4, 665, 367	50. 51
17–18	1. 94	92, 199	179	92, 110	4, 573, 086	49. 60
18–19	2. 11	92, 020	194	91, 923	4, 480, 976	48. 70
19–20	2. 28	91, 826	209	91, 721	4, 389, 053	47. 80
20–21	2. 46	91, 617	225	91, 504	4, 297, 332	46. 91
21–22	2. 61	91, 392	239	91, 273	4, 205, 828	46. 02
22–23	2. 74	91, 153	250	91, 028	4, 114, 555	45. 14
23–24	2. 83	90, 903	257	90, 774	4, 023, 527	44. 26
24–25	2. 88	90, 646	261	90, 516	3, 932, 753	43. 39
25–26	2. 92	90, 385	264	90, 253	3, 842, 237	42. 51
26–27	2. 97	90, 121	267	89, 988	3, 751, 984	41. 63
27–28	3. 04	89, 854	273	89, 717	3, 661, 996	40. 75
28–29	3. 14	89, 581	281	89, 440	3, 572, 279	39. 88
29–30	3. 25	89, 300	291	89, 155	3, 482, 839	39. 00
30–31	3. 38	89, 009	300	88, 859	3, 393, 684	38. 13
31–32	3. 52	88, 709	312	88, 553	3, 304, 825	37. 25
32–33	3. 69	88, 397	326	88, 233	3, 216, 272	36. 38
33–34	3. 88	88, 071	341	87, 900	3, 128, 039	35. 52
34–35	4. 09	87, 730	359	87, 551	3, 040, 139	34. 65
35–36	4. 33	87, 371	378	87, 182	2, 952, 588	33. 79
36–37	4. 59	86, 993	399	86, 793	2, 865, 406	32. 94
37–38	4. 88	86, 594	423	86, 382	2, 778, 613	32. 09
38–39	5. 20	86, 171	449	85, 946	2, 692, 231	31. 24
39–40	5. 56	85, 722	476	85, 484	2, 606, 285	30. 40
40–41	5. 95	85, 246	507	84, 993	2, 520, 801	29. 57
41–42	6. 39	84, 739	542	84, 467	2, 435, 808	28. 74
42–43	6. 87	84, 197	578	83, 909	2, 351, 341	27. 93
43–44	7. 40	83, 619	619	83, 309	2, 267, 432	27. 12
44–45	7. 99	83, 000	664	82, 668	2, 184, 123	26. 31
45–46	8. 63	82, 336	710	81, 981	2, 101, 455	25. 52
46–47	9. 32	81, 626	761	81, 245	2, 019, 474	24. 74
47–48	10. 06	80, 865	814	80, 458	1, 938, 229	23. 97
48–49	10. 86	80, 051	869	79, 617	1, 857, 771	23. 21
49–50	11. 72	79, 182	928	78, 718	1, 778, 154	22. 46
50–51	12. 64	78, 254	989	77, 759	1, 699, 436	21. 72
51–52	13. 64	77, 265	1, 054	76, 738	1, 621, 677	20. 99
52–53	14. 72	76, 211	1, 122	75, 650	1, 544, 939	20. 27
53–54	15. 90	75, 089	1, 194	74, 492	1, 469, 289	19. 57
54–55	17. 16	73, 895	1, 268	73, 261	1, 394, 797	18. 88

TABLE 2.—LIFE TABLE FOR TOTAL MALES IN THE UNITED STATES: 1939–1941—Continued

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+1$	$1,000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
55–56	18. 50	72, 627	1, 344	71, 955	1, 321, 536	18. 20
56–57	19. 93	71, 283	1, 420	70, 573	1, 249, 581	17. 53
57–58	21. 44	69, 863	1, 498	69, 114	1, 179, 008	16. 88
58–59	23. 02	68, 365	1, 574	67, 579	1, 109, 894	16. 23
59–60	24. 69	66, 791	1, 649	65, 966	1, 042, 315	15. 61
60–61	26. 47	65, 142	1, 724	64, 280	976, 349	14. 99
61–62	28. 37	63, 418	1, 800	62, 518	912, 069	14. 38
62–63	30. 41	61, 618	1, 873	60, 682	849, 551	13. 79
63–64	32. 60	59, 745	1, 948	58, 771	788, 869	13. 20
64–65	34. 97	57, 797	2, 021	56, 787	730, 098	12. 63
65–66	37. 55	55, 776	2, 094	54, 729	673, 311	12. 07
66–67	40. 37	53, 682	2, 167	52, 599	618, 582	11. 52
67–68	43. 47	51, 515	2, 239	50, 395	565, 983	10. 99
68–69	46. 87	49, 276	2, 310	48, 121	515, 588	10. 46
69–70	50. 62	46, 966	2, 378	45, 777	467, 467	9. 95
70–71	54. 77	44, 588	2, 442	43, 367	421, 690	9. 46
71–72	59. 36	42, 146	2, 502	40, 895	378, 323	8. 98
72–73	64. 44	39, 644	2, 555	38, 367	337, 428	8. 51
73–74	70. 05	37, 089	2, 598	35, 791	299, 061	8. 06
74–75	76. 18	34, 491	2, 627	33, 177	263, 270	7. 63
75–76	82. 84	31, 864	2, 640	30, 544	230, 093	7. 22
76–77	90. 02	29, 224	2, 631	27, 908	199, 549	6. 83
77–78	97. 70	26, 593	2, 598	25, 295	171, 641	6. 45
78–79	105. 90	23, 995	2, 541	22, 724	146, 346	6. 10
79–80	114. 61	21, 454	2, 459	20, 225	123, 622	5. 76
80–81	123. 86	18, 995	2, 353	17, 818	103, 397	5. 44
81–82	133. 67	16, 642	2, 224	15, 530	85, 579	5. 14
82–83	144. 04	14, 418	2, 077	13, 380	70, 049	4. 86
83–84	154. 98	12, 341	1, 912	11, 384	56, 669	4. 59
84–85	166. 43	10, 429	1, 736	9, 561	45, 285	4. 34
85–86	178. 31	8, 693	1, 550	7, 918	35, 724	4. 11
86–87	190. 55	7, 143	1, 361	6, 463	27, 806	3. 89
87–88	203. 08	5, 782	1, 174	5, 194	21, 343	3. 69
88–89	215. 82	4, 608	995	4, 111	16, 149	3. 50
89–90	228. 71	3, 613	826	3, 200	12, 038	3. 33
90–91	241. 68	2, 787	674	2, 450	8, 838	3. 17
91–92	254. 68	2, 113	538	1, 844	6, 388	3. 02
92–93	267. 63	1, 575	421	1, 364	4, 544	2. 88
93–94	280. 66	1, 154	324	992	3, 180	2. 76
94–95	293. 62	830	244	708	2, 188	2. 64
95–96	306. 49	586	179	496	1, 480	2. 52
96–97	319. 29	407	130	342	984	2. 42
97–98	332. 09	277	92	231	642	2. 32
98–99	344. 97	185	64	153	411	2. 23
99–100	358. 06	121	43	99	258	2. 13
100–101	371. 53	78	29	63	159	2. 05
101–102	385. 57	49	19	40	96	1. 96
102–103	400. 33	30	12	24	56	1. 88
103–104	415. 94	18	7	14	32	1. 79
104–105	432. 43	11	5	8	18	1. 71
105–106	449. 65	6	3	5	10	1. 64
106–107	467. 23	3	1	2	5	1. 57
107–108	484. 46	2	1	2	3	1. 51
108–109	500. 29	1	1	1	1	1. 46

NOTE.—Rates of mortality at ages above 92 are not based on actual statistics at these ages, but have been obtained by mathematical extrapolation from mortality rates at younger ages. Other life table functions at these ages are based on the extrapolated rates of mortality, and may not necessarily represent actual conditions.

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TABLE 3.—LIFE TABLE FOR TOTAL FEMALES IN THE UNITED STATES: 1939–1941

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+1$	$1{,}000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
0–1	41. 52	100, 000	4, 152	96, 549	6, 588, 801	65. 89
1–2	4. 89	95, 848	469	95, 571	6, 492, 252	67. 73
2–3	2. 44	95, 379	232	95, 256	6, 396, 681	67. 07
3–4	1. 74	95, 147	166	95, 061	6, 301, 425	66. 23
4–5	1. 40	94, 981	133	94, 912	6, 206, 364	65. 34
5–6	1. 20	94, 848	114	94, 791	6, 111, 452	64. 43
6–7	1. 03	94, 734	97	94, 685	6, 016, 661	63. 51
7–8	. 90	94, 637	86	94, 594	5, 921, 976	62. 58
8–9	. 82	94, 551	77	94, 513	5, 827, 382	61. 63
9–10	. 76	94, 474	72	94, 438	5, 732, 869	60. 68
10–11	. 75	94, 402	70	94, 367	5, 638, 431	59. 73
11–12	. 76	94, 332	72	94, 296	5, 544, 064	58. 77
12–13	. 81	94, 260	76	94, 222	5, 449, 768	57. 82
13–14	. 90	94, 184	85	94, 141	5, 355, 546	56. 86
14–15	1. 04	94, 099	99	94, 049	5, 261, 405	55. 91
15–16	1. 21	94, 000	113	93, 944	5, 167, 356	54. 97
16–17	1. 38	93, 887	130	93, 822	5, 073, 412	54. 04
17–18	1. 53	93, 757	143	93, 686	4, 979, 590	53. 11
18–19	1. 65	93, 614	155	93, 536	4, 885, 904	52. 19
19–20	1. 78	93, 459	166	93, 377	4, 792, 368	51. 28
20–21	1. 90	93, 293	177	93, 204	4, 698, 991	50. 37
21–22	2. 01	93, 116	186	93, 024	4, 605, 787	49. 46
22–23	2. 11	92, 930	196	92, 831	4, 512, 763	48. 56
23–24	2. 19	92, 734	203	92, 633	4, 419, 932	47. 66
24–25	2. 26	92, 531	209	92, 427	4, 327, 299	46. 77
25–26	2. 32	92, 322	214	92, 214	4, 234, 872	45. 87
26–27	2. 39	92, 108	221	91, 998	4, 142, 658	44. 98
27–28	2. 47	91, 887	227	91, 774	4, 050, 660	44. 08
28–29	2. 57	91, 660	235	91, 542	3, 958, 886	43. 19
29–30	2. 66	91, 425	243	91, 304	3, 867, 344	42. 30
30–31	2. 77	91, 182	253	91, 055	3, 776, 040	41. 41
31–32	2. 89	90, 929	262	90, 798	3, 684, 985	40. 53
32–33	3. 01	90, 667	274	90, 530	3, 594, 187	39. 64
33–34	3. 15	90, 393	285	90, 251	3, 503, 657	38. 76
34–35	3. 31	90, 108	298	89, 959	3, 413, 406	37. 88
35–36	3. 47	89, 810	311	89, 655	3, 323, 447	37. 01
36–37	3. 65	89, 499	327	89, 335	3, 233, 792	36. 13
37–38	3. 84	89, 172	342	89, 001	3, 144, 457	35. 26
38–39	4. 05	88, 830	360	88, 650	3, 055, 456	34. 40
39–40	4. 27	88, 470	378	88, 281	2, 966, 806	33. 53
40–41	4. 52	88, 092	398	87, 893	2, 878, 525	32. 68
41–42	4. 79	87, 694	420	87, 484	2, 790, 632	31. 82
42–43	5. 10	87, 274	445	87, 052	2, 703, 148	30. 97
43–44	5. 43	86, 829	471	86, 593	2, 616, 096	30. 13
44–45	5. 80	86, 358	502	86, 107	2, 529, 503	29. 29
45–46	6. 21	85, 856	533	85, 590	2, 443, 396	28. 46
46–47	6. 65	85, 323	567	85, 040	2, 357, 806	27. 63
47–48	7. 12	84, 756	604	84, 454	2, 272, 766	26. 82
48–49	7. 63	84, 152	641	83, 831	2, 188, 312	26. 00
49–50	8. 17	83, 511	683	83, 169	2, 104, 481	25. 20
50–51	8. 76	82, 828	725	82, 466	2, 021, 312	24. 40
51–52	9. 40	82, 103	772	81, 717	1, 938, 846	23. 61
52–53	10. 09	81, 331	820	80, 921	1, 857, 129	22. 83
53–54	10. 85	80, 511	874	80, 074	1, 776, 208	22. 06
54–55	11. 67	79, 637	929	79, 173	1, 696, 134	21. 30

TABLE 3.—LIFE TABLE FOR TOTAL FEMALES IN THE UNITED STATES: 1939–1941—Continued

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+1$	$1{,}000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
55–56	12.57	78,708	989	78,213	1,616,961	20.54
56–57	13.54	77,719	1,052	77,193	1,538,748	19.80
57–58	14.60	76,667	1,120	76,107	1,461,555	19.06
58–59	15.75	75,547	1,190	74,952	1,385,448	18.34
59–60	17.00	74,357	1,264	73,726	1,310,496	17.62
60–61	18.37	73,093	1,342	72,421	1,236,770	16.92
61–62	19.85	71,751	1,425	71,039	1,164,349	16.23
62–63	21.47	70,326	1,510	69,571	1,093,310	15.55
63–64	23.24	68,816	1,599	68,016	1,023,739	14.88
64–65	25.19	67,217	1,694	66,370	955,723	14.22
65–66	27.36	65,523	1,792	64,627	889,353	13.57
66–67	29.78	63,731	1,899	62,782	824,726	12.94
67–68	32.50	61,832	2,009	60,828	761,944	12.32
68–69	35.54	59,823	2,127	58,759	701,116	11.72
69–70	38.95	57,696	2,247	56,573	642,357	11.13
70–71	42.74	55,449	2,370	54,264	585,784	10.56
71–72	46.96	53,079	2,493	51,833	531,520	10.01
72–73	51.63	50,586	2,612	49,280	479,687	9.48
73–74	56.79	47,974	2,724	46,612	430,407	8.97
74–75	62.43	45,250	2,825	43,838	383,795	8.48
75–76	68.56	42,425	2,909	40,971	339,957	8.01
76–77	75.19	39,516	2,971	38,031	298,986	7.57
77–78	82.33	36,545	3,009	35,041	260,955	7.14
78–79	89.97	33,536	3,017	32,027	225,914	6.74
79–80	98.14	30,519	2,995	29,022	193,887	6.35
80–81	106.87	27,524	2,942	26,053	164,865	5.99
81–82	116.18	24,582	2,856	23,154	138,812	5.65
82–83	126.09	21,726	2,739	20,357	115,658	5.32
83–84	136.62	18,987	2,594	17,690	95,301	5.02
84–85	147.72	16,393	2,421	15,182	77,611	4.73
85–86	159.32	13,972	2,226	12,859	62,429	4.47
86–87	171.38	11,746	2,013	10,739	49,570	4.22
87–88	183.83	9,733	1,790	8,838	38,831	3.99
88–89	196.61	7,943	1,561	7,163	29,993	3.78
89–90	209.67	6,382	1,338	5,712	22,830	3.58
90–91	222.96	5,044	1,125	4,482	17,118	3.39
91–92	236.44	3,919	927	3,456	12,636	3.22
92–93	250.05	2,992	748	2,618	9,180	3.07
93–94	263.53	2,244	591	1,948	6,562	2.92
94–95	276.92	1,653	458	1,424	4,614	2.79
95–96	290.19	1,195	347	1,022	3,190	2.67
96–97	303.27	848	257	720	2,168	2.56
97–98	316.13	591	187	497	1,448	2.45
98–99	328.79	404	133	338	951	2.35
99–100	341.27	271	92	225	613	2.26
100–101	353.68	179	63	147	388	2.17
101–102	366.19	116	43	94	241	2.09
102–103	379.03	73	28	60	147	2.00
103–104	392.49	45	17	36	87	1.92
104–105	406.91	28	12	22	51	1.83
105–106	422.58	16	7	13	29	1.75
106–107	439.78	9	4	8	16	1.67
107–108	458.69	5	2	4	8	1.58
108–109	479.41	3	2	2	4	1.49
109–110	501.93	1	0	1	2	1.41
110–111	526.10	1	1	1	1	1.33

NOTE.—Rates of mortality at ages above 87 are not based on actual statistics at these ages, but have been obtained by mathematical extrapolation from mortality rates younger ages. Other life table functions at these ages are based on the extrapolated rates of mortality, and may not necessarily represent actual conditions.

TABLE 4.—LIFE TABLE FOR TOTAL WHITES IN THE UNITED STATES: 1939–1941

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+1$,	$1{,}000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
0–1	43.15	100,000	4,315	96,354	6,492,419	64.92
1–2	4.60	95,685	440	95,425	6,396,065	66.84
2–3	2.43	95,245	231	95,123	6,300,640	66.15
3–4	1.76	95,014	167	94,927	6,205,517	65.31
4–5	1.41	94,847	134	94,777	6,110,590	64.43
5–6	1.24	94,713	118	94,654	6,015,813	63.52
6–7	1.10	94,595	104	94,543	5,921,159	62.59
7–8	1.00	94,491	94	94,444	5,826,616	61.66
8–9	.92	94,397	87	94,353	5,732,172	60.72
9–10	.87	94,310	82	94,269	5,637,819	59.78
10–11	.85	94,228	81	94,187	5,543,550	58.83
11–12	.86	94,147	81	94,107	5,449,363	57.88
12–13	.89	94,066	83	94,025	5,355,256	56.93
13–14	.96	93,983	91	93,937	5,261,231	55.98
14–15	1.07	93,892	100	93,842	5,167,294	55.03
15–16	1.20	93,792	113	93,735	5,073,452	54.09
16–17	1.33	93,679	124	93,617	4,979,717	53.16
17–18	1.45	93,555	136	93,487	4,886,100	52.23
18–19	1.56	93,419	146	93,347	4,792,613	51.30
19–20	1.67	93,273	156	93,195	4,699,266	50.38
20–21	1.78	93,117	166	93,034	4,606,071	49.47
21–22	1.88	92,951	175	92,864	4,513,037	48.55
22–23	1.97	92,776	182	92,685	4,420,173	47.64
23–24	2.03	92,594	189	92,499	4,327,488	46.74
24–25	2.08	92,405	192	92,310	4,234,989	45.83
25–26	2.12	92,213	195	92,115	4,142,679	44.92
26–27	2.16	92,018	199	91,919	4,050,564	44.02
27–28	2.23	91,819	204	91,717	3,958,645	43.11
28–29	2.30	91,615	212	91,509	3,866,928	42.21
29–30	2.39	91,403	218	91,294	3,775,419	41.31
30–31	2.49	91,185	228	91,071	3,684,125	40.40
31–32	2.60	90,957	236	90,839	3,593,054	39.50
32–33	2.73	90,721	248	90,597	3,502,215	38.60
33–34	2.87	90,473	259	90,343	3,411,618	37.71
34–35	3.03	90,214	273	90,077	3,321,275	36.82
35–36	3.20	89,941	288	89,797	3,231,198	35.93
36–37	3.40	89,653	305	89,500	3,141,401	35.04
37–38	3.61	89,348	322	89,187	3,051,901	34.16
38–39	3.85	89,026	343	88,855	2,962,714	33.28
39–40	4.11	88,683	365	88,501	2,873,859	32.41
40–41	4.41	88,318	389	88,123	2,785,358	31.54
41–42	4.74	87,929	416	87,721	2,697,235	30.68
42–43	5.11	87,513	447	87,289	2,609,514	29.82
43–44	5.52	87,066	480	86,826	2,522,225	28.97
44–45	5.97	86,586	517	86,327	2,435,399	28.13
45–46	6.46	86,069	557	85,791	2,349,072	27.29
46–47	7.00	85,512	598	85,213	2,263,281	26.47
47–48	7.59	84,914	645	84,591	2,178,068	25.65
48–49	8.22	84,269	693	83,923	2,093,477	24.84
49–50	8.90	83,576	743	83,204	2,009,554	24.04
50–51	9.64	82,833	799	82,434	1,926,350	23.26
51–52	10.45	82,034	857	81,605	1,843,916	22.48
52–53	11.32	81,177	919	80,717	1,762,311	21.71
53–54	12.28	80,258	985	79,766	1,681,594	20.95
54–55	13.31	79,273	1,055	78,745	1,601,828	20.21

TABLE 4.—LIFE TABLE FOR TOTAL WHITES IN THE UNITED STATES: 1939–1941—Continued

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+1$	$1,000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
55–56	14. 43	78, 218	1, 129	77, 653	1, 523, 083	19. 47
56–57	15. 63	77, 089	1, 205	76, 487	1, 445, 430	18. 75
57–58	16. 92	75, 884	1, 284	75, 242	1, 368, 943	18. 04
58–59	18. 31	74, 600	1, 365	73, 918	1, 293, 701	17. 34
59–60	19. 79	73, 235	1, 450	72, 510	1, 219, 783	16. 66
60–61	21. 40	71, 785	1, 536	71, 017	1, 147, 273	15. 98
61–62	23. 12	70, 249	1, 624	69, 437	1, 076, 256	15. 32
62–63	24. 99	68, 625	1, 715	67, 767	1, 006, 819	14. 67
63–64	27. 01	66, 910	1, 807	66, 007	939, 052	14. 03
64–65	29. 22	65, 103	1, 902	64, 151	873, 045	13. 41
65–66	31. 64	63, 201	2, 000	62, 201	808, 894	12. 80
66–67	34. 33	61, 201	2, 101	60, 150	746, 693	12. 20
67–68	37. 31	59, 100	2, 206	57, 997	686, 543	11. 62
68–69	40. 63	56, 894	2, 311	55, 739	628, 546	11. 05
69–70	44. 31	54, 583	2, 418	53, 374	572, 807	10. 49
70–71	48. 39	52, 165	2, 524	50, 903	519, 433	9. 96
71–72	52. 90	49, 641	2, 626	48, 328	468, 530	9. 44
72–73	57. 88	47, 015	2, 721	45, 654	420, 202	8. 94
73–74	63. 36	44, 294	2, 807	42, 890	374, 548	8. 46
74–75	69. 34	41, 487	2, 877	40, 049	331, 658	7. 99
75–76	75. 83	38, 610	2, 927	37, 146	291, 609	7. 55
76–77	82. 82	35, 683	2, 955	34, 206	254, 463	7. 13
77–78	90. 31	32, 728	2, 956	31, 249	220, 257	6. 73
78–79	98. 32	29, 772	2, 927	28, 309	189, 008	6. 35
79–80	106. 87	26, 845	2, 869	25, 410	160, 699	5. 99
80–81	115. 99	23, 976	2, 781	22, 585	135, 289	5. 64
81–82	125. 73	21, 195	2, 665	19, 863	112, 704	5. 32
82–83	136. 12	18, 530	2, 522	17, 268	92, 841	5. 01
83–84	147. 17	16, 008	2, 356	14, 830	75, 573	4. 72
84–85	158. 85	13, 652	2, 169	12, 568	60, 743	4. 45
85–86	171. 09	11, 483	1, 964	10, 500	48, 175	4. 20
86–87	183. 84	9, 519	1, 750	8, 644	37, 675	3. 96
87–88	197. 03	7, 769	1, 531	7, 003	29, 031	3. 74
88–89	210. 61	6, 238	1, 314	5, 581	22, 028	3. 53
89–90	224. 53	4, 924	1, 105	4, 372	16, 447	3. 34
90–91	238. 74	3, 819	912	3, 363	12, 075	3. 16
91–92	253. 20	2, 907	736	2, 539	8, 712	3. 00
92–93	267. 84	2, 171	582	1, 880	6, 173	2. 84
93–94	282. 74	1, 589	449	1, 364	4, 293	2. 70
94–95	297. 77	1, 140	339	971	2, 929	2. 57
95–96	312. 88	801	251	675	1, 958	2. 45
96–97	328. 03	550	180	460	1, 283	2. 33
97–98	343. 18	370	127	306	823	2. 23
98–99	358. 27	243	87	199	517	2. 13
99–100	373. 27	156	58	127	318	2. 04
100–101	388. 11	98	38	79	191	1. 95
101–102	402. 76	60	24	48	112	1. 88
102–103	417. 14	36	15	28	64	1. 81
103–104	431. 21	21	9	16	36	1. 74
104–105	444. 89	12	5	9	20	1. 68
105–106	458. 10	7	3	5	11	1. 62
106–107	470. 78	4	2	3	6	1. 57
107–108	482. 81	2	1	2	3	1. 53
108–109	494. 08	1	1	1	1	1. 48

NOTE.—Rates of mortality at ages above 92 are not based on actual statistics at these ages, but have been obtained by mathematical extrapolation from mortality rates at younger ages. Other life table functions at these ages are based on the extrapolated rates of mortality, and may not necessarily represent actual conditions.

TABLE 5.—LIFE TABLE FOR WHITE MALES IN THE UNITED STATES: 1939–1941

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+1$	$1,000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
0–1	48. 12	100, 000	4, 812	95, 913	6, 281, 188	62. 81
1–2	4. 87	95, 188	464	94, 914	6, 185, 275	64. 98
2–3	2. 65	94, 724	250	94, 592	6, 090, 361	64. 30
3–4	1. 90	94, 474	179	94, 381	5, 995, 769	63. 46
4–5	1. 53	94, 295	145	94, 219	5, 901, 388	62. 58
5–6	1. 38	94, 150	130	94, 085	5, 807, 169	61. 68
6–7	1. 24	94, 020	116	93, 962	5, 713, 084	60. 76
7–8	1. 14	93, 904	108	93, 850	5, 619, 122	59. 84
8–9	1. 06	93, 796	99	93, 747	5, 525, 272	58. 91
9–10	1. 02	93, 697	96	93, 649	5, 431, 525	57. 97
10–11	1. 00	93, 601	93	93, 554	5, 337, 876	57. 03
11–12	1. 01	93, 508	95	93, 460	5, 244, 322	56. 08
12–13	1. 06	93, 413	99	93, 364	5, 150, 862	55. 14
13–14	1. 14	93, 314	106	93, 261	5, 057, 498	54. 20
14–15	1. 27	93, 208	119	93, 148	4, 964, 237	53. 26
15–16	1. 43	93, 089	133	93, 023	4, 871, 089	52. 33
16–17	1. 58	92, 956	147	92, 882	4, 778, 066	51. 40
17–18	1. 72	92, 809	160	92, 729	4, 685, 184	50. 48
18–19	1. 86	92, 649	172	92, 563	4, 592, 455	49. 57
19–20	1. 99	92, 477	184	92, 385	4, 499, 892	48. 66
20–21	2. 12	92, 293	195	92, 195	4, 407, 507	47. 76
21–22	2. 23	92, 098	205	91, 996	4, 315, 312	46. 86
22–23	2. 32	91, 893	214	91, 785	4, 223, 316	45. 96
23–24	2. 38	91, 679	218	91, 571	4, 131, 531	45. 07
24–25	2. 41	91, 461	220	91, 351	4, 039, 960	44. 17
25–26	2. 43	91, 241	222	91, 130	3, 948, 609	43. 28
26–27	2. 45	91, 019	223	90, 908	3, 857, 479	42. 38
27–28	2. 51	90, 796	228	90, 682	3, 766, 571	41. 48
28–29	2. 59	90, 568	234	90, 451	3, 675, 889	40. 59
29–30	2. 68	90, 334	242	90, 212	3, 585, 438	39. 69
30–31	2. 79	90, 092	251	89, 967	3, 495, 226	38. 80
31–32	2. 91	89, 841	262	89, 709	3, 405, 259	37. 90
32–33	3. 06	89, 579	274	89, 443	3, 315, 550	37. 01
33–34	3. 23	89, 305	288	89, 161	3, 226, 107	36. 12
34–35	3. 42	89, 017	304	88, 865	3, 136, 946	35. 24
35–36	3. 63	88, 713	322	88, 552	3, 048, 081	34. 36
36–37	3. 87	88, 391	342	88, 220	2, 959, 529	33. 48
37–38	4. 14	88, 049	364	87, 867	2, 871, 309	32. 61
38–39	4. 43	87, 685	389	87, 490	2, 783, 442	31. 74
39–40	4. 76	87, 296	416	87, 088	2, 695, 952	30. 88
40–41	5. 13	86, 880	446	86, 657	2, 608, 864	30. 03
41–42	5. 54	86, 434	479	86, 195	2, 522, 207	29. 18
42–43	6. 00	85, 955	515	85, 698	2, 436, 012	28. 34
43–44	6. 50	85, 440	555	85, 162	2, 350, 314	27. 51
44–45	7. 06	84, 885	600	84, 585	2, 265, 152	26. 69
45–46	7. 66	84, 285	646	83, 962	2, 180, 567	25. 87
46–47	8. 33	83, 639	696	83, 292	2, 096, 605	25. 07
47–48	9. 04	82, 943	750	82, 568	2, 013, 313	24. 27
48–49	9. 81	82, 193	806	81, 790	1, 930, 745	23. 49
49–50	10. 64	81, 387	866	80, 954	1, 848, 955	22. 72
50–51	11. 55	80, 521	930	80, 056	1, 768, 001	21. 96
51–52	12. 53	79, 591	997	79, 092	1, 687, 945	21. 21
52–53	13. 60	78, 594	1, 069	78, 059	1, 608, 853	20. 47
53–54	14. 76	77, 525	1, 145	76, 953	1, 530, 794	19. 75
54–55	16. 02	76, 380	1, 224	75, 768	1, 453, 841	19. 03

TABLE 5.—LIFE TABLE FOR WHITE MALES IN THE UNITED STATES: 1939–1941—Continued

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+1$	$1,000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
55–56	17. 37	75, 156	1, 305	74, 504	1, 378, 073	18. 34
56–57	18. 81	73, 851	1, 390	73, 156	1, 303, 569	17. 65
57–58	20. 34	72, 461	1, 473	71, 724	1, 230, 413	16. 98
58–59	21. 95	70, 988	1, 558	70, 209	1, 158, 689	16. 32
59–60	23. 66	69, 430	1, 643	68, 609	1, 088, 480	15. 68
60–61	25. 48	67, 787	1, 727	66, 923	1, 019, 871	15. 05
61–62	27. 43	66, 060	1, 813	65, 153	952, 948	14. 43
62–63	29. 52	64, 247	1, 896	63, 299	887, 795	13. 82
63–64	31. 77	62, 351	1, 981	61, 361	824, 496	13. 22
64–65	34. 20	60, 370	2, 065	59, 337	763, 135	12. 64
65–66	36. 85	58, 305	2, 148	57, 232	703, 798	12. 07
66–67	39. 75	56, 157	2, 232	55, 041	646, 566	11. 51
67–68	42. 93	53, 925	2, 315	52, 767	591, 525	10. 97
68–69	46. 43	51, 610	2, 396	50, 412	538, 758	10. 44
69–70	50. 28	49, 214	2, 475	47, 976	488, 346	9. 92
70–71	54. 54	46, 739	2, 549	45, 465	440, 370	9. 42
71–72	59. 24	44, 190	2, 618	42, 881	394, 905	8. 94
72–73	64. 43	41, 572	2, 678	40, 233	352, 024	8. 47
73–74	70. 14	38, 894	2, 728	37, 530	311, 791	8. 02
74–75	76. 37	36, 166	2, 762	34, 784	274, 261	7. 58
75–76	83. 13	33, 404	2, 777	32, 016	239, 477	7. 17
76–77	90. 40	30, 627	2, 769	29, 243	207, 461	6. 77
77–78	98. 18	27, 858	2, 735	26, 490	178, 218	6. 40
78–79	106. 47	25, 123	2, 675	23, 786	151, 728	6. 04
79–80	115. 30	22, 448	2, 588	21, 155	127, 942	5. 70
80–81	124. 71	19, 860	2, 477	18, 621	106, 787	5. 38
81–82	134. 72	17, 383	2, 341	16, 213	88, 166	5. 07
82–83	145. 37	15, 042	2, 187	13, 948	71, 953	4. 78
83–84	156. 68	12, 855	2, 014	11, 848	58, 005	4. 51
84–85	168. 59	10, 841	1, 828	9, 927	46, 157	4. 26
85–86	181. 04	9, 013	1, 631	8, 198	36, 230	4. 02
86–87	193. 95	7, 382	1, 432	6, 665	28, 032	3. 80
87–88	207. 27	5, 950	1, 233	5, 334	21, 367	3. 59
88–89	220. 91	4, 717	1, 042	4, 195	16, 033	3. 40
89–90	234. 82	3, 675	863	3, 244	11, 838	3. 22
90–91	248. 94	2, 812	700	2, 461	8, 594	3. 06
91–92	263. 22	2, 112	556	1, 834	6, 133	2. 90
92–93	277. 60	1, 556	432	1, 340	4, 299	2. 76
93–94	292. 02	1, 124	328	960	2, 959	2. 63
94–95	306. 42	796	244	674	1, 999	2. 51
95–96	320. 76	552	177	464	1, 325	2. 40
96–97	334. 96	375	126	312	861	2. 30
97–98	348. 98	249	87	205	549	2. 20
98–99	362. 75	162	59	133	344	2. 12
99–100	376. 23	103	38	84	211	2. 04
100–101	389. 35	65	26	52	127	1. 96
101–102	402. 05	39	15	32	75	1. 90
102–103	414. 29	24	10	18	43	1. 84
103–104	425. 99	14	6	11	25	1. 78
104–105	437. 12	8	4	6	14	1. 73
105–106	447. 60	4	2	4	8	1. 68
106–107	457. 38	2	1	2	4	1. 64
107–108	466. 40	1	0	1	2	1. 61
108–109	474. 62	1	1	1	1	1. 57

NOTE.—Rates of mortality at ages above 92 are not based on actual statistics at these ages, but have been obtained by mathematical extrapolation from mortality rates at younger ages Other life table functions at these ages are based on the extrapolated rates of mortality, and may not necessarily represent actual conditions.

TABLE 6.—LIFE TABLE FOR WHITE FEMALES IN THE UNITED STATES: 1939–1941

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+1$	$1{,}000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
0–1	37.89	100,000	3,789	96,822	6,728,965	67.29
1–2	4.32	96,211	415	95,966	6,632,143	68.93
2–3	2.20	95,796	211	95,684	6,536,177	68.23
3–4	1.61	95,585	154	95,505	6,440,493	67.38
4–5	1.28	95,431	122	95,367	6,344,988	66.49
5–6	1.10	95,309	106	95,256	6,249,621	65.57
6–7	.96	95,203	91	95,158	6,154,365	64.64
7–8	.85	95,112	80	95,072	6,059,207	63.71
8–9	.77	95,032	74	94,995	5,964,135	62.76
9–10	.72	94,958	68	94,924	5,869,140	61.81
10–11	.70	94,890	66	94,857	5,774,216	60.85
11–12	.70	94,824	66	94,791	5,679,359	59.89
12–13	.72	94,758	69	94,723	5,584,568	58.94
13–14	.77	94,689	73	94,653	5,489,845	57.98
14–15	.86	94,616	82	94,575	5,395,192	57.02
15–16	.96	94,534	91	94,489	5,300,617	56.07
16–17	1.07	94,443	101	94,392	5,206,128	55.12
17–18	1.17	94,342	111	94,287	5,111,736	54.18
18–19	1.26	94,231	119	94,172	5,017,449	53.25
19–20	1.36	94,112	128	94,048	4,923,277	52.31
20–21	1.45	93,984	136	93,916	4,829,229	51.38
21–22	1.54	93,848	145	93,776	4,735,313	50.46
22–23	1.62	93,703	152	93,627	4,641,537	49.53
23–24	1.70	93,551	159	93,472	4,547,910	48.61
24–25	1.76	93,392	164	93,310	4,454,438	47.70
25–26	1.82	93,228	169	93,144	4,361,128	46.78
26–27	1.88	93,059	175	92,972	4,267,984	45.86
27–28	1.95	92,884	181	92,793	4,175,012	44.95
28–29	2.03	92,703	188	92,610	4,082,219	44.04
29–30	2.11	92,515	195	92,417	3,989,609	43.12
30–31	2.20	92,320	204	92,218	3,897,192	42.21
31–32	2.30	92,116	212	92,010	3,804,974	41.31
32–33	2.40	91,904	220	91,794	3,712,964	40.40
33–34	2.52	91,684	231	91,568	3,621,170	39.50
34–35	2.64	91,453	242	91,332	3,529,602	38.59
35–36	2.78	91,211	253	91,085	3,438,270	37.70
36–37	2.92	90,958	266	90,825	3,347,185	36.80
37–38	3.09	90,692	280	90,552	3,256,360	35.91
38–39	3.26	90,412	295	90,265	3,165,808	35.02
39–40	3.46	90,117	312	89,961	3,075,543	34.13
40–41	3.68	89,805	330	89,640	2,985,582	33.25
41–42	3.93	89,475	352	89,299	2,895,942	32.37
42–43	4.20	89,123	374	88,936	2,806,643	31.49
43–44	4.51	88,749	400	88,549	2,717,707	30.62
44–45	4.85	88,349	429	88,134	2,629,158	29.76
45–46	5.23	87,920	460	87,690	2,541,024	28.90
46–47	5.64	87,460	493	87,214	2,453,334	28.05
47–48	6.08	86,967	528	86,703	2,366,120	27.21
48–49	6.55	86,439	566	86,156	2,279,417	26.37
49–50	7.06	85,873	606	85,570	2,193,261	25.54
50–51	7.62	85,267	650	84,942	2,107,691	24.72
51–52	8.22	84,617	695	84,269	2,022,749	23.90
52–53	8.88	83,922	746	83,549	1,938,480	23.10
53–54	9.61	83,176	799	82,777	1,854,931	22.30
54–55	10.40	82,377	857	81,948	1,772,154	21.51

TABLE 6.—LIFE TABLE FOR WHITE FEMALES IN THE UNITED STATES: 1939–1941—Continued

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+1$	$1,000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
55–56	11. 28	81, 520	919	81, 060	1, 690, 206	20. 73
56–57	12. 24	80, 601	987	80, 108	1, 609, 146	19. 96
57–58	13. 30	79, 614	1, 059	79, 084	1, 529, 038	19. 21
58–59	14. 46	78, 555	1, 136	77, 987	1, 449, 954	18. 46
59–60	15. 74	77, 419	1, 219	76, 809	1, 371, 967	17. 72
60–61	17. 14	76, 200	1, 306	75, 547	1, 295, 158	17. 00
61–62	18. 67	74, 894	1, 399	74, 195	1, 219, 611	16. 28
62–63	20. 35	73, 495	1, 495	72, 748	1, 145, 416	15. 58
63–64	22. 17	72, 000	1, 596	71, 202	1, 072, 668	14. 90
64–65	24. 19	70, 404	1, 703	69, 552	1, 001, 466	14. 22
65–66	26. 43	68, 701	1, 816	67, 793	931, 914	13. 56
66–67	28. 93	66, 885	1, 935	65, 918	864, 121	12. 92
67–68	31. 74	64, 950	2, 061	63, 920	798, 203	12. 29
68–69	34. 89	62, 889	2, 194	61, 791	734, 283	11. 68
69–70	38. 41	60, 695	2, 332	59, 529	672, 492	11. 08
70–71	42. 33	58, 363	2, 470	57, 128	612, 963	10. 50
71–72	46. 69	55, 893	2, 610	54, 588	555, 835	9. 94
72–73	51. 50	53, 283	2, 744	51, 911	501, 247	9. 41
73–74	56. 80	50, 539	2, 870	49, 104	449, 336	8. 89
74–75	62. 59	47, 669	2, 984	46, 177	400, 232	8. 40
75–76	68. 89	44, 685	3, 078	43, 146	354, 055	7. 92
76–77	75. 69	41, 607	3, 149	40, 032	310, 909	7. 47
77–78	83. 00	38, 458	3, 192	36, 862	270, 877	7. 04
78–79	90. 83	35, 266	3, 203	33, 664	234, 015	6. 64
79–80	99. 21	32, 063	3, 181	30, 472	200, 351	6. 25
80–81	108. 19	28, 882	3, 125	27, 320	169, 879	5. 88
81–82	117. 80	25, 757	3, 034	24, 240	142, 559	5. 53
82–83	128. 09	22, 723	2, 911	21, 267	118, 319	5. 21
83–84	139. 06	19, 812	2, 755	18, 435	97, 052	4. 90
84–85	150. 70	17, 057	2, 570	15, 772	78, 617	4. 61
85–86	162. 94	14, 487	2, 361	13, 306	62, 845	4. 34
86–87	175. 73	12, 126	2, 131	11, 061	49, 539	4. 09
87–88	189. 02	9, 995	1, 889	9, 051	38, 478	3. 85
88–89	202. 76	8, 106	1, 644	7, 284	29, 427	3. 63
89–90	216. 90	6, 462	1, 401	5, 762	22, 143	3. 43
90–91	231. 41	5, 061	1, 171	4, 475	16, 381	3. 24
91–92	246. 24	3, 890	958	3, 411	11, 906	3. 06
92–93	261. 36	2, 932	766	2, 548	8, 495	2. 90
93–94	276. 71	2, 166	600	1, 866	5, 947	2. 75
94–95	292. 26	1, 566	457	1, 338	4, 081	2. 61
95–96	307. 96	1, 109	342	938	2, 743	2. 47
96–97	323. 79	767	248	643	1, 805	2. 35
97–98	339. 68	519	176	430	1, 162	2. 24
98–99	355. 61	343	122	282	732	2. 14
99–100	371. 52	221	82	180	450	2. 04
100–101	387. 39	139	54	111	270	1. 95
101–102	403. 16	85	34	68	159	1. 87
102–103	418. 80	51	22	40	91	1. 79
103–104	434. 27	29	12	24	51	1. 72
104–105	449. 51	17	8	12	27	1. 65
105–106	464. 50	9	4	7	15	1. 59
106–107	479. 19	5	2	4	8	1. 53
107–108	493. 53	3	2	2	4	1. 47
108–109	507. 50	1	0	1	2	1. 42
109–110	521. 04	1	1	1	1	1. 37

NOTE.—Rates of mortality at ages above 92 are not based on actual statistics at these ages, but have been obtained by mathematical extrapolation from mortality rates at younger ages. Other life table functions at these ages are based on the extrapolated rates of mortality, and may not necessarily represent actual conditions.

TABLE 7.—LIFE TABLE FOR TOTAL NEGROES IN THE UNITED STATES: 1939–1941

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+1$	$1{,}000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
0–1	74. 16	100, 000	7, 416	93, 960	5, 385, 044	53. 85
1–2	8. 67	92, 584	803	92, 110	5, 291, 084	57. 15
2–3	4. 02	91, 781	368	91, 586	5, 198, 974	56. 65
3–4	2. 58	91, 413	237	91, 290	5, 107, 388	55. 87
4–5	2. 12	91, 176	193	91, 076	5, 016, 098	55. 02
5–6	1. 80	90, 983	164	90, 901	4, 925, 022	54. 13
6–7	1. 55	90, 819	140	90, 749	4, 834, 121	53. 23
7–8	1. 35	90, 679	123	90, 617	4, 743, 372	52. 31
8–9	1. 22	90, 556	111	90, 501	4, 652, 755	51. 38
9–10	1. 17	90, 445	106	90, 392	4, 562, 254	50. 44
10–11	1. 20	90, 339	109	90, 284	4, 471, 862	49. 50
11–12	1. 32	90, 230	119	90, 171	4, 381, 578	48. 56
12–13	1. 54	90, 111	139	90, 041	4, 291, 407	47. 62
13–14	1. 88	89, 972	169	89, 888	4, 201, 366	46. 70
14–15	2. 36	89, 803	212	89, 696	4, 111, 478	45. 78
15–16	2. 91	89, 591	261	89, 461	4, 021, 782	44. 89
16–17	3. 47	89, 330	310	89, 175	3, 932, 321	44. 02
17–18	3. 97	89, 020	354	88, 843	3, 843, 146	43. 17
18–19	4. 44	88, 666	393	88, 470	3, 754, 303	42. 34
19–20	4. 91	88, 273	434	88, 055	3, 665, 833	41. 53
20–21	5. 37	87, 839	472	87, 603	3, 577, 778	40. 73
21–22	5. 78	87, 367	505	87, 115	3, 490, 175	39. 95
22–23	6. 14	86, 862	533	86, 595	3, 403, 060	39. 18
23–24	6. 40	86, 329	553	86, 052	3, 316, 465	38. 42
24–25	6. 60	85, 776	566	85, 493	3, 230, 413	37. 66
25–26	6. 76	85, 210	576	84, 922	3, 144, 920	36. 91
26–27	6. 93	84, 634	586	84, 341	3, 059, 998	36. 16
27–28	7. 14	84, 048	600	83, 747	2, 975, 657	35. 40
28–29	7. 40	83, 448	618	83, 139	2, 891, 910	34. 66
29–30	7. 68	82, 830	636	82, 512	2, 808, 771	33. 91
30–31	7. 97	82, 194	655	81, 867	2, 726, 259	33. 17
31–32	8. 30	81, 539	677	81, 201	2, 644, 392	32. 43
32–33	8. 66	80, 862	700	80, 512	2, 563, 191	31. 70
33–34	9. 05	80, 162	725	79, 799	2, 482, 679	30. 97
34–35	9. 48	79, 437	754	79, 060	2, 402, 880	30. 25
35–36	9. 94	78, 683	781	78, 293	2, 323, 820	29. 53
36–37	10. 42	77, 902	812	77, 496	2, 245, 527	28. 83
37–38	10. 93	77, 090	842	76, 669	2, 168, 031	28. 12
38–39	11. 46	76, 248	874	75, 810	2, 091, 362	27. 43
39–40	12. 04	75, 374	908	74, 920	2, 015, 552	26. 74
40–41	12. 68	74, 466	944	73, 994	1, 940, 632	26. 06
41–42	13. 40	73, 522	985	73, 029	1, 866, 638	25. 39
42–43	14. 21	72, 537	1, 031	72, 022	1, 793, 609	24. 73
43–44	15. 15	71, 506	1, 083	70, 964	1, 721, 587	24. 08
44–45	16. 18	70, 423	1, 139	69, 853	1, 650, 623	23. 44
45–46	17. 30	69, 284	1, 199	68, 685	1, 580, 770	22. 82
46–47	18. 49	68, 085	1, 259	67, 456	1, 512, 085	22. 21
47–48	19. 73	66, 826	1, 318	66, 167	1, 444, 629	21. 62
48–49	21. 00	65, 508	1, 376	64, 820	1, 378, 462	21. 04
49–50	22. 31	64, 132	1, 430	63, 417	1, 313, 642	20. 48
50–51	23. 65	62, 702	1, 483	61, 960	1, 250, 225	19. 94
51–52	25. 01	61, 219	1, 532	60, 453	1, 188, 265	19. 41
52–53	26. 40	59, 687	1, 575	58, 900	1, 127, 812	18. 90
53–54	27. 80	58, 112	1, 616	57, 304	1, 068, 912	18. 39
54–55	29. 21	56, 496	1, 650	55, 670	1, 011, 608	17. 91

TABLE 7.—LIFE TABLE FOR TOTAL NEGROES IN THE UNITED STATES: 1939–1941—Continued

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+1$	$1,000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
55–56	30. 60	54, 846	1, 679	54, 007	955, 938	17. 43
56–57	31. 96	53, 167	1, 699	52, 318	901, 931	16. 96
57–58	33. 28	51, 468	1, 713	50, 611	849, 613	16. 51
58–59	34. 54	49, 755	1, 718	48, 896	799, 002	16. 06
59–60	35. 77	48, 037	1, 719	47, 178	750, 106	15. 62
60–61	37. 00	46, 318	1, 714	45, 461	702, 928	15. 18
61–62	38. 25	44, 604	1, 706	43, 751	657, 467	14. 74
62–63	39. 56	42, 898	1, 697	42, 050	613, 716	14. 31
63–64	40. 95	41, 201	1, 687	40, 358	571, 666	13. 87
64–65	42. 43	39, 514	1, 676	38, 675	531, 308	13. 45
65–66	44. 00	37, 838	1, 665	37, 006	492, 633	13. 02
66–67	45. 67	36, 173	1, 652	35, 347	455, 627	12. 60
67–68	47. 44	34, 521	1, 638	33, 702	420, 280	12. 17
68–69	49. 34	32, 883	1, 622	32, 072	386, 578	11. 76
69–70	51. 41	31, 261	1, 607	30, 457	354, 506	11. 34
70–71	53. 71	29, 654	1, 593	28, 858	324, 049	10. 93
71–72	56. 32	28, 061	1, 580	27, 271	295, 191	10. 52
72–73	59. 29	26, 481	1, 570	25, 695	267, 920	10. 12
73–74	62. 68	24, 911	1, 562	24, 130	242, 225	9. 72
74–75	66. 43	23, 349	1, 551	22, 574	218, 095	9. 34
75–76	70. 49	21, 798	1, 536	21, 030	195, 521	8. 97
76–77	74. 81	20, 262	1, 516	19, 504	174, 491	8. 61
77–78	79. 31	18, 746	1, 487	18, 002	154, 987	8. 27
78–79	83. 95	17, 259	1, 449	16, 535	136, 985	7. 94
79–80	88. 72	15, 810	1, 402	15, 109	120, 450	7. 62
80–81	93. 61	14, 408	1, 349	13, 733	105, 341	7. 31
81–82	98. 61	13, 059	1, 288	12, 415	91, 608	7. 01
82–83	103. 71	11, 771	1, 221	11, 161	79, 193	6. 73
83–84	108. 93	10, 550	1, 149	9, 976	68, 032	6. 45
84–85	114. 34	9, 401	1, 075	8, 864	58, 056	6. 18
85–86	120. 01	8, 326	999	7, 826	49, 192	5. 91
86–87	126. 03	7, 327	923	6, 865	41, 366	5. 65
87–88	132. 48	6, 404	849	5, 980	34, 501	5. 39
88–89	139. 51	5, 555	775	5, 167	28, 521	5. 13
89–90	147. 12	4, 780	703	4, 429	23, 354	4. 89
90–91	155. 38	4, 077	634	3, 760	18, 925	4. 64
91–92	164. 37	3, 443	566	3, 161	15, 165	4. 40
92–93	174. 14	2, 877	501	2, 627	12, 004	4. 17
93–94	184. 70	2, 376	439	2, 156	9, 377	3. 95
94–95	196. 19	1, 937	380	1, 748	7, 221	3. 73
95–96	208. 68	1, 557	325	1, 395	5, 473	3. 51
96–97	222. 22	1, 232	273	1, 095	4, 078	3. 31
97–98	236. 85	959	228	845	2, 983	3. 11
98–99	252. 63	731	184	639	2, 138	2. 92
99–100	269. 58	547	148	473	1, 499	2. 74
100–101	287. 75	399	115	342	1, 026	2. 57
101–102	307. 15	284	87	241	684	2. 40
102–103	327. 79	197	65	165	443	2. 25
103–104	349. 68	132	46	109	278	2. 10
104–105	372. 80	86	32	70	169	1. 96
105–106	397. 13	54	21	43	99	1. 83
106–107	422. 63	33	14	26	56	1. 71
107–108	449. 24	19	9	15	30	1. 59
108–109	476. 94	10	5	8	15	1. 49
109–110	505. 68	5	2	4	7	1. 38
110–111	535. 48	3	2	2	3	1. 29
111–112	566. 42	1	0	0	1	1. 20
112–113	598. 66	1	1	1	1	1. 10

NOTE.—Rates of mortality at ages above 87 are not based on actual statistics at these ages, but have been obtained by mathematical extrapolation from mortality rates at younger ages. Other life table functions at these ages are based on the extrapolated rates of mortality, and may not necessarily represent actual conditions.

TABLE 8.—LIFE TABLE FOR NEGRO MALES IN THE UNITED STATES: 1939–1941

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+1$	$1{,}000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
0–1	82. 28	100, 000	8, 228	93, 282	5, 225, 657	52. 26
1–2	9. 37	91, 772	860	91, 265	5, 132, 375	55. 93
2–3	4. 32	90, 912	392	90, 704	5, 041, 110	55. 45
3–4	2. 69	90, 520	244	90, 393	4, 950, 406	54. 69
4–5	2. 16	90, 276	194	90, 175	4, 860, 013	53. 83
5–6	1. 86	90, 082	168	89, 998	4, 769, 838	52. 95
6–7	1. 63	89, 914	147	89, 841	4, 679, 840	52. 05
7–8	1. 47	89, 767	132	89, 701	4, 589, 999	51. 13
8–9	1. 37	89, 635	123	89, 573	4, 500, 298	50. 21
9–10	1. 34	89, 512	119	89, 453	4, 410, 725	49. 28
10–11	1. 38	89, 393	123	89, 331	4, 321, 272	48. 34
11–12	1. 49	89, 270	133	89, 204	4, 231, 941	47. 41
12–13	1. 67	89, 137	149	89, 062	4, 142, 737	46. 48
13–14	1. 94	88, 988	173	88, 902	4, 053, 675	45. 55
14–15	2. 31	88, 815	205	88, 713	3, 964, 773	44. 64
15–16	2. 74	88, 610	242	88, 489	3, 876, 060	43. 74
16–17	3. 20	88, 368	283	88, 226	3, 787, 571	42. 86
17–18	3. 69	88, 085	325	87, 922	3, 699, 345	42. 00
18–19	4. 22	87, 760	371	87, 575	3, 611, 423	41. 15
19–20	4. 83	87, 389	421	87, 179	3, 523, 848	40. 32
20–21	5. 44	86, 968	474	86, 731	3, 436, 669	39. 52
21–22	6. 02	86, 494	520	86, 234	3, 349, 938	38. 73
22–23	6. 50	85, 974	558	85, 695	3, 263, 704	37. 96
23–24	6. 85	85, 416	585	85, 123	3, 178, 009	37. 21
24–25	7. 11	84, 831	604	84, 529	3, 092, 886	36. 46
25–26	7. 33	84, 227	617	83, 919	3, 008, 357	35. 72
26–27	7. 54	83, 610	631	83, 294	2, 924, 438	34. 98
27–28	7. 80	82, 979	647	82, 656	2, 841, 144	34. 24
28–29	8. 10	82, 332	667	81, 999	2, 758, 488	33. 50
29–30	8. 40	81, 665	686	81, 322	2, 676, 489	32. 77
30–31	8. 72	80, 979	706	80, 625	2, 595, 167	32. 05
31–32	9. 06	80, 273	728	79, 910	2, 514, 542	31. 32
32–33	9. 43	79, 545	749	79, 170	2, 434, 632	30. 61
33–34	9. 83	78, 796	775	78, 408	2, 355, 462	29. 89
34–35	10. 25	78, 021	800	77, 622	2, 277, 054	29. 19
35–36	10. 71	77, 221	827	76, 807	2, 199, 432	28. 48
36–37	11. 21	76, 394	856	75, 966	2, 122, 625	27. 79
37–38	11. 74	75, 538	887	75, 095	2, 046, 659	27. 09
38–39	12. 30	74, 651	918	74, 191	1, 971, 564	26. 41
39–40	12. 93	73, 733	953	73, 256	1, 897, 373	25. 73
40–41	13. 62	72, 780	992	72, 284	1, 824, 117	25. 06
41–42	14. 40	71, 788	1, 033	71, 272	1, 751, 833	24. 40
42–43	15. 28	70, 755	1, 082	70, 214	1, 680, 561	23. 75
43–44	16. 29	69, 673	1, 135	69, 106	1, 610, 347	23. 11
44–45	17. 40	68, 538	1, 192	67, 942	1, 541, 241	22. 49
45–46	18. 59	67, 346	1, 252	66, 721	1, 473, 299	21. 88
46–47	19. 86	66, 094	1, 313	65, 437	1, 406, 578	21. 28
47–48	21. 18	64, 781	1, 372	64, 096	1, 341, 141	20. 70
48–49	22. 55	63, 409	1, 430	62, 694	1, 277, 045	20. 14
49–50	23. 94	61, 979	1, 484	61, 237	1, 214, 351	19. 59
50–51	25. 36	60, 495	1, 534	59, 728	1, 153, 114	19. 06
51–52	26. 79	58, 961	1, 579	58, 172	1, 093, 386	18. 54
52–53	28. 23	57, 382	1, 620	56, 571	1, 035, 214	18. 04
53–54	29. 66	55, 762	1, 654	54, 935	978, 643	17. 55
54–55	31. 08	54, 108	1, 682	53, 267	923, 708	17. 07

TABLE 8.—LIFE TABLE FOR NEGRO MALES IN THE UNITED STATES: 1939–1941—Continued

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+1$	$1,000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
55–56	32. 48	52, 426	1, 703	51, 575	870, 441	16. 60
56–57	33. 86	50, 723	1, 717	49, 865	818, 866	16. 14
57–58	35. 20	49, 006	1, 725	48, 143	769, 001	15. 69
58–59	36. 50	47, 281	1, 726	46, 417	720, 858	15. 25
59–60	37. 79	45, 555	1, 722	44, 694	674, 441	14. 81
60–61	39. 10	43, 833	1, 714	42, 976	629, 747	14. 37
61–62	40. 45	42, 119	1, 704	41, 268	586, 771	13. 93
62–63	41. 89	40, 415	1, 693	39, 569	545, 503	13. 50
63–64	43. 43	38, 722	1, 681	37, 881	505, 934	13. 07
64–65	45. 08	37, 041	1, 670	36, 206	468, 053	12. 64
65–66	46. 85	35, 371	1, 657	34, 543	431, 847	12. 21
66–67	48. 75	33, 714	1, 644	32, 892	397, 304	11. 78
67–68	50. 77	32, 070	1, 628	31, 256	364, 412	11. 36
68–69	52. 94	30, 442	1, 611	29, 636	333, 156	10. 94
69–70	55. 32	28, 831	1, 595	28, 033	303, 520	10. 53
70–71	57. 99	27, 236	1, 580	26, 446	275, 487	10. 11
71–72	61. 04	25, 656	1, 566	24, 874	249, 041	9. 71
72–73	64. 55	24, 090	1, 555	23, 312	224, 167	9. 31
73–74	68. 57	22, 535	1, 545	21, 763	200, 855	8. 91
74–75	73. 09	20, 990	1, 534	20, 223	179, 092	8. 53
75–76	78. 03	19, 456	1, 518	18, 696	158, 869	8. 17
76–77	83. 36	17, 938	1, 496	17, 190	140, 173	7. 81
77–78	89. 02	16, 442	1, 463	15, 711	122, 983	7. 48
78–79	94. 95	14, 979	1, 423	14, 267	107, 272	7. 16
79–80	101. 07	13, 556	1, 370	12, 871	93, 005	6. 86
80–81	107. 30	12, 186	1, 307	11, 533	80, 134	6. 58
81–82	113. 53	10, 879	1, 235	10, 261	68, 601	6. 31
82–83	119. 69	9, 644	1, 155	9, 067	58, 340	6. 05
83–84	125. 73	8, 489	1, 067	7, 955	49, 273	5. 80
84–85	131. 73	7, 422	978	6, 933	41, 318	5. 57
85–86	137. 83	6, 444	888	6, 001	34, 385	5. 34
86–87	144. 15	5, 556	801	5, 155	28, 384	5. 11
87–88	150. 83	4, 755	717	4, 397	23, 229	4. 89
88–89	157. 99	4, 038	638	3, 719	18, 832	4. 66
89–90	165. 74	3, 400	564	3, 118	15, 113	4. 45
90–91	174. 17	2, 836	494	2, 589	11, 995	4. 23
91–92	183. 40	2, 342	429	2, 128	9, 406	4. 02
92–93	193. 52	1, 913	370	1, 728	7, 278	3. 80
93–94	204. 63	1, 543	316	1, 384	5, 550	3. 60
94–95	216. 85	1, 227	266	1, 094	4, 166	3. 39
95–96	230. 27	961	221	851	3, 072	3. 20
96–97	245. 00	740	182	649	2, 221	3. 00
97–98	261. 13	558	145	485	1, 572	2. 82
98–99	278. 77	413	115	355	1, 087	2. 63
99–100	298. 02	298	89	254	732	2. 46
100–101	319. 00	209	67	175	478	2. 29
101–102	341. 78	142	48	118	303	2. 13
102–103	366. 49	94	35	77	185	1. 97
103–104	393. 22	59	23	47	108	1. 83
104–105	422. 08	36	15	29	61	1. 69
105–106	453. 17	21	10	16	32	1. 56
106–107	486. 58	11	5	8	16	1. 43
107–108	522. 44	6	3	5	8	1. 31
108–109	560. 82	3	2	2	3	1. 20
109–110	601. 85	1	1	1	1	1. 10

NOTE.—Rates of mortality at ages above 92 are not based on actual statistics at these ages, but have been obtained by mathematical extrapolation from mortality rates at younger ages. Other life table functions at these ages are based on the extrapolated rates of mortality, and may not necessarily represent actual conditions.

TABLE 9.—LIFE TABLE FOR NEGRO FEMALES IN THE UNITED STATES: 1939–1941

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+1$	$1,000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
0–1	65. 84	100, 000	6, 584	94, 657	5, 556, 051	55. 56
1–2	7. 96	93, 416	744	92, 977	5, 461, 394	58. 46
2–3	3. 72	92, 672	345	92, 489	5, 368, 417	57. 93
3–4	2. 48	92, 327	228	92, 208	5, 275, 928	57. 14
4–5	2. 09	92, 099	193	91, 999	5, 183, 720	56. 28
5–6	1. 75	91, 906	160	91, 826	5, 091, 721	55. 40
6–7	1. 46	91, 746	134	91, 679	4, 999, 895	54. 50
7–8	1. 23	91, 612	113	91, 556	4, 908, 216	53. 58
8–9	1. 08	91, 499	99	91, 450	4, 816, 660	52. 64
9–10	1. 01	91, 400	92	91, 354	4, 725, 210	51. 70
10–11	1. 04	91, 308	95	91, 260	4, 633, 856	50. 75
11–12	1. 16	91, 213	106	91, 160	4, 542, 596	49. 80
12–13	1. 40	91, 107	128	91, 042	4, 451, 436	48. 86
13–14	1. 82	90, 979	166	90, 896	4, 360, 394	47. 93
14–15	2. 41	90, 813	219	90, 704	4, 269, 498	47. 01
15–16	3. 07	90, 594	278	90, 456	4, 178, 794	46. 13
16–17	3. 71	90, 316	335	90, 149	4, 088, 338	45. 27
17–18	4. 24	89, 981	381	89, 790	3, 998, 189	44. 43
18–19	4. 65	89, 600	417	89, 391	3, 908, 399	43. 62
19–20	5. 01	89, 183	447	88, 959	3, 819, 008	42. 82
20–21	5. 32	88, 736	472	88, 500	3, 730, 049	42. 04
21–22	5. 59	88, 264	494	88, 017	3, 641, 549	41. 26
22–23	5. 83	87, 770	512	87, 515	3, 553, 532	40. 49
23–24	6. 03	87, 258	525	86, 995	3, 466, 017	39. 72
24–25	6. 16	86, 733	535	86, 465	3, 379, 022	38. 96
25–26	6. 27	86, 198	540	85, 928	3, 292, 557	38. 20
26–27	6. 40	85, 658	548	85, 384	3, 206, 629	37. 44
27–28	6. 57	85, 110	559	84, 831	3, 121, 245	36. 67
28–29	6. 80	84, 551	575	84, 263	3, 036, 414	35. 91
29–30	7. 05	83, 976	592	83, 680	2, 952, 151	35. 15
30–31	7. 33	83, 384	611	83, 079	2, 868, 471	34. 40
31–32	7. 64	82, 773	632	82, 457	2, 785, 392	33. 65
32–33	7. 99	82, 141	656	81, 813	2, 702, 935	32. 91
33–34	8. 37	81, 485	682	81, 144	2, 621, 122	32. 17
34–35	8. 80	80, 803	711	80, 447	2, 539, 978	31. 43
35–36	9. 24	80, 092	740	79, 722	2, 459, 531	30. 71
36–37	9. 71	79, 352	771	78, 966	2, 379, 809	29. 99
37–38	10. 20	78, 581	801	78, 181	2, 300, 843	29. 28
38–39	10. 70	77, 780	832	77, 363	2, 222, 662	28. 58
39–40	11. 23	76, 948	864	76, 516	2, 145, 299	27. 88
40–41	11. 81	76, 084	898	75, 635	2, 068, 783	27. 19
41–42	12. 46	75, 186	937	74, 717	1, 993, 148	26. 51
42–43	13. 20	74, 249	980	73, 759	1, 918, 431	25. 84
43–44	14. 05	73, 269	1, 029	72, 754	1, 844, 672	25. 18
44–45	14. 99	72, 240	1, 083	71, 698	1, 771, 918	24. 53
45–46	16. 02	71, 157	1, 140	70, 587	1, 700, 220	23. 89
46–47	17. 11	70, 017	1, 198	69, 418	1, 629, 633	23. 27
47–48	18. 24	68, 819	1, 255	68, 191	1, 560, 215	22. 67
48–49	19. 42	67, 564	1, 312	66, 908	1, 492, 024	22. 08
49–50	20. 62	66, 252	1, 367	65, 568	1, 425, 116	21. 51
50–51	21. 87	64, 885	1, 419	64, 176	1, 359, 548	20. 95
51–52	23. 15	63, 466	1, 469	62, 732	1, 295, 372	20. 41
52–53	24. 47	61, 997	1, 517	61, 238	1, 232, 640	19. 88
53–54	25. 83	60, 480	1, 563	59, 699	1, 171, 402	19. 37
54–55	27. 21	58, 917	1, 603	58, 115	1, 111, 703	18. 87

TABLE 9.—LIFE TABLE FOR NEGRO FEMALES IN THE UNITED STATES: 1939–1941—Continued

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+1$	$1{,}000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
55–56	28.58	57,314	1,638	56,495	1,053,588	18.38
56–57	29.92	55,676	1,666	54,843	997,093	17.91
57–58	31.21	54,010	1,686	53,168	942,250	17.45
58–59	32.42	52,324	1,696	51,476	889,082	16.99
59–60	33.58	50,628	1,700	49,778	837,606	16.54
60–61	34.72	48,928	1,699	48,078	787,828	16.10
61–62	35.86	47,229	1,694	46,382	739,750	15.66
62–63	37.03	45,535	1,686	44,692	693,368	15.23
63–64	38.25	43,849	1,678	43,010	648,676	14.79
64–65	39.54	42,171	1,667	41,338	605,666	14.36
65–66	40.90	40,504	1,656	39,675	564,328	13.93
66–67	42.33	38,848	1,645	38,026	524,653	13.51
67–68	43.84	37,203	1,630	36,388	486,627	13.08
68–69	45.44	35,573	1,617	34,764	450,239	12.66
69–70	47.18	33,956	1,602	33,155	415,475	12.24
70–71	49.12	32,354	1,589	31,560	382,320	11.82
71–72	51.29	30,765	1,578	29,975	350,760	11.40
72–73	53.76	29,187	1,569	28,403	320,785	10.99
73–74	56.55	27,618	1,562	26,837	292,382	10.59
74–75	59.63	26,056	1,554	25,279	265,545	10.19
75–76	62.94	24,502	1,542	23,731	240,266	9.81
76–77	66.41	22,960	1,525	22,198	216,535	9.43
77–78	69.98	21,435	1,500	20,685	194,337	9.07
78–79	73.62	19,935	1,468	19,201	173,652	8.71
79–80	77.37	18,467	1,428	17,753	154,451	8.36
80–81	81.27	17,039	1,385	16,347	136,698	8.02
81–82	85.40	15,654	1,337	14,985	120,351	7.69
82–83	89.81	14,317	1,286	13,674	105,366	7.36
83–84	94.57	13,031	1,232	12,415	91,692	7.04
84–85	99.71	11,799	1,177	11,211	79,277	6.72
85–86	105.29	10,622	1,118	10,063	68,066	6.41
86–87	111.35	9,504	1,058	8,975	58,003	6.10
87–88	117.93	8,446	996	7,948	49,028	5.81
88–89	125.09	7,450	932	6,983	41,080	5.51
89–90	132.87	6,518	866	6,085	34,097	5.23
90–91	141.32	5,652	799	5,252	28,012	4.96
91–92	150.48	4,853	730	4,488	22,760	4.69
92–93	160.40	4,123	662	3,792	18,272	4.43
93–94	171.12	3,461	592	3,166	14,480	4.18
94–95	182.70	2,869	524	2,607	11,314	3.94
95–96	195.17	2,345	458	2,116	8,707	3.71
96–97	208.58	1,887	393	1,690	6,591	3.49
97–98	222.99	1,494	333	1,327	4,901	3.28
98–99	238.43	1,161	277	1,022	3,574	3.08
99–100	254.96	884	225	772	2,552	2.89
100–101	272.61	659	180	568	1,780	2.70
101–102	291.43	479	140	410	1,212	2.53
102–103	311.48	339	105	286	802	2.36
103–104	332.80	234	78	195	516	2.21
104–105	355.43	156	56	128	321	2.06
105–106	379.41	100	38	82	193	1.92
106–107	404.81	62	25	49	111	1.79
107–108	431.65	37	16	29	62	1.66
108–109	460.00	21	10	17	33	1.54
109–110	489.88	11	5	8	16	1.43
110–111	521.36	6	3	5	8	1.33
111–112	554.48	3	2	2	3	1.23
112–113	589.28	1	0	0	1	1.13
113–114	625.81	1	1	1	1	1.04

NOTE.—Rates of mortality at ages above 87 are not based on actual statistics at these ages, but have been obtained by mathematical extrapolation from mortality rates at younger ages. Other life table functions at these ages are based on the extrapolated rates of mortality, and may not necessarily represent actual conditions.

TABLE 10.—LIFE TABLE FOR TOTAL OTHER RACES[1] IN THE UNITED STATES: 1939–1941

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to x+1	$1,000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
0–1	93. 03	100, 000	9, 303	93, 110	5, 435, 389	54. 35
1–2	20. 37	90, 697	1, 847	89, 607	5, 342, 279	58. 90
2–3	9. 73	88, 850	864	88, 392	5, 252, 672	59. 12
3–4	5. 31	87, 986	467	87, 743	5, 164, 280	58. 69
4–5	3. 91	87, 519	342	87, 340	5, 076, 537	58. 01
5–6	3. 20	87, 177	279	87, 038	4, 989, 197	57. 23
6–7	2. 67	86, 898	233	86, 781	4, 902, 159	56. 41
7–8	2. 30	86, 665	199	86, 566	4, 815, 378	55. 56
8–9	2. 08	86, 466	180	86, 376	4, 728, 812	54. 69
9–10	1. 97	86, 286	170	86, 200	4, 642, 436	53. 80
10–11	1. 98	86, 116	171	86, 031	4, 556, 236	52. 91
11–12	2. 08	85, 945	178	85, 856	4, 470, 205	52. 01
12–13	2. 24	85, 767	193	85, 670	4, 384, 349	51. 12
13–14	2. 52	85, 574	215	85, 467	4, 298, 679	50. 23
14–15	2. 92	85, 359	249	85, 234	4, 213, 212	49. 36
15–16	3. 38	85, 110	288	84, 966	4, 127, 978	48. 50
16–17	3. 86	84, 822	328	84, 658	4, 043, 012	47. 66
17–18	4. 28	84, 494	362	84, 313	3, 958, 354	46. 85
18–19	4. 69	84, 132	394	83, 935	3, 874, 041	46. 05
19–20	5. 10	83, 738	427	83, 525	3, 790, 106	45. 26
20–21	5. 50	83, 311	458	83, 082	3, 706, 581	44. 49
21–22	5. 83	82, 853	483	82, 611	3, 623, 499	43. 73
22–23	6. 07	82, 370	501	82, 120	3, 540, 888	42. 99
23–24	6. 19	81, 869	506	81, 616	3, 458, 768	42. 25
24–25	6. 19	81, 363	504	81, 111	3, 377, 152	41. 51
25–26	6. 15	80, 859	497	80, 610	3, 296, 041	40. 76
26–27	6. 10	80, 362	491	80, 117	3, 215, 431	40. 01
27–28	6. 12	79, 871	488	79, 627	3, 135, 314	39. 25
28–29	6. 18	79, 383	491	79, 137	3, 055, 687	38. 49
29–30	6. 28	78, 892	495	78, 644	2, 976, 550	37. 73
30–31	6. 38	78, 397	501	78, 147	2, 897, 906	36. 96
31–32	6. 50	77, 896	506	77, 643	2, 819, 759	36. 20
32–33	6. 63	77, 390	513	77, 133	2, 742, 116	35. 43
33–34	6. 75	76, 877	520	76, 617	2, 664, 983	34. 67
34–35	6. 89	76, 357	526	76, 094	2, 588, 366	33. 90
35–36	7. 04	75, 831	534	75, 565	2, 512, 272	33. 13
36–37	7. 21	75, 297	543	75, 026	2, 436, 707	32. 36
37–38	7. 42	74, 754	554	74, 476	2, 361, 681	31. 59
38–39	7. 65	74, 200	568	73, 916	2, 287, 205	30. 82
39–40	7. 93	73, 632	584	73, 340	2, 213, 289	30. 06
40–41	8. 23	73, 048	601	72, 748	2, 139, 949	29. 30
41–42	8. 58	72, 447	622	72, 136	2, 067, 201	28. 53
42–43	8. 96	71, 825	643	71, 504	1, 995, 065	27. 78
43–44	9. 38	71, 182	667	70, 848	1, 923, 561	27. 02
44–45	9. 84	70, 515	694	70, 168	1, 852, 713	26. 27
45–46	10. 37	69, 821	724	69, 459	1, 782, 545	25. 53
46–47	10. 96	69, 097	757	68, 718	1, 713, 086	24. 79
47–48	11. 64	68, 340	796	67, 942	1, 644, 368	24. 06
48–49	12. 40	67, 544	837	67, 126	1, 576, 426	23. 34
49–50	13. 24	66, 707	883	66, 265	1, 509, 300	22. 63
50–51	14. 16	65, 824	932	65, 358	1, 443, 035	21. 92
51–52	15. 14	64, 892	983	64, 400	1, 377, 677	21. 23
52–53	16. 17	63, 909	1, 033	63, 392	1, 313, 277	20. 55
53–54	17. 25	62, 876	1, 085	62, 334	1, 249, 885	19. 88
54–55	18. 40	61, 791	1, 137	61, 222	1, 187, 551	19. 22

[1] All except white and Negro.

TABLE 10.—LIFE TABLE FOR TOTAL OTHER RACES [1] IN THE UNITED STATES: 1939–1941—Continued

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+1$	$1{,}000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
55–56	19. 63	60, 654	1, 191	60, 058	1, 126, 329	18. 57
56–57	20. 96	59, 463	1, 246	58, 840	1, 066, 271	17. 93
57–58	22. 41	58, 217	1, 305	57, 565	1, 007, 431	17. 30
58–59	24. 00	56, 912	1, 366	56, 229	949, 866	16. 69
59–60	25. 71	55, 546	1, 428	54, 832	893, 637	16. 09
60–61	27. 52	54, 118	1, 489	53, 374	838, 805	15. 50
61–62	29. 41	52, 629	1, 548	51, 854	785, 431	14. 92
62–63	31. 37	51, 081	1, 603	50, 280	733, 577	14. 36
63–64	33. 38	49, 478	1, 651	48, 653	683, 297	13. 81
64–65	35. 48	47, 827	1, 697	46, 978	634, 644	13. 27
65–66	37. 71	46, 130	1, 739	45, 261	587, 666	12. 74
66–67	40. 12	44, 391	1, 781	43, 500	542, 405	12. 22
67–68	42. 76	42, 610	1, 822	41, 699	498, 905	11. 71
68–69	45. 67	40, 788	1, 863	39, 857	457, 206	11. 21
69–70	48. 91	38, 925	1, 903	37, 973	417, 349	10. 72
70–71	52. 52	37, 022	1, 945	36, 049	379, 376	10. 25
71–72	56. 56	35, 077	1, 984	34, 086	343, 327	9. 79
72–73	61. 08	33, 093	2, 021	32, 083	309, 241	9. 34
73–74	66. 09	31, 072	2, 053	30, 045	277, 158	8. 92
74–75	71. 47	29, 019	2, 074	27, 982	247, 113	8. 52
75–76	77. 02	26, 945	2, 076	25, 907	219, 131	8. 13
76–77	82. 59	24, 869	2, 054	23, 842	193, 224	7. 77
77–78	88. 00	22, 815	2, 007	21, 811	169, 382	7. 42
78–79	93. 13	20, 808	1, 938	19, 839	147, 571	7. 09
79–80	98. 13	18, 870	1, 852	17, 944	127, 732	6. 77
80–81	103. 20	17, 018	1, 756	16, 140	109, 788	6. 45
81–82	108. 54	15, 262	1, 657	14, 433	93, 648	6. 14
82–83	114. 36	13, 605	1, 556	12, 828	79, 215	5. 82
83–84	120. 86	12, 049	1, 456	11, 321	66, 387	5. 51
84–85	128. 22	10, 593	1, 358	9, 914	55, 066	5. 20
85–86	136. 62	9, 235	1, 262	8, 604	45, 152	4. 89
86–87	146. 24	7, 973	1, 166	7, 390	36, 548	4. 58
87–88	157. 26	6, 807	1, 070	6, 272	29, 158	4. 28
88–89	169. 86	5, 737	975	5, 249	22, 886	3. 99
89–90	184. 22	4, 762	877	4, 323	17, 637	3. 70
90–91	200. 51	3, 885	779	3, 496	13, 314	3. 43
91–92	218. 92	3, 106	680	2, 766	9, 818	3. 16
92–93	239. 61	2, 426	581	2, 135	7, 052	2. 91
93–94	262. 54	1, 845	485	1, 603	4, 917	2. 67
94–95	288. 19	1, 360	392	1, 164	3, 314	2. 44
95–96	316. 71	968	306	815	2, 150	2. 22
96–97	348. 25	662	231	546	1, 335	2. 02
97–98	382. 96	431	165	349	789	1. 83
98–99	420. 90	266	112	210	440	1. 65
99–100	462. 08	154	71	119	230	1. 49
100–101	506. 41	83	42	62	111	1. 34
101–102	553. 64	41	23	29	49	1. 21
102–103	603. 27	18	11	13	20	1. 09
103–104	654. 47	7	4	5	7	. 98
104–105	705. 98	3	2	1	2	. 88
105–106	756. 23	1	1	1	1	. 80

[1] All except white and Negro.

NOTE.—Rates of mortality at ages above 87 are not based on actual statistics at these ages, but have been obtained by mathematical extrapolation from mortality rates at younger ages. Other life table functions at these ages are based on the extrapolated rates of mortality, and may not necessarily represent actual conditions.

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TABLE 11.—LIFE TABLE FOR OTHER RACES,[1] MALES IN THE UNITED STATES: 1939–1941

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+1$	$1,000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
0–1	98.63	100,000	9,864	92,589	5,356,374	53.56
1–2	20.36	90,136	1,835	89,054	5,263,785	58.40
2–3	9.59	88,302	848	87,852	5,174,731	58.60
3–4	5.16	87,454	451	87,220	5,086,879	58.17
4–5	3.89	87,003	338	86,827	4,999,659	57.47
5–6	3.30	86,665	286	86,522	4,912,832	56.69
6–7	2.84	86,379	246	86,257	4,826,310	55.87
7–8	2.49	86,133	214	86,026	4,740,053	55.03
8–9	2.25	85,919	194	85,822	4,654,027	54.17
9–10	2.12	85,725	181	85,634	4,568,205	53.29
10–11	2.07	85,544	178	85,455	4,482,571	52.40
11–12	2.12	85,366	180	85,276	4,397,116	51.51
12–13	2.24	85,186	191	85,091	4,311,840	50.62
13–14	2.48	84,995	211	84,889	4,226,749	49.73
14–15	2.85	84,784	242	84,663	4,141,860	48.85
15–16	3.29	84,542	278	84,403	4,057,197	47.99
16–17	3.72	84,264	314	84,107	3,972,794	47.15
17–18	4.11	83,950	344	83,778	3,888,687	46.32
18–19	4.45	83,606	372	83,420	3,804,909	45.51
19–20	4.79	83,234	399	83,034	3,721,489	44.71
20–21	5.11	82,835	423	82,624	3,638,455	43.92
21–22	5.37	82,412	443	82,190	3,555,831	43.15
22–23	5.55	81,969	455	81,742	3,473,641	42.38
23–24	5.60	81,514	456	81,286	3,391,899	41.61
24–25	5.54	81,058	449	80,833	3,310,613	40.84
25–26	5.43	80,609	438	80,391	3,229,780	40.07
26–27	5.35	80,171	429	79,956	3,149,389	39.28
27–28	5.37	79,742	428	79,529	3,069,433	38.49
28–29	5.48	79,314	434	79,097	2,989,904	37.70
29–30	5.66	78,880	447	78,656	2,910,807	36.90
30–31	5.88	78,433	462	78,202	2,832,151	36.11
31–32	6.13	77,971	477	77,733	2,753,949	35.32
32–33	6.37	77,494	494	77,247	2,676,216	34.53
33–34	6.60	77,000	508	76,746	2,598,969	33.75
34–35	6.83	76,492	522	76,231	2,522,223	32.97
35–36	7.08	75,970	538	75,701	2,445,992	32.20
36–37	7.34	75,432	554	75,156	2,370,291	31.42
37–38	7.63	74,878	571	74,592	2,295,135	30.65
38–39	7.96	74,307	592	74,011	2,220,543	29.88
39–40	8.32	73,715	613	73,409	2,146,532	29.12
40–41	8.72	73,102	637	72,784	2,073,123	28.36
41–42	9.16	72,465	664	72,133	2,000,339	27.60
42–43	9.63	71,801	692	71,455	1,928,206	26.85
43–44	10.15	71,109	722	70,748	1,856,751	26.11
44–45	10.72	70,387	754	70,010	1,786,003	25.37
45–46	11.35	69,633	790	69,238	1,715,993	24.64
46–47	12.03	68,843	829	68,429	1,646,755	23.92
47–48	12.78	68,014	869	67,579	1,578,326	23.21
48–49	13.61	67,145	914	66,689	1,510,747	22.50
49–50	14.50	66,231	960	65,751	1,444,058	21.80
50–51	15.44	65,271	1,008	64,767	1,378,307	21.12
51–52	16.43	64,263	1,056	63,734	1,313,540	20.44
52–53	17.45	63,207	1,103	62,656	1,249,806	19.77
53–54	18.49	62,104	1,148	61,530	1,187,150	19.12
54–55	19.60	60,956	1,195	60,358	1,125,620	18.47

[1] All except white and Negro.

TABLE 11.—LIFE TABLE FOR OTHER RACES,[1] MALES IN THE UNITED STATES: 1939–1941—Continued

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+1$	$1,000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
55–56	20. 79	59, 761	1, 243	59, 140	1, 065, 262	17. 83
56–57	22. 12	58, 518	1, 294	57, 871	1, 006, 122	17. 19
57–58	23. 61	57, 224	1, 351	56, 548	948, 251	16. 57
58–59	25. 29	55, 873	1, 414	55, 166	891, 703	15. 96
59–60	27. 15	54, 459	1, 478	53, 721	836, 537	15. 36
60–61	29. 14	52, 981	1, 544	52, 209	782, 816	14. 78
61–62	31. 23	51, 437	1, 606	50, 634	730, 607	14. 20
62–63	33. 40	49, 831	1, 664	48, 999	679, 973	13. 65
63–64	35. 62	48, 167	1, 716	47, 310	630, 974	13. 10
64–65	37. 94	46, 451	1, 762	45, 570	583, 664	12. 57
65–66	40. 43	44, 689	1, 807	43, 786	538, 094	12. 04
66–67	43. 14	42, 882	1, 849	41, 957	494, 308	11. 53
67–68	46. 14	41, 033	1, 894	40, 086	452, 351	11. 02
68–69	49. 49	39, 139	1, 937	38, 171	412, 265	10. 53
69–70	53. 22	37, 202	1, 979	36, 212	374, 094	10. 06
70–71	57. 36	35, 223	2, 021	34, 213	337, 882	9. 59
71–72	61. 96	33, 202	2, 057	32, 173	303, 669	9. 15
72–73	67. 04	31, 145	2, 088	30, 101	271, 496	8. 72
73–74	72. 60	29, 057	2, 110	28, 002	241, 395	8. 31
74–75	78. 54	26, 947	2, 116	25, 890	213, 393	7. 92
75–76	84. 70	24, 831	2, 103	23, 779	187, 503	7. 55
76–77	90. 93	22, 728	2, 067	21, 695	163, 724	7. 20
77–78	97. 09	20, 661	2, 006	19, 658	142, 029	6. 87
78–79	103. 09	18, 655	1, 923	17, 693	122, 371	6. 56
79–80	109. 04	16, 732	1, 824	15, 820	104, 678	6. 26
80–81	115. 11	14, 908	1, 716	14, 050	88, 858	5. 96
81–82	121. 47	13, 192	1, 603	12, 390	74, 808	5. 67
82–83	128. 28	11, 589	1, 486	10, 846	62, 418	5. 39
83–84	135. 72	10, 103	1, 372	9, 417	51, 572	5. 10
84–85	143. 92	8, 731	1, 256	8, 103	42, 155	4. 83
85–86	153. 01	7, 475	1, 144	6, 903	34, 052	4. 56
86–87	163. 12	6, 331	1, 033	5, 815	27, 149	4. 29
87–88	174. 38	5, 298	924	4, 837	21, 334	4. 03
88–89	186. 94	4, 374	817	3, 965	16, 497	3. 77
89–90	200. 91	3, 557	715	3, 200	12, 532	3. 52
90–91	216. 43	2, 842	615	2, 534	9, 332	3. 28
91–92	233. 63	2, 227	520	1, 967	6, 798	3. 05
92–93	252. 64	1, 707	431	1, 491	4, 831	2. 83
93–94	273. 60	1, 276	349	1, 101	3, 340	2. 62
94–95	296. 64	927	275	789	2, 239	2. 42
95–96	321. 89	652	210	547	1, 450	2. 22
96–97	349. 48	442	155	365	903	2. 04
97–98	379. 54	287	109	233	538	1. 87
98–99	412. 21	178	73	141	305	1. 71
99–100	447. 61	105	47	82	164	1. 56
100–101	485. 88	58	28	44	82	1. 42
101–102	527. 15	30	16	22	38	1. 29
102–103	571. 55	14	8	10	16	1. 16
103–104	619. 21	6	4	4	6	1. 05
104–105	670. 28	2	1	1	2	. 94
105–106	724. 86	1	1	1	1	. 84

[1] All except white and Negro.

NOTE.—Rates of mortality at ages above 87 are not based on actual statistics at these ages, but have been obtained by mathematical extrapolation from mortality rates at younger ages. Other life table functions at these ages are based on the extrapolated rates of mortality, and may not necessarily represent actual conditions.

TABLE 12.—LIFE TABLE FOR OTHER RACES,[1] FEMALES IN THE UNITED STATES: 1939–1941

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+1$	$1,000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
0–1	87. 17	100, 000	8, 717	93, 654	5, 583, 750	55. 84
1–2	20. 38	91, 283	1, 861	90, 185	5, 490, 096	60. 14
2–3	9. 86	89, 422	882	88, 955	5, 399, 911	60. 39
3–4	5. 46	88, 540	483	88, 289	5, 310, 956	59. 98
4–5	3. 93	88, 057	346	87, 876	5, 222, 667	59. 31
5–6	3. 10	87, 711	272	87, 575	5, 134, 791	58. 54
6–7	2. 51	87, 439	219	87, 329	5, 047, 216	57. 72
7–8	2. 12	87, 220	185	87, 128	4, 959, 887	56. 87
8–9	1. 90	87, 035	165	86, 952	4, 872, 759	55. 99
9–10	1. 83	86, 870	160	86, 790	4, 785, 807	55. 09
10–11	1. 89	86, 710	163	86, 629	4, 699, 017	54. 19
11–12	2. 03	86, 547	176	86, 459	4, 612, 388	53. 29
12–13	2. 25	86, 371	194	86, 274	4, 525, 929	52. 40
13–14	2. 56	86, 177	221	86, 066	4, 439, 655	51. 52
14–15	2. 99	85, 956	257	85, 828	4, 353, 589	50. 65
15–16	3. 49	85, 699	298	85, 550	4, 267, 761	49. 80
16–17	4. 00	85, 401	342	85, 230	4, 182, 211	48. 97
17–18	4. 48	85, 059	381	84, 868	4, 096, 981	48. 17
18–19	4. 95	84, 678	420	84, 468	4, 012, 113	47. 38
19–20	5. 45	84, 258	459	84, 028	3, 927, 645	46. 61
20–21	5. 93	83, 799	497	83, 551	3, 843, 617	45. 87
21–22	6. 36	83, 302	529	83, 038	3, 760, 066	45. 14
22–23	6. 70	82, 773	554	82, 496	3, 677, 028	44. 42
23–24	6. 93	82, 219	571	81, 933	3, 594, 532	43. 72
24–25	7. 09	81, 648	579	81, 359	3, 512, 599	43. 02
25–26	7. 20	81, 069	583	80, 778	3, 431, 240	42. 32
26–27	7. 26	80, 486	585	80, 193	3, 350, 462	41. 63
27–28	7. 31	79, 901	584	79, 609	3, 270, 269	40. 93
28–29	7. 33	79, 317	581	79, 027	3, 190, 660	40. 23
29–30	7. 30	78, 736	575	78, 449	3, 111, 633	39. 52
30–31	7. 25	78, 161	567	77, 877	3, 033, 184	38. 81
31–32	7. 19	77, 594	558	77, 316	2, 955, 307	38. 09
32–33	7. 12	77, 036	549	76, 761	2, 877, 991	37. 36
33–34	7. 06	76, 487	540	76, 218	2, 801, 230	36. 62
34–35	7. 01	75, 947	532	75, 681	2, 725, 012	35. 88
35–36	6. 97	75, 415	526	75, 152	2, 649, 331	35. 13
36–37	6. 96	74, 889	521	74, 629	2, 574, 179	34. 37
37–38	6. 99	74, 368	520	74, 108	2, 499, 550	33. 61
38–39	7. 06	73, 848	521	73, 587	2, 425, 442	32. 84
39–40	7. 17	73, 327	526	73, 064	2, 351, 855	32. 07
40–41	7. 32	72, 801	533	72, 535	2, 278, 791	31. 30
41–42	7. 51	72, 268	543	71, 997	2, 206, 256	30. 53
42–43	7. 73	71, 725	554	71, 448	2, 134, 259	29. 76
43–44	7. 99	71, 171	569	70, 886	2, 062, 811	28. 98
44–45	8. 28	70, 602	585	70, 310	1, 991, 925	28. 21
45–46	8. 64	70, 017	605	69, 714	1, 921, 615	27. 44
46–47	9. 07	69, 412	629	69, 098	1, 851, 901	26. 68
47–48	9. 59	68, 783	660	68, 453	1, 782, 803	25. 92
48–49	10. 20	68, 123	694	67, 776	1, 714, 350	25. 17
49–50	10. 91	67, 429	736	67, 061	1, 646, 574	24. 42
50–51	11. 71	66, 693	781	66, 302	1, 579, 513	23. 68
51–52	12. 60	65, 912	830	65, 497	1, 513, 211	22. 96
52–53	13. 58	65, 082	884	64, 640	1, 447, 714	22. 24
53–54	14. 64	64, 198	940	63, 728	1, 383, 074	21. 54
54–55	15. 77	63, 258	997	62, 760	1, 319, 346	20. 86

[1] All except white and Negro.

TABLE 12.—LIFE TABLE FOR OTHER RACES,[1] FEMALES IN THE UNITED STATES: 1939–1941—Continued

YEAR OF AGE	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of year of age	Number living at beginning of year of age	Number dying during year of age	In year of age	In year of age and all later years	Average number of years of life remaining at beginning of year of age
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+1$	$1,000q_x$	l_x	d_x	L_x	T_x	$\mathring{e}_x$
55–56	16.96	62,261	1,056	61,734	1,256,586	20.18
56–57	18.19	61,205	1,113	60,648	1,194,852	19.52
57–58	19.47	60,092	1,170	59,507	1,134,204	18.87
58–59	20.78	58,922	1,225	58,310	1,074,697	18.24
59–60	22.13	57,697	1,276	57,059	1,016,387	17.62
60–61	23.53	56,421	1,328	55,757	959,328	17.00
61–62	24.99	55,093	1,376	54,405	903,571	16.40
62–63	26.52	53,717	1,425	53,004	849,166	15.81
63–64	28.14	52,292	1,472	51,556	796,162	15.23
64–65	29.87	50,820	1,517	50,061	744,606	14.65
65–66	31.71	49,303	1,564	48,521	694,545	14.09
66–67	33.70	47,739	1,609	46,935	646,024	13.53
67–68	35.85	46,130	1,654	45,303	599,089	12.99
68–69	38.20	44,476	1,699	43,627	553,786	12.45
69–70	40.80	42,777	1,745	41,904	510,159	11.93
70–71	43.73	41,032	1,794	40,135	468,255	11.41
71–72	47.08	39,238	1,847	38,315	428,120	10.91
72–73	50.92	37,391	1,904	36,438	389,805	10.43
73–74	55.27	35,487	1,962	34,506	353,367	9.96
74–75	59.98	33,525	2,011	32,520	318,861	9.51
75–76	64.85	31,514	2,043	30,493	286,341	9.09
76–77	69.66	29,471	2,053	28,444	255,848	8.68
77–78	74.21	27,418	2,035	26,401	227,404	8.29
78–79	78.37	25,383	1,989	24,388	201,003	7.92
79–80	82.32	23,394	1,926	22,431	176,615	7.55
80–81	86.33	21,468	1,853	20,541	154,184	7.18
81–82	90.66	19,615	1,778	18,726	133,643	6.81
82–83	95.56	17,837	1,705	16,984	114,917	6.44
83–84	101.31	16,132	1,634	15,315	97,933	6.07
84–85	108.10	14,498	1,567	13,714	82,618	5.70
85–86	116.15	12,931	1,502	12,180	68,904	5.33
86–87	125.67	11,429	1,437	10,711	56,724	4.96
87–88	136.86	9,992	1,367	9,308	46,013	4.60
88–89	149.93	8,625	1,293	7,979	36,705	4.26
89–90	165.08	7,332	1,211	6,726	28,726	3.92
90–91	182.53	6,121	1,117	5,563	22,000	3.59
91–92	202.49	5,004	1,013	4,497	16,437	3.28
92–93	225.15	3,991	899	3,542	11,940	2.99
93–94	250.73	3,092	775	2,705	8,398	2.72
94–95	279.43	2,317	647	1,993	5,693	2.46
95–96	311.46	1,670	520	1,409	3,700	2.22
96–97	347.03	1,150	399	950	2,291	1.99
97–98	386.35	751	290	606	1,341	1.79
98–99	429.62	461	198	362	735	1.60
99–100	477.05	263	126	200	373	1.42
100–101	528.84	137	72	101	173	1.26
101–102	585.21	65	38	46	72	1.11
102–103	646.37	27	18	18	26	.98
103–104	712.51	9	6	6	8	.86
104–105	783.85	3	2	2	2	.75
105–106	860.59	1	1	0	0	.64

[1] All except white and Negro.

NOTE.—Rates of mortality at ages above 87 are not based on actual statistics at these ages, but have been obtained by mathematical extrapolation from mortality rates at younger ages. Other life table functions at these ages are based on the extrapolated rates of mortality, and may not necessarily represent actual conditions.

TABLE 13.—LIFE TABLE FUNCTIONS FOR THE FIRST YEAR OF LIFE, IN THE UNITED STATES: 1939–1941

AGE INTERVAL	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of age interval	Number alive at beginning of age interval	Number dying during age interval	In the age interval	In this and all subsequent age intervals	Average number of years of life remaining at beginning of age interval
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+t$	$_tq_x$	l_x	l_x-l_{x+t}	T_x-T_{x+t}	T_x	$\mathring{e}_x$
TOTAL POPULATION						
0–1 day	13. 97	100, 000	1, 397	271	6, 362, 494	63. 62
1–2 days	3. 67	98, 603	362	269	6, 362, 223	64. 52
2–3 days	2. 32	98, 241	228	269	6, 361, 954	64. 76
3 days to 1 week	3. 71	98, 013	364	1, 071	6, 361, 685	64. 91
1–2 weeks	2. 60	97, 649	254	1, 869	6, 360, 614	65. 14
2–3 weeks	1. 70	97, 395	166	1, 865	6, 358, 745	65. 29
3 weeks to 1 month	1. 46	97, 229	142	2, 512	6, 356, 880	65. 38
0–1 month	29. 13	100, 000	2, 913	8, 126	6, 362, 494	63. 62
1–2 months	3. 64	97, 087	353	8, 076	6, 354, 368	65. 45
2–3 months	2. 90	96, 734	281	8, 049	6, 346, 292	65. 61
3–4 months	2. 41	96, 453	232	8, 028	6, 338, 243	65. 71
4–5 months	1. 95	96, 221	188	8, 011	6, 330, 215	65. 79
5–6 months	1. 65	96, 033	158	7, 996	6, 322, 204	65. 83
6–7 months	1. 42	95, 875	136	7, 984	6, 314, 208	65. 86
7–8 months	1. 20	95, 739	115	7, 973	6, 306, 224	65. 87
8–9 months	1. 06	95, 624	101	7, 964	6, 298, 251	65. 86
9–10 months	. 92	95, 523	88	7, 957	6, 290, 287	65. 85
10–11 months	. 78	95, 435	74	7, 950	6, 282, 330	65. 83
11–12 months	. 74	95, 361	71	7, 944	6, 274, 380	65. 80
TOTAL MALES						
0–1 day	15. 71	100, 000	1, 571	271	6, 160, 087	61. 60
1–2 days	4. 19	98, 429	412	269	6, 159, 816	62. 58
2–3 days	2. 75	98, 017	270	268	6, 159, 547	62. 84
3 days to 1 week	4. 27	97, 747	417	1, 068	6, 159, 279	63. 01
1–2 weeks	2. 85	97, 330	277	1, 862	6, 158, 211	63. 27
2–3 weeks	1. 85	97, 053	180	1, 858	6, 156, 349	63. 43
3 weeks to 1 month	1. 62	96, 873	157	2, 502	6, 154, 491	63. 53
0–1 month	32. 84	100, 000	3, 284	8, 098	6, 160, 087	61. 60
1–2 months	4. 06	96, 716	393	8, 043	6, 151, 989	63. 61
2–3 months	3. 19	96, 323	307	8, 014	6, 143, 946	63. 78
3–4 months	2. 61	96, 016	251	7, 991	6, 135, 932	63. 91
4–5 months	2. 12	95, 765	203	7, 972	6, 127, 941	63. 99
5–6 months	1. 79	95, 562	171	7, 956	6, 119, 969	64. 04
6–7 months	1. 51	95, 391	144	7, 943	6, 112, 013	64. 07
7–8 months	1. 32	95, 247	126	7, 932	6, 104, 070	64. 09
8–9 months	1. 16	95, 121	110	7, 922	6, 096, 138	64. 09
9–10 months	. 98	95, 011	93	7, 914	6, 088, 216	64. 08
10–11 months	. 85	94, 918	81	7, 906	6, 080, 302	64. 06
11–12 months	. 79	94, 837	75	7, 900	6, 072, 396	64. 03
TOTAL FEMALES						
0–1 day	12. 14	100, 000	1, 214	272	6, 588, 801	65. 89
1–2 days	3. 14	98, 786	310	270	6, 588, 529	66. 69
2–3 days	1. 88	98, 476	185	269	6, 588, 259	66. 90
3 days to 1 week	3. 13	98, 291	308	1, 075	6, 587, 990	67. 03
1–2 weeks	2. 34	97, 983	229	1, 875	6, 586, 915	67. 23
2–3 weeks	1. 53	97, 754	150	1, 872	6, 585, 040	67. 36
3 weeks to 1 month	1. 28	97, 604	125	2, 522	6, 583, 168	67. 45
0–1 month	25. 21	100, 000	2, 521	8, 155	6, 588, 801	65. 89
1–2 months	3. 18	97, 479	310	8, 110	6, 580, 646	67. 51
2–3 months	2. 60	97, 169	253	8, 087	6, 572, 536	67. 64
3–4 months	2. 20	96, 916	213	8, 067	6, 564, 449	67. 73
4–5 months	1. 79	96, 703	173	8, 051	6, 556, 382	67. 80
5–6 months	1. 49	96, 530	144	8, 038	6, 548, 331	67. 84
6–7 months	1. 32	96, 386	127	8, 027	6, 540, 293	67. 86
7–8 months	1. 08	96, 259	104	8, 017	6, 532, 266	67. 86
8–9 months	. 95	96, 155	91	8, 009	6, 524, 249	67. 85
9–10 months	. 85	96, 064	82	8, 002	6, 516, 240	67. 83
10–11 months	. 70	95, 982	67	7, 996	6, 508, 238	67. 81
11–12 months	. 70	95, 915	67	7, 990	6, 500, 242	67. 77

TABLE 13.—LIFE TABLE FUNCTIONS FOR THE FIRST YEAR OF LIFE, IN THE UNITED STATES: 1939–1941—Continued

AGE INTERVAL	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of age interval	Number alive at beginning of age interval	Number dying during age interval	In the age interval	In this and all subsequent age intervals	Average number of years of life remaining at beginning of age interval
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+t$	$_tq_x$	l_x	l_x-l_{x+t}	T_x-T_{x+t}	T_x	$\mathring{e}_x$
TOTAL WHITES						
0–1 day	13.67	100,000	1,367	271	6,492,419	64.92
1–2 days	3.52	98,633	347	270	6,492,148	65.82
2–3 days	2.18	98,286	214	269	6,491,878	66.05
3 days to 1 week	3.35	98,072	329	1,072	6,491,609	66.19
1–2 weeks	2.26	97,743	221	1,871	6,490,537	66.40
2–3 weeks	1.50	97,522	146	1,867	6,488,666	66.54
3 weeks to 1 month	1.28	97,376	125	2,516	6,486,799	66.62
0–1 month	27.49	100,000	2,749	8,136	6,492,419	64.92
1–2 months	3.18	97,251	309	8,091	6,484,283	66.68
2–3 months	2.56	96,942	248	8,068	6,476,192	66.80
3–4 months	2.09	96,694	202	8,049	6,468,124	66.89
4–5 months	1.67	96,492	161	8,034	6,460,075	66.95
5–6 months	1.42	96,331	137	8,022	6,452,041	66.98
6–7 months	1.20	96,194	115	8,011	6,444,019	66.99
7–8 months	1.04	96,079	100	8,002	6,436,008	66.99
8–9 months	.92	95,979	88	7,995	6,428,006	66.97
9–10 months	.80	95,891	77	7,988	6,420,011	66.95
10–11 months	.69	95,814	66	7,982	6,412,023	66.92
11–12 months	.66	95,748	63	7,976	6,404,041	66.88
WHITE MALES						
0–1 day	15.38	100,000	1,538	271	6,281,188	62.81
1–2 days	4.03	98,462	397	269	6,280,917	63.79
2–3 days	2.57	98,065	252	268	6,280,648	64.05
3 days to 1 week	3.84	97,813	376	1,069	6,280,380	64.21
1–2 weeks	2.49	97,437	243	1,865	6,279,311	64.44
2–3 weeks	1.64	97,194	159	1,861	6,277,446	64.59
3 weeks to 1 month	1.44	97,035	140	2,507	6,275,585	64.67
0–1 month	31.05	100,000	3,105	8,110	6,281,188	62.81
1–2 months	3.59	96,895	348	8,060	6,273,078	64.74
2–3 months	2.83	96,547	273	8,034	6,265,018	64.89
3–4 months	2.27	96,274	219	8,014	6,256,984	64.99
4–5 months	1.81	96,055	174	7,997	6,248,970	65.06
5–6 months	1.54	95,881	148	7,984	6,240,973	65.09
6–7 months	1.26	95,733	121	7,973	6,232,989	65.11
7–8 months	1.14	95,612	109	7,963	6,225,016	65.11
8–9 months	.99	95,503	95	7,955	6,217,053	65.10
9–10 months	.85	95,408	81	7,947	6,209,098	65.08
10–11 months	.76	95,327	72	7,941	6,201,151	65.05
11–12 months	.70	95,255	67	7,935	6,193,210	65.02
WHITE FEMALES						
0–1 day	11.87	100,000	1,187	272	6,728,965	67.29
1–2 days	2.98	98,813	294	270	6,728,693	68.10
2–3 days	1.76	98,519	173	269	6,728,423	68.30
3 days to 1 week	2.85	98,346	280	1,075	6,728,154	68.41
1–2 weeks	2.02	98,066	198	1,877	6,727,079	68.60
2–3 weeks	1.35	97,868	132	1,874	6,725,202	68.72
3 weeks to 1 month	1.11	97,736	108	2,525	6,723,328	68.79
0–1 month	23.72	100,000	2,372	8,162	6,728,965	67.29
1–2 months	2.75	97,628	268	8,125	6,720,803	68.84
2–3 months	2.28	97,360	222	8,104	6,712,678	68.95
3–4 months	1.89	97,138	184	8,087	6,704,574	69.02
4–5 months	1.53	96,954	148	8,073	6,696,487	69.07
5–6 months	1.28	96,806	124	8,062	6,688,414	69.09
6–7 months	1.12	96,682	108	8,052	6,680,352	69.10
7–8 months	.94	96,574	91	8,044	6,672,300	69.09
8–9 months	.84	96,483	81	8,037	6,664,256	69.07
9–10 months	.74	96,402	71	8,031	6,656,219	69.05
10–11 months	.62	96,331	60	8,025	6,648,188	69.01
11–12 months	.62	96,271	60	8,020	6,640,163	68.97

TABLE 13.—LIFE TABLE FUNCTIONS FOR THE FIRST YEAR OF LIFE, IN THE UNITED STATES: 1939–1941—Continued

AGE INTERVAL	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of age interval	Number alive at beginning of age interval	Number dying during age interval	In the age interval	In this and all subsequent age intervals	Average number of years of life remaining at beginning of age interval
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to $x+t$	${}_tq_x$	l_x	l_x-l_{x+t}	T_x-T_{x+t}	T_x	$\mathring{e}_x$
TOTAL NEGROES						
0–1 day	17.06	100,000	1,706	271	5,385,044	53.85
1–2 days	5.01	98,294	492	268	5,384,773	54.78
2–3 days	3.49	97,802	341	267	5,384,505	55.06
3 days to 1 week	6.19	97,461	603	1,064	5,384,238	55.25
1–2 weeks	4.89	96,858	474	1,851	5,383,174	55.58
2–3 weeks	3.09	96,384	298	1,844	5,381,323	55.83
3 weeks to 1 month	2.67	96,086	257	2,481	5,379,479	55.99
0–1 month	41.71	100,000	4,171	8,046	5,385,044	53.85
1–2 months	6.65	95,829	637	7,959	5,376,998	56.11
2–3 months	5.14	95,192	489	7,912	5,369,039	56.40
3–4 months	4.47	94,703	423	7,874	5,361,127	56.61
4–5 months	3.80	94,280	358	7,842	5,353,253	56.78
5–6 months	3.06	93,922	287	7,815	5,345,411	56.91
6–7 months	2.83	93,635	265	7,792	5,337,596	57.00
7–8 months	2.23	93,370	208	7,772	5,329,804	57.08
8–9 months	1.94	93,162	181	7,756	5,322,032	57.13
9–10 months	1.68	92,981	156	7,742	5,314,276	57.15
10–11 months	1.30	92,825	121	7,730	5,306,534	57.17
11–12 months	1.29	92,704	120	7,720	5,298,804	57.16
NEGRO MALES						
0–1 day	19.21	100,000	1,921	271	5,225,657	52.26
1–2 days	5.54	98,079	543	268	5,225,386	53.28
2–3 days	4.17	97,536	407	266	5,225,118	53.57
3 days to 1 week	7.26	97,129	705	1,060	5,224,852	53.79
1–2 weeks	5.29	96,424	510	1,843	5,223,792	54.18
2–3 weeks	3.36	95,914	322	1,835	5,221,949	54.44
3 weeks to 1 month	2.89	95,592	276	2,468	5,220,114	54.61
0–1 month	46.84	100,000	4,684	8,011	5,225,657	52.26
1–2 months	7.33	95,316	699	7,914	5,217,646	54.74
2–3 months	5.60	94,617	530	7,863	5,209,732	55.06
3–4 months	4.80	94,087	452	7,822	5,201,869	55.29
4–5 months	4.18	93,635	391	7,787	5,194,047	55.47
5–6 months	3.34	93,244	311	7,757	5,186,260	55.62
6–7 months	3.14	92,933	292	7,732	5,178,503	55.72
7–8 months	2.50	92,641	232	7,710	5,170,771	55.82
8–9 months	2.24	92,409	207	7,692	5,163,061	55.87
9–10 months	1.80	92,202	166	7,677	5,155,369	55.91
10–11 months	1.47	92,036	135	7,664	5,147,692	55.93
11–12 months	1.40	91,901	129	7,653	5,140,028	55.93
NEGRO FEMALES						
0–1 day	14.86	100,000	1,486	271	5,556,051	55.56
1–2 days	4.48	98,514	441	269	5,555,780	56.40
2–3 days	2.77	98,073	272	268	5,555,511	56.65
3 days to 1 week	5.10	97,801	499	1,068	5,555,243	56.80
1–2 weeks	4.50	97,302	438	1,860	5,554,175	57.08
2–3 weeks	2.82	96,864	273	1,853	5,552,315	57.32
3 weeks to 1 month	2.45	96,591	237	2,494	5,550,462	57.46
0–1 month	36.46	100,000	3,646	8,083	5,556,051	55.56
1–2 months	5.96	96,354	574	8,006	5,547,968	57.58
2–3 months	4.68	95,780	448	7,963	5,539,962	57.84
3–4 months	4.12	95,332	393	7,928	5,531,999	58.03
4–5 months	3.41	94,939	324	7,898	5,524,071	58.19
5–6 months	2.76	94,615	261	7,874	5,516,173	58.30
6–7 months	2.52	94,354	238	7,853	5,508,299	58.38
7–8 months	1.94	94,116	183	7,835	5,500,446	58.44
8–9 months	1.64	93,933	154	7,821	5,492,611	58.47
9–10 months	1.56	93,779	146	7,809	5,484,790	58.49
10–11 months	1.13	93,633	106	7,798	5,476,981	58.49
11–12 months	1.19	93,527	111	7,789	5,469,183	58.48

TABLE 13.—LIFE TABLE FUNCTIONS FOR THE FIRST YEAR OF LIFE, IN THE UNITED STATES: 1939–1941—Continued

AGE INTERVAL	MORTALITY RATE	OF 100,000 BORN ALIVE		STATIONARY POPULATION		AVERAGE FUTURE LIFETIME
Period of life between two exact ages stated	Number dying per 1,000 alive at beginning of age interval	Number alive at beginning of age interval	Number dying during age interval	In the age interval	In this and all subsequent age intervals	Average number of years of life remaining at beginning of age interval
(1)	(2)	(3)	(4)	(5)	(6)	(7)
x to x+t	$_tq_x$	l_x	l_x-l_{x+t}	T_x-T_{x+t}	T_x	$\mathring{e}_x$
TOTAL OTHER RACES[1]						
0–1 day	14.09	100,000	1,409	271	5,435,389	54.35
1–2 days	3.37	98,591	332	269	5,435,118	55.13
2–3 days	3.10	98,259	305	269	5,434,849	55.31
3 days to 1 week	6.75	97,954	661	1,069	5,434,580	55.48
1–2 weeks	5.24	97,293	510	1,859	5,433,511	55.85
2–3 weeks	3.35	96,783	324	1,851	5,431,652	56.12
3 weeks to 1 month	3.40	96,459	328	2,489	5,429,801	56.29
0–1 month	38.69	100,000	3,869	8,077	5,435,389	54.35
1–2 months	9.20	96,131	884	7,974	5,427,312	56.46
2–3 months	8.18	95,247	779	7,905	5,419,338	56.90
3–4 months	7.26	94,468	686	7,844	5,411,433	57.28
4–5 months	6.39	93,782	599	7,790	5,403,589	57.62
5–6 months	5.59	93,183	521	7,744	5,395,799	57.91
6–7 months	4.87	92,662	451	7,703	5,388,055	58.15
7–8 months	4.23	92,211	390	7,668	5,380,352	58.35
8–9 months	3.69	91,821	339	7,638	5,372,684	58.51
9–10 months	3.21	91,482	294	7,611	5,365,046	58.65
10–11 months	2.84	91,188	259	7,588	5,357,435	58.75
11–12 months	2.55	90,929	232	7,568	5,349,847	58.84
OTHER RACES,[1] MALES						
0–1 day	15.41	100,000	1,541	271	5,356,374	53.56
1–2 days	3.46	98,459	341	269	5,356,103	54.40
2–3 days	3.22	98,118	316	268	5,355,834	54.59
3 days–1 week	8.18	97,802	800	1,066	5,355,566	54.76
1–2 weeks	6.01	97,002	583	1,853	5,354,500	55.20
2–3 weeks	3.76	96,419	363	1,844	5,352,647	55.51
3 weeks–1 month	3.94	96,056	378	2,478	5,350,803	55.71
0–1 month	43.22	100,000	4,322	8,049	5,356,374	53.56
1–2 months	9.35	95,678	895	7,936	5,348,325	55.90
2–3 months	8.40	94,783	796	7,865	5,340,389	56.34
3–4 months	7.50	93,987	705	7,803	5,332,524	56.74
4–5 months	6.65	93,282	620	7,748	5,324,721	57.08
5–6 months	5.86	92,662	543	7,699	5,316,973	57.38
6–7 months	5.10	92,119	470	7,657	5,309,274	57.63
7–8 months	4.41	91,649	404	7,621	5,301,617	57.85
8–9 months	3.82	91,245	349	7,589	5,293,996	58.02
9–10 months	3.27	90,896	297	7,562	5,286,407	58.16
10–11 months	2.76	90,599	250	7,540	5,278,845	58.27
11–12 months	2.36	90,349	213	7,520	5,271,305	58.34
OTHER RACES,[1] FEMALES						
0–1 day	12.70	100,000	1,270	271	5,583,750	55.84
1–2 days	3.25	98,730	321	270	5,583,479	56.55
2–3 days	3.00	98,409	295	269	5,583,209	56.73
3 days to 1 week	5.27	98,114	517	1,071	5,582,940	56.90
1–2 weeks	4.43	97,597	432	1,866	5,581,869	57.19
2–3 weeks	2.91	97,165	283	1,859	5,580,003	57.43
3 weeks to 1 month	2.85	96,882	276	2,501	5,578,144	57.58
0–1 month	33.94	100,000	3,394	8,107	5,583,750	55.84
1–2 months	9.03	96,606	872	8,014	5,575,643	57.72
2–3 months	7.96	95,734	762	7,946	5,567,629	58.16
3–4 months	7.01	94,972	666	7,887	5,559,683	58.54
4–5 months	6.13	94,306	578	7,835	5,551,796	58.87
5–6 months	5.30	93,728	497	7,790	5,543,961	59.15
6–7 months	4.63	93,231	432	7,751	5,536,171	59.38
7–8 months	4.04	92,799	375	7,718	5,528,420	59.57
8–9 months	3.56	92,424	329	7,688	5,520,702	59.73
9–10 months	3.16	92,095	291	7,662	5,513,014	59.86
10–11 months	2.92	91,804	268	7,639	5,505,352	59.97
11–12 months	2.76	91,536	253	7,617	5,497,713	60.06

[1] All except white and Negro.

PART III

ACTUARIAL TABLES

Scope of the actuarial tables

The actuarial functions included in this volume are based on the 1939–1941 life tables for white males and white females in the United States, and on a Makeham graduation of the life table for total whites, which was prepared in order to facilitate the calculation of values of annuities and other benefits involving two or more joint lives. In addition to the elementary life table values, the functions tabulated on the basis of the white males and white females tables are the usual commutation columns (C, D, M, N, R, and S), whole life immediate annuity values, and both single and annual premiums for whole life assurances. These are given at five interest rates: 2, 2½, 3, 3½, and 4 percent.

The functions tabulated for the makehamized mortality table [1] are the elementary values including the force of mortality, single whole life immediate annuities, and equal age whole life immediate annuities for two, three, and four joint lives. The annuity values are shown for four interest rates: 2, 2½, 3, and 4 percent. A table of the Makeham constants and their common logarithms, and a table of uniform seniority for two lives are also included. This mortality table follows Makeham's law at ages 17 and over. An auxiliary table, to facilitate the approximate computation of joint life annuity values when one or more of the lives are under age 17, is given on page 96.

Comparison with mortality tables based on the experience of insured lives

It is interesting to compare the life tables for which actuarial functions are tabulated here with those based on recent life insurance experience.

Among such tables, the greatest interest attaches to the Commissioners 1941 Standard Ordinary Mortality table [2] which has now (August 1945) been recognized by law in 25 States [3] including the 23 which have enacted the Standard Non-Forfeiture and Valuation Laws [4] recommended by the National Association of Insurance Commissioners in December 1942. However, this table cannot be regarded as reflecting current life insurance experience, since the rates of mortality include adjustments which are considered sufficient to provide "reasonable margins for adverse fluctuations in mortality and for contingencies," together with an additional factor of conservatism in the calculation of premiums.[5] However, the underlying experience table, excluding these margins, which is known as the 1930–1940 Experience table, is also available.[6] In table N, the rates of mortality for white males in the United States, both in 1929–1931 and in 1939–1941, are compared with those of both the Commissioners 1941 Standard Ordinary Mortality table and the 1930–1940 Experience table. The corresponding values from the makehamized mortality table for total whites (table 38 of this volume) are also shown. The 1930–1940 Experience table is based primarily on the experience during the decade of 16 life insurance companies (15 United States companies and 1 Canadian company) which include the 13 largest companies in the United States and Canada.[7]

TABLE N.—ANNUAL RATE OF MORTALITY PER 1,000 AT SELECTED AGES FROM CERTAIN UNITED STATES LIFE TABLES FOR 1929–1931 AND 1939–1941, AND FROM MORTALITY TABLES BASED ON RECENT LIFE INSURANCE EXPERIENCE IN THE UNITED STATES AND CANADA

AGE	United States white males, 1929–1931	United States white males, 1939–1941	United States total whites, 1939–1941, makehamized	Commissioners 1941 Standard Ordinary	1930–1940 Experience
0	62.32	48.12	43.15	[1] 22.58	[1] 21.82
1	9.93	4.87	4.60	5.77	5.01
5	2.66	1.38	1.24	2.76	1.96
10	1.47	1.00	.85	1.97	1.11
15	2 13	1.43	1.30	2.15	1.30
20	3.18	2.12	1.65	2.43	1.67
25	3 71	2.43	1.98	2.88	2.01
30	4.13	2.79	2.49	3.56	2.22
35	5.10	3.63	3.29	4.59	2.79
40	6.79	5.13	4.53	6.18	4.06
45	9.29	7.66	6.46	8.61	6.24
50	12.78	11.55	9.45	12.32	9.76
55	18.19	17.37	14.08	17.98	15.40
60	26.44	25.48	21.25	26.59	23.69
65	38.65	36.85	32.30	39.64	36.13
70	57.96	54.54	49.26	59.30	54.25
75	85.26	83.13	75.06	88.64	81.05
80	129.97	124.71	113.82	131.85	121.06
85	184.68	181.04	170.94	194.13	178.98
90	245.50	248.94	252.61	280.99	265.23

[1] Extension to age 0 by Malvin E. Davis. (See Transactions, Actuarial Society of America, vol. 43, Part 1, No. 107, p. 103, May 1942.) These rates include a relatively small proportion of experience in the first week of life, where the mortality rate is high.

[1] The term "mortality table," which is the name customarily applied by actuaries, is a more appropriate one to describe the makehamized table, since this table does not include values of the average future lifetime or of the functions relating to the stationary population.

[2] National Association of Insurance Commissioners, *Report of the Committee To Study Non-Forfeiture Benefits and Related Matters*, p. 186, 1941.

[3] California, Colorado, Connecticut, Delaware, Illinois, Indiana, Kentucky, Maine, Maryland, Massachusetts, Michigan, Missouri, Montana, Nebraska, New Hampshire, New Jersey, New Mexico, North Carolina, Oregon, Pennsylvania, Tennessee, Vermont, Virginia, West Virginia, and Wisconsin. Colorado and Connecticut have recognized the new table but have not enacted the standard laws.

[4] National Association of Insurance Commissioners, *Report of the Committee of Commissioners Appointed To Consider and To Make Recommendations on the Report of the Committee To Study Non-Forfeiture Benefits and Related Matters*, Exhibits A and B, 1942.

[5] Thompson, John S., *The Commissioners 1941 Standard Ordinary Mortality Table*, Transactions, Actuarial Society of America, vol. 42, Part 2, No. 106, pp. 314–340, September 1941.

[6] Thompson, *op. cit.*, p. 325. This article gives a complete account of the method of construction of both tables.

[7] *Report of Joint Committee on Mortality*, Transactions, Actuarial Society of America, vol. 35, Part 2, No. 92, pp. 353–356, October 1934, and vol. 42, Part 1, No. 105, pp. 140–149, May 1941.

It can therefore be considered representative of recent life insurance experience in the two countries.

In comparing this table with those based on the experience of the general population, several points should be kept in mind. In the first place, although the insurance experience included insurance on the lives of women as well as men, policies on the lives of men are far more numerous. In addition, tabulation was on the basis of amounts of insurance rather than lives, so that the death of an individual having $10,000 of insurance has the same effect as 10 deaths of persons with $1,000 policies. As men, in general, carry much larger amounts of insurance than women, it is clear that the total experience reflects the mortality of males to a much greater extent than that of females. In the second place, all group and industrial insurance were excluded from the experience, which in view of the tabulation by amounts rather than lives insured, suggests that a very substantial proportion of the total insurance represented was held by persons in the higher income levels. In the third place, the experience was limited to persons who had undergone a medical examination at the time of issuance of the policy, and also (with some exceptions at the oldest and the youngest ages) to those policies issued in 1925 or later, while at the same time all experience during the first 5 years of the existence of a particular policy was excluded.[8] This means that the experience consisted, for the most part, of persons who had been medically examined between 5 and 15 years prior to the time of exposure.

[8] Thompson, *op. cit.*, pp. 316, 322–327.

Table N shows that mortality rates under the 1930–1940 Experience table are lower at most ages than those of white males in the United States in 1939–1941. This is probably due primarily to the influence of the greater weight given, in the insurance experience, to persons in the higher income brackets, and to the inclusion of a number of female lives, and only slightly due, if at all, to the medical examination, the effect of which would be expected to have largely worn off after 5 to 15 years. The difference is actually somewhat greater than the figures indicate, since the insurance experience covers a somewhat earlier period. The mortality rates in the Commissioners table are, in general, intermediate between those for white males in 1929–1931 and 1939–1941 to about age 60, after which they are higher than either. When the mortality rates of the 1930–1940 Experience table are compared with those of the makehamized mortality table for total whites in 1939–1941, it is found that the insurance table shows lower rates at ages 13 to 19 and 26 to 47 and higher rates elsewhere. At ages above 47 the difference increases rapidly. However, if comparison is made with the unmakehamized mortality rates for total whites in 1939–1941 (table 4 in this volume), the ages at which the life insurance mortality is lower are 18 to 48.

Table O gives a similar comparison of net values of immediate whole life annuities and single and annual net premiums for whole life insurance at 3 percent interest. In this case, the life table for white males in the United States in 1929–1931 is not included in the comparison, as commutation columns on this basis are not available. These premiums and values are based on interest and mortality only, and include no

TABLE O.—IMMEDIATE WHOLE LIFE ANNUITY VALUES AND SINGLE AND ANNUAL NET PREMIUMS [1] AT 3 PERCENT INTEREST, DERIVED FROM CERTAIN UNITED STATES LIFE TABLES FOR 1939–1941 AND FROM MORTALITY TABLES BASED ON RECENT LIFE INSURANCE EXPERIENCE IN THE UNITED STATES AND CANADA

AGE	VALUE OF IMMEDIATE WHOLE LIFE ANNUITY OF ONE PER ANNUM				NET SINGLE PREMIUM FOR WHOLE LIFE INSURANCE OF ONE UNIT				NET ANNUAL PREMIUM FOR WHOLE LIFE INSURANCE OF ONE UNIT			
	United States white males, 1939–1941	United States total whites, 1939–1941, makehamized	Commissioners 1941 Standard Ordinary	1930–1940 Experience	United States white males, 1939–1941	United States total whites, 1939–1941, makehamized	Commissioners 1941 Standard Ordinary	1930–1940 Experience	United States white males, 1939–1941	United States total whites, 1939–1941, makehamized	Commissioners 1941 Standard Ordinary	1930–1940 Experience
0	26.2661	26.7047	[2] 26.3093	[2] 27.1190	0.20584	0.19307	[2] 0.20196	[2] 0.18100	0.00755	0.00697	[2] 0.00737	[2] 0.00644
1	27.4216	27.7461	26.8195	27.5557	.17218	.16274	.18972	.16828	.00606	.00566	.00682	.00589
5	27.0080	27.3545	26.4770	27.2266	.18423	.17415	.19970	.17787	.00658	.00614	.00727	.00630
10	26.1725	26.5553	25.7391	26.4962	.20857	.19742	.22119	.19914	.00768	.00716	.00827	.00724
15	25.1862	25.6216	24.8033	25.5628	.23730	.22462	.24845	.22632	.00906	.00844	.00963	.00852
20	24.1201	24.5966	23.7453	24.5249	.26835	.25447	.27926	.25656	.01068	.00994	.01129	.01005
25	22.9490	23.4392	22.5489	23.3616	.30246	.28818	.31411	.29044	.01263	.01179	.01334	.01192
30	21.6056	22.1352	21.2078	22.0339	.34158	.32616	.35317	.32911	.01511	.01410	.01590	.01429
35	20.0917	20.6798	19.7207	20.5160	.38568	.36855	.39649	.37332	.01829	.01700	.01913	.01735
40	18.4247	19.0744	18.0928	18.8243	.43422	.41531	.44390	.42259	.02235	.02069	.02325	.02132
45	16.6360	17.3295	16.3393	16.9913	.48634	.46613	.49497	.47598	.02758	.02543	.02855	.02646
50	14.7687	15.4660	14.4864	15.0544	.54074	.52041	.54894	.53239	.03429	.03161	.03545	.03316
55	12.8673	13.5186	12.5730	13.0686	.59612	.57713	.60467	.59023	.04299	.03975	.04455	.04195
60	10.9775	11.5350	10.6494	11.0926	.65112	.63491	.66070	.64779	.05436	.05065	.05672	.05357
65	9.1155	9.5745	8.7742	9.1781	.70538	.69201	.71531	.70355	.06973	.06544	.07318	.06912
70	7.3134	7.7029	7.0091	7.3785	.75787	.74652	.76673	.75597	.09116	.08578	.09573	.09023
75	5.6634	5.9845	5.4104	5.7324	.80592	.79657	.81329	.80391	.12095	.11405	.12687	.11941
80	4.2681	4.4730	4.0210	4.2787	.84655	.84059	.85376	.84625	.16069	.15359	.17004	.16031
85	3.1562	3.2026	2.8633	3.0444	.87896	.87760	.88748	.88221	.21148	.20882	.22972	.21813
90	2.3350	2.1840	1.9290	1.9084	.90285	.90726	.91469	.91529	.27072	.28494	.31228	.31470

[1] These premiums and values are based on interest and mortality only, and include no allowance for operating expenses, taxes, or contingencies. They are not to be compared with the gross premium rates actually charged by life insurance companies.

[2] Based on Davis extension. See footnote to table N.

allowance for operating expenses, taxes, or contingencies. They are not to be compared with the gross premium rates actually charged by life insurance companies.

Uses of the actuarial tables

The actuarial tables based on the 1939–1941 United States life tables for white males and white females can be used in making valuations and cost estimates for pension schemes and collective plans for providing benefits to dependent survivors, when the covered group can be considered representative of the general population of the Nation. This implies, in the case of death benefits, that the members of the group have not been selected primarily on the basis of physical fitness, economic status, or any other characteristic which would materially affect their mortality prospects; and, in the case of annuities, that there has not been a strong element of self-selection, such as is commonly exercised by annuitants of life insurance companies. An example would be a social insurance coverage which applies on a compulsory basis to all persons engaged in specified occupations. Of course, groups in particular occupations involving a special hazard could not be considered representative of the general population.

The actuarial tables can also be used in courts of law in damage suits involving loss of income through death or disablement, and in all other cases in which a lump sum payment is to replace a series of periodic payments during the life of an individual, and vice versa. Similar, but frequently more complicated, problems arise in the valuation of estates, particularly when two or more different heirs have an immediate or contingent interest in the same property. The tables might also be used, in some cases, in determining the value of life annuities payable under workmen's compensation laws.

It would be outside the scope of this volume to enter into any discussion of the technicalities of these various uses. However, they all involve the calculation of present values of life annuities or net premiums for life insurance benefits. The basic mathematical theory underlying these calculations is presented from an elementary standpoint on pages 85 to 92 of part IV; and specific instructions in the use of the actuarial tables in this volume, together with numerical examples, are given on pages 92 to 99. For the reader who is already conversant with the general theory, but wishes to acquaint himself with the particular arrangement of tables adopted in this volume, the following summary may be helpful.

Auxiliary tables intended for use in connection with the actuarial tables

Subject	Table	Page
Reference lists of formulas:		
For single life annuities	P	87
For single life assurance benefits	Q	88
For annuities and assurance benefits involving two or more joint lives	R	91
Auxiliary tables for use in special calculations:		
In computing values of joint life annuities involving ages under 17:		
Present value of one due in 1 to 17 years at 2, 2½, 3, and 4 percent interest	S	95
Adjustment factor r for approximating values of joint life annuities involving ages under 17	U	96
In computing assurance premiums involving two or more joint lives:		
Values of the rate of discount for various rates of interest	Y	97
In estimating joint life annuity values based on the separate life tables for white males and white females:		
Adjusted ages for use in the rougher method of approximation	Z	98

Mathematical notation employed in the actuarial tables

The symbols used in the headings of the actuarial tables conform to standard actuarial practice except that the simpler forms N_x and S_x are employed instead of $\mathbb{N}_x$ and $\mathbb{S}_x$. The special open-face symbols have never served any real need except in England,[9] and their use seems to have been almost wholly confined to English-speaking countries. The usage adopted in this volume, besides conforming to general practice outside the English-speaking world, has been recommended for adoption by a subcommittee designated by the Permanent Committee of the International Congresses of Actuaries to study the revision of the international actuarial notation.[10] In order to avoid any possible confusion, the definitions of the symbols N_x and S_x as used in this volume are given at the bottom of each page of tables in which they appear.

[9] See *Notation Internationale,* pamphlet issued by the Comité Permanent des Congrès Internationaux d'Actuaires, p. 4, Bruxelles, Février 1939; also E. F. Spurgeon, *Life Contingencies,* third edition, pp. 35, 36, 69, Cambridge University Press, London, 1938.

[10] *Notation Internationale* (previously cited), p. 10. The fact that the change was actually proposed by the British actuarial bodies shows that there is general agreement as to its desirability.

Table 14.—United States White Males: 1939–1941—Elementary Values

[In the interest of internal consistency within the actuarial tables, certain of these values have been altered very slightly from those appearing in table 5, p. 34. For explanation, see text, p. 137]

Age	Of 100,000 born alive: Number surviving to each age	Of 100,000 born alive: Number dying in each year of age	Probability of surviving 1 year at each age	Probability of dying in each year of age	Force of mortality at each age	Age	Of 100,000 born alive: Number surviving to each age	Of 100,000 born alive: Number dying in each year of age	Probability of surviving 1 year at each age	Probability of dying in each year of age	Force of mortality at each age
x	l_x	d_x	p_x	q_x	μ_x	x	l_x	d_x	p_x	q_x	μ_x
0	100,000	4,812	0.95188	0.04812	10.29757	55	75,156	1,305	0.98264	0.01736	0.01682
1	95,188	464	.99513	.00487	.00813	56	73,851	1,390	.98118	.01882	.01824
2	94,724	250	.99736	.00264	.00300	57	72,461	1,473	.97967	.02033	.01975
3	94,474	179	.99811	.00189	.00212	58	70,988	1,558	.97805	.02195	.02135
4	94,295	145	.99846	.00154	.00167	59	69,430	1,643	.97634	.02366	.02305
5	94,150	130	.99862	.00138	.00144	60	67,787	1,727	.97452	.02548	.02486
6	94,020	116	.99877	.00123	.00131	61	66,060	1,813	.97256	.02744	.02679
7	93,904	108	.99885	.00115	.00119	62	64,247	1,896	.97049	.02951	.02887
8	93,796	99	.99894	.00106	.00110	63	62,351	1,981	.96823	.03177	.03109
9	93,697	96	.99898	.00102	.00104	64	60,370	2,065	.96579	.03421	.03350
10	93,601	93	.99901	.00099	.00100	65	58,305	2,148	.96316	.03684	.03613
11	93,508	95	.99898	.00102	.00100	66	56,157	2,232	.96025	.03975	.03900
12	93,413	99	.99894	.00106	.00103	67	53,925	2,315	.95707	.04293	.04217
13	93,314	106	.99886	.00114	.00109	68	51,610	2,396	.95357	.04643	.04565
14	93,208	119	.99872	.00128	.00120	69	49,214	2,475	.94971	.05029	.04950
15	93,089	133	.99857	.00143	.00135	70	46,739	2,549	.94546	.05454	.05376
16	92,956	147	.99842	.00158	.00151	71	44,190	2,618	.94076	.05924	.05849
17	92,809	160	.99828	.00172	.00166	72	41,572	2,678	.93558	.06442	.06374
18	92,649	172	.99814	.00186	.00179	73	38,894	2,728	.92986	.07014	.06956
19	92,477	184	.99801	.00199	.00192	74	36,166	2,762	.92363	.07637	.07598
20	92,293	195	.99789	.00211	.00206	75	33,404	2,777	.91687	.08313	.08301
21	92,098	205	.99777	.00223	.00218	76	30,627	2,769	.90959	.09041	.09066
22	91,893	214	.99767	.00233	.00228	77	27,858	2,735	.90182	.09818	.09894
23	91,679	218	.99762	.00238	.00236	78	25,123	2,675	.89352	.10648	.10785
24	91,461	220	.99759	.00241	.00240	79	22,448	2,588	.88471	.11529	.11742
25	91,241	222	.99757	.00243	.00242	80	19,860	2,477	.87528	.12472	.12772
26	91,019	223	.99755	.00245	.00244	81	17,383	2,341	.86533	.13467	.13880
27	90,796	228	.99749	.00251	.00248	82	15,042	2,187	.85461	.14539	.15074
28	90,568	234	.99742	.00258	.00255	83	12,855	2,014	.84333	.15667	.16359
29	90,334	242	.99732	.00268	.00264	84	10,841	1,828	.83138	.16862	.17737
30	90,092	251	.99721	.00279	.00273	85	9,013	1,631	.81904	.18096	.19203
31	89,841	262	.99708	.00292	.00285	86	7,382	1,432	.80601	.19399	.20754
32	89,579	274	.99694	.00306	.00299	87	5,950	1,233	.79277	.20723	.22382
33	89,305	288	.99678	.00322	.00314	88	4,717	1,042	.77910	.22090	.24084
34	89,017	304	.99658	.00342	.00332	89	3,675	863	.76517	.23483	.25854
35	88,713	322	.99637	.00363	.00352	90	2,812	700	.75107	.24893	.27687
36	88,391	342	.99613	.00387	.00375	91	2,112	556	.73674	.26326	.29580
37	88,049	364	.99587	.00413	.00401	92	1,556	432	.72237	.27763	.31526
38	87,685	389	.99556	.00444	.00429	93	1,124	328.2	.70801	.29199	.33521
39	87,296	416	.99523	.00477	.00460	94	795.8	243.9	.69352	.30648	.35558
40	86,880	446	.99487	.00513	.00495	95	551.9	177.0	.67929	.32071	.37631
41	86,434	479	.99446	.00554	.00534	96	374.9	125.6	.66498	.33502	.39732
42	85,955	515	.99401	.00599	.00578	97	249.3	87.0	.65102	.34898	.41853
43	85,440	555	.99350	.00650	.00626	98	162.3	58.9	.63709	.36291	.43985
44	84,885	600	.99293	.00707	.00679	99	103.4	38.88	.62398	.37602	.46120
45	84,285	646	.99234	.00766	.00738	100	64.52	25.12	.61066	.38934	.48245
46	83,639	696	.99168	.00832	.00802	101	39.40	15.84	.59797	.40203	.50351
47	82,943	750	.99096	.00904	.00871	102	23.56	9.76	.58574	.41426	.52425
48	82,193	806	.99019	.00981	.00946	103	13.80	5.880	.57391	.42609	.54453
49	81,387	866	.98936	.01064	.01027	104	7.920	3.462	.56288	.43712	.56422
50	80,521	930	.98845	.01155	.01114	105	4.458	1.995	.55249	.44751	.58317
51	79,591	997	.98747	.01253	.01210	106	2.463	1.127	.54243	.45757	.60122
52	78,594	1,069	.98640	.01360	.01314	107	1.336	.6229	.53376	.46624	.61822
53	77,525	1,145	.98523	.01477	.01427	108	.7131	.7131	.00000	1.00000	----------
54	76,380	1,224	.98397	.01603	.01550						

TABLE 15.—UNITED STATES WHITE MALES: 1939–1941—COMMUTATION COLUMNS AT 2 PERCENT INTEREST

x	D_x	N_x	S_x	C_x	M_x	R_x	x	D_x	N_x	S_x	C_x	M_x	R_x
0	100,000	3,458,807	94,230,796	4,717.6	32,180.2	1,611,142.8	55	25,290	385,621	4,252,498	430.53	17,729.07	302,239.59
1	93,322	3,358,807	90,771,989	445.98	27,462.56	1,578,962.58	56	24,364	360,331	3,866,877	449.58	17,298.54	284,510.52
2	91,046	3,265,485	87,413,182	235.58	27,016.58	1,551,500.02	57	23,437	335,967	3,506,546	467.08	16,848.96	267,211.98
3	89,025	3,174,439	84,147,697	165.37	26,781.00	1,524,483.44	58	22,510	312,530	3,170,579	484.35	16,381.88	250,363.02
4	87,114	3,085,414	80,973,258	131.33	26,615.63	1,497,702.44	59	21,584	290,020	2,858,049	500.76	15,897.53	233,981.14
5	85,275	2,998,300	77,887,844	115.44	26,484.30	1,471,086.81	60	20,660	268,436	2,568,029	516.04	15,396.77	218,083.61
6	83,487	2,913,025	74,889,544	100.98	26,368.86	1,444,602.51	61	19,739	247,776	2,299,593	531.11	14,880.73	202,686.84
7	81,749	2,829,538	71,976,519	92.177	26,267.88	1,418,233.65	62	18,821	228,037	2,051,817	544.54	14,349.62	187,806.11
8	80,054	2,747,789	69,146,981	82.839	26,175.70	1,391,965.77	63	17,907	209,216	1,823,780	557.79	13,805.08	173,456.49
9	78,401	2,667,735	66,399,192	78.753	26,092.86	1,365,790.07	64	16,998	191,309	1,614,564	570.04	13,247.29	159.651.41
10	76,785	2,589,334	63,732,457	74.796	26,014.11	1,339,697.21	65	16,095	174,311	1,423,255	581.33	12,677.25	146,404.12
11	75,205	2,512,549	61,142,123	74.907	25,939.32	1,313,683.10	66	15,198	158,216	1,248,944	592.22	12,095.92	133,726.87
12	73,656	2,437,344	58,629,574	76.530	25,864.41	1,287,743.78	67	14,308	143,018	1,090,728	602.20	11,503.70	121,630.95
13	72,135	2,363,688	56,192,230	80.335	25,787.88	1,261,879.37	68	13,425	128,710	947,710	611.05	10,901.50	110,127.25
14	70,640	2,291,553	53,828,542	88.419	25,707.54	1,236,091.49	69	12,551	115,285	819,000	618.82	10,290.45	99,225.75
15	69,166	2,220,913	51,536,989	96.883	25,619.12	1,210,383.95	70	11,686	102,734	703,715	624.82	9,671.63	88,935.30
16	67,713	2,151,747	49,316,076	104.98	25,522.24	1,184,764.83	71	10,832	91,048	600,981	629.15	9,046.81	79,263.67
17	66,281	2,084,034	47,164,329	112.03	25,417.26	1,159,242.59	72	9,990.5	80,215.8	509,933.3	630.95	8,417.66	70,216.86
18	64,869	2,017,753	45,080,295	118.07	25,305.23	1,133,825.33	73	9,163.7	70,225.3	429,717.5	630.13	7,786.71	61,799.20
19	63,479	1,952,884	43,062,542	123.83	25,187.16	1,108,520.10	74	8,353.9	61,061.6	359,492.2	625.48	7,156.58	54,012.49
20	62,111	1,889,405	41,109,658	128.66	25,063.33	1,083,332.94	75	7,564.6	52,707.7	298,430.6	616.54	6,531.10	46,855.91
21	60,764	1,827,294	39,220,253	132.60	24,934.67	1,058,269.61	76	6,799.7	45,143.1	245,722.9	602.71	5,914.56	40,324.81
22	59,440	1,766,530	37,392,959	135.71	24,802.07	1,033,334.94	77	6,063.7	38,343.4	200,579.8	583.64	5,311.85	34,410.25
23	58,139	1,707,090	35,626,429	135.54	24,666.36	1,008,532.87	78	5,361.2	32,279.7	162,236.4	559.64	4,728.21	29,098.40
24	56,863	1,648,951	33,919,339	134.10	24,530.82	983,866.51	79	4,696.4	26,918.5	129,956.7	530.82	4,168.57	24,370.19
25	55,614	1,592,088	32,270,388	132.66	24,396.72	959,335.69	80	4,073.5	22,222.1	103,038.2	498.09	3,637.75	20,201.62
26	54,391	1,536,474	30,678,300	130.65	24,204.06	934,938.97	81	3,495.5	18,148.6	80,816.1	461.52	3,139.66	16,563.87
27	53,194	1,482,083	29,141,826	130.96	24,133.41	910,674.91	82	2,965.5	14,653.1	62,667.5	422.70	2,678.14	13,424.21
28	52,020	1,428,889	27,659,743	131.77	24,002.45	886,541.50	83	2,484.6	11,687.6	48,014.4	381.63	2,255.44	10,746.07
29	50,868	1,376,869	26,230,854	133.60	23,870.68	862,539.05	84	2,054.3	9,203.0	36,326.8	339.60	1,873.81	8,490.63
30	49,737	1,326,001	24,853,985	135.85	23,737.08	838,668.37	85	1,674.4	7,148.7	27,123.8	297.06	1,534.21	6,616.82
31	48,626	1,276,264	23,527,984	139.03	23,601.23	814,931.29	86	1,344.5	5,474.3	19,975.1	255.70	1,237.15	5,082.61
32	47,534	1,227,638	22,251,720	142.54	23,462.20	791,330.06	87	1,062.4	4,129.8	14,500.8	215.85	981.45	3,845.46
33	46,459	1,180,104	21,024,082	146.89	23,319.66	767,867.86	88	825.75	3,067.39	10,371.01	178.83	765.60	2,864.01
34	45,401	1,133,645	19,843,978	152.01	23,172.77	744,548.20	89	630.73	2,241.64	7,303.62	145.21	586.77	2,098.41
35	44,359	1,088,244	18,710,333	157.85	23,020.76	721,375.43	90	473.15	1,610.91	5,061.98	115.47	441.56	1,511.64
36	43,331	1,043,885	17,622,089	164.37	22,862.91	698,354.67	91	348.40	1,137.76	3,451.07	89.921	326.091	1,070.080
37	42,317	1,000,554	16,578,204	171.51	22,698.54	675,491.76	92	251.65	789.36	2,313.31	68.496	236.170	743.989
38	41,316	958,237	15,577,650	179.70	22,527.03	652,793.22	93	178.22	537.71	1,523.95	51.018	167.674	507.819
39	40,326	916,921	14,619,413	188.40	22,347.33	630,266.19	94	123.71	359.49	986.24	37.170	116.656	340.145
40	39,347	876,595	13,702,492	198.03	22,158.93	607,918.86	95	84.109	235.779	626.748	26.446	79.486	223.489
41	38,378	837,248	12,825,897	208.51	21,960.90	585,759.93	96	56.014	151.670	390.969	18.398	53.040	144.003
42	37,417	798,870	11,988,649	219.79	21,752,39	563,799.03	97	36.518	95.656	239.299	12.494	34.642	90.963
43	36,463	761,453	11,189,779	232.21	21,532.60	542,046.64	98	23.308	59.138	143.643	8.2927	22.1481	56.3212
44	35,516	724,990	10,428,326	246.12	21,300.39	520,514.04	99	14.558	35.830	84.505	5.3667	13.8554	34.1731
45	34,573	689,474	9,703,336	259.79	21,054.27	499,213.65	100	8.9059	21.2724	48.6749	3.3994	8.4887	20.3177
46	33,636	654,901	9,013,862	274.41	20,794.48	478,159.38	101	5.3319	12.3665	27.4025	2.1015	5.0893	11.8290
47	32,702	621,265	8,358,961	289.90	20,520.07	457,364.90	102	3.1258	7.0346	15.0360	1.2695	2.9878	6.7397
48	31,771	588,563	7,737,696	305.44	20,230.17	436,844,83	103	1.7950	3.9088	8.0014	.74982	1.71835	3.75188
49	30,842	556,792	7,149,133	321.74	19,924.73	416,614.66	104	1.0100	2.1138	4.0926	.43282	.96853	2.03353
50	29,916	525,950	6,592,341	338.75	19,602.99	396,689.93	105	.55734	1.10378	1.97878	.24453	.53571	1.06500
51	28,990	496,034	6,066,391	356.03	19,264.24	377,086.94	106	.30189	.54644	.87500	.13543	.29118	.52929
52	28,066	467,044	5,570,357	374.26	18,908.21	357,822.70	107	.16054	.24455	.32856	.073384	.155747	.238110
53	27,141	438,978	5,103,313	393.00	18,533.95	338,914.49	108	.084010	.084010	.084010	.082363	.082363	.082363
54	26,216	411,837	4,664,335	411.88	18,140.95	320,380.54							

$N_x = D_x + D_{x+1} + \dots$ $S_x = N_x + N_{x+1} + \dots$

TABLE 16.—UNITED STATES WHITE MALES: 1939–1941—COMMUTATION COLUMNS AT 2½ PERCENT INTEREST

x	D_x	N_x	S_x	C_x	M_x	R_x	x	D_x	N_x	S_x	C_x	M_x	R_x
0	100,000	3,056,502	77,493,609	4094.6	25,451.3	1,166,413.7	55	19,326	280,794	3,012,161	327.40	12,477.85	207,325.11
1	92,866	2,956,502	74,437,167	441.64	20,756.69	1,140,962.43	56	18,528	261,468	2,731,367	340.22	12,150.45	194,847.26
2	90,160	2,863,636	71,480,665	232.15	20,315.05	1,120,205.74	57	17,736	242,940	2,469,899	351.74	11,810.23	182,696.81
3	87,729	2,773,476	68,617,029	162.17	20,082.90	1,099,890.69	58	16,951	225,204	2,226,959	362.96	11,458.49	170,886.58
4	85,427	2,685,747	65,843,553	128.16	19,920.73	1,079,807.79	59	16,175	208,253	2,001,755	373.43	11,095.53	159,428.09
5	83,215	2,600,320	63,157,806	112.10	19,792.57	1,059,887.06	60	15,407	192,078	1,703,502	382.95	10,722.10	148,332.56
6	81,073	2,517,105	60,557,486	97.587	19,680.47	1,040,094.49	61	14,648	176,071	1,601,424	392.21	10,339.15	137,610.46
7	78,998	2,436,032	58,040,381	88.641	19,582.88	1,020,414.02	62	13,899	162,023	1,424,753	400.16	9,946.94	127,271.31
8	76,983	2,357,034	55,604,349	79.272	19,494.24	1,000,831.14	63	13,160	148,124	1,262,730	407.90	9,546.78	117,324.37
9	75,026	2,280,051	53,247,315	74.095	19,414.07	981,336.90	64	12,431	134,964	1,114,606	414.83	9,138.88	107,777.59
10	73,121	2,205,025	50,967,264	70.879	19,339.97	961,921.93	65	11,713	122,533	979,642	420.98	8,724.05	98,638.71
11	71,267	2,131,904	48,762,239	70.638	19,269.10	942,581.96	66	11,006	110,820	857,109	426.77	8,303.07	89,914.66
12	69,458	2,060,637	46,630,335	71.817	19,198.46	923,312.80	67	10,311	99,814	746,289	431.85	7,876.30	81,611.59
13	67,692	1,991,179	44,569,698	75.019	19,126.64	904,114.40	68	9,627.4	89,502.9	640,474.6	436.05	7,444.45	73,735.29
14	65,966	1,923,487	42,578,519	82.165	19,051.62	884,987.76	69	8,956.6	79,875.5	556,971.7	439.45	7,008.40	66,290.84
15	64,275	1,857,521	40,655,032	89.592	18,969.46	865,936.14	70	8,298.7	70,918.9	477,096.2	441.55	6,568.95	59,282.44
16	62,617	1,793,246	38,797,511	96.608	18,879.86	846,966.68	71	7,654.7	62,620.2	406,177.3	442.44	6,127.40	52,713.49
17	60,994	1,730,629	37,004,265	102.59	18,783.26	828,086.82	72	7,025.6	54,965.5	343,557.1	441.54	5,684.96	46,586.09
18	59,403	1,669,635	35,273,636	107.59	18,680.67	809,303.56	73	6,412.7	47,939.9	288,591.6	438.81	5,243.42	40,901.13
19	57,847	1,610,232	33,604,001	112.29	18,573.08	790,622.89	74	5,817.5	41,527.2	240,651.7	433.44	4,804.61	85,657.71
20	56,324	1,552,385	31,993,769	116.10	18,460.79	772,049.81	75	5,242.1	35,709.7	199,124.5	425.17	4,371.17	30,853.10
21	54,834	1,496,001	30,441,384	119.08	18,344.69	753,589.02	76	4,689.1	30,467.6	163,414.8	413.60	3,946.00	26,481.93
22	53,377	1,441,227	28,945,323	121.27	18,225.61	735,244.33	77	4,161.1	25,778.5	132,947.2	398.56	3,532.40	22,535.93
23	51,954	1,387,850	27,504,096	120.53	18,104.34	717,018.72	78	3,661.1	21,617.4	107,168.7	380.31	3,133.84	19,003.53
24	50,567	1,335,896	26,116,246	118.67	17,983.81	698,914.38	79	3,191.5	17,956.3	85,551.3	358.97	2,753.53	15,869.69
25	49,215	1,285,329	24,780,350	116.82	17,865.14	680,930.57	80	2,754.7	14,764.8	67,595.0	335.19	2,394.56	13,116.16
26	47,897	1,236,114	23,495,021	114.49	17,748.32	663,005.43	81	2,352.3	12,010.1	52,830.2	309.06	2,059.37	10,721.60
27	46,615	1,188,217	22,258,907	114.20	17,633.83	645,317.11	82	1,985.9	9,657.8	40,820.1	281.69	1,750.31	8,662.23
28	45,363	1,141,602	21,070,690	114.35	17,519.63	627,683.28	83	1,655.7	7,671.9	31,162.3	253.08	1,468.02	6,911.92
29	44,143	1,096,239	19,929,088	115.37	17,405.28	610,163.65	84	1,362.3	6,016.2	23,490.4	224.10	1,215.54	5,443.30
30	42,951	1,052,096	18,832,849	116.74	17,289.91	592,758.37	85	1,104.9	4,653.9	17,474.2	195.07	991.44	4,227.76
31	41,786	1,009,145	17,780,753	118.89	17,173.17	575,468.40	86	882.92	3,549.01	12,820.29	167.10	796.37	3,236.32
32	40,648	967,359	16,771,608	121.30	17,054.28	558,295.29	87	694.29	2,666.09	9,271.28	140.37	629.27	2,439.95
33	39,536	926,711	15,804,249	124.39	16,932.98	541,241.01	88	536.99	1,971.80	6,605.19	115.73	488.90	1,810.68
34	38,447	887,175	14,877,538	128.10	16,808.59	524,308.03	89	408.16	1,434.81	4,633.39	93.511	373.167	1,321.785
35	37,381	848,728	13,990,363	132.37	16,680.49	507,499.44	90	304.70	1,026.65	3,108.58	73.999	279.656	948.618
36	36,337	811,347	13,141,635	137.16	16,548.12	490,818.95	91	223.27	721.95	2,171.93	57.343	205.657	668.962
37	35,314	775,010	12,330,288	142.43	16,410.96	474,270.83	92	160.48	498.68	1,449.98	43.467	148.314	463.305
38	34,310	739,696	11,555,278	148.50	16,268.53	457,859.87	93	113.10	338.20	951.30	32.218	104.847	314.991
39	33,324	705,386	10,815,582	154.93	16,120.03	441,591.34	94	78.120	225.099	613.104	23.358	72.629	210.144
40	32,357	672,062	10,110,196	162.05	15,965.10	425,471.31	95	52.856	146.979	388.005	16.538	49.271	137.515
41	31,406	639,705	9,438,134	169.80	15,803.05	409,506.21	96	35.029	94.123	241.026	11.449	32.733	88.244
42	30,470	608,299	8,798,429	178.11	15,633.25	393,703.16	97	22.725	59.094	146.903	7.7371	21.2839	55.5110
43	29,548	577,829	8,190,130	187.26	15,455.14	378,069.91	98	14.434	36.369	87.809	5.1104	13.5468	34.2271
44	28,641	548,281	7,612,301	197.50	15,267.88	362,614.77	99	8.9714	21.9348	51.4404	3.2011	8.4364	20.6803
45	27,744	519,640	7,064,020	207.46	15,070.38	347,346.89	100	5.4614	12.9634	29.5056	2.0745	5.1453	12.2439
46	26,860	491,896	6,544,380	218.07	14,862.92	332,276.51	101	3.2538	7.5020	16.5422	1.2762	3.0708	7.0986
47	25,987	465,036	6,052,484	229.25	14,644.85	317,413.59	102	1.8982	4.2482	9.0402	.76717	1.79458	4.02777
48	25,124	439,049	5,587,448	240.36	14,415.60	302,768.74	103	1.0847	2.3500	4.7920	.45092	1.02741	2.23319
49	24,271	413,925	5,148,399	251.96	14,175.24	288,353.14	104	.60736	1.26535	2.44202	.25901	.57649	1.20578
50	23,427	389,654	4,734,474	263.98	13,923.28	274,177.90	105	.33353	.65799	1.17667	.14562	.31748	.62929
51	22,592	366,227	4,344,820	276.09	13,659.30	260,254.62	106	.17978	.32446	.51868	.080255	.171864	.311807
52	21,764	343,635	3,978,593	288.81	13,383.21	246,595.32	107	.095138	.144680	.194222	.043275	.091609	.139943
53	20,945	321,871	3,634,958	301.80	13,094.40	233,212.11	108	.049542	.049542	.049542	.048334	.048334	.048334
54	20,132	300,926	3,313,087	314.75	12,792.60	220,117.71							

$N_x = D_x + D_{x+1} + \ldots$ $S_x = N_x + N_{x+1} + \ldots$

TABLE 17.—UNITED STATES WHITE MALES: 1939–1941—COMMUTATION COLUMNS AT 3 PERCENT INTEREST

x	D_x	N_x	S_x	C_x	M_x	R_x	x	D_x	N_x	S_x	C_x	M_x	R_x
0	100,000	2,726,608	64,373,119	4,671.8	20,584.1	851,664.1	55	14,788	205,069	2,140,522	249.30	8,815.38	142,724.09
1	92,416	2,626,608	61,646,511	437.36	15,912.27	831,080.01	56	14,108	190,281	1,935,453	257.81	8,566.08	133,908.71
2	89,286	2,534,192	59,019,903	228.79	15,474.91	815,167.74	57	13,439	176,173	1,745,172	265.24	8,308.27	125,342.63
3	86,457	2,444,906	56,485,711	159.04	15,246.12	799,692.83	58	12,783	162,734	1,568,999	272.38	8,043.03	117,034.36
4	83,780	2,358,449	54,040,805	125.08	15,087.08	784,446.71	59	12,138	149,951	1,406,265	278.87	7,770.65	108,991.33
5	81,215	2,274,669	51,682,356	108.87	14,962.00	769,359.63	60	11,506	137,813	1,256,314	284.59	7,491.78	101,220.68
6	78,740	2,193,454	49,407,687	94.319	14,853.13	754,397.63	61	10,886	126,307	1,118,501	290.06	7,207.19	93,728.90
7	76,353	2,114,714	47,214,233	85.256	14,758.81	739,544.50	62	10,279	115,421	992,194	294.51	6,917.13	86,521.71
8	74,043	2,038,361	45,099,519	75.875	14,673.56	724,785.69	63	9,685.0	105,141.5	876,773.4	298.75	6,622.62	79,604.58
9	71,811	1,964,318	43,061,158	71.433	14,597.68	710,112.13	64	9,104.1	95,456.5	771,631.9	302.34	6,323.87	72,981.96
10	69,648	1,892,507	41,096,840	67.185	14,526.25	695,514.45	65	8,536 6	86,352.4	676,175.4	305.34	6,021.53	66,658.09
11	67,552	1,822,859	39,204,333	66.631	14,459.06	680,988.20	66	7,982.6	77,815 8	589,823.0	308.04	5,716.19	60,636.56
12	65,518	1,755,307	37,381,474	67.414	14,392.43	666,529.14	67	7,442.1	69,833.2	512,007.2	310.18	5,408.15	54,920.37
13	63,542	1,689,789	35,626,167	70.078	14,325.02	652,136.71	68	6,915.2	62,391.1	442,174.0	311.69	5,097.97	49,512.22
14	61,621	1,626,247	33,936,378	76.382	14,254.94	637,811.69	69	6,402.1	55,475.9	379,782.9	312.59	4,786.28	44,414.25
15	59,750	1,564,626	32,310,131	82.881	14,178.56	623,556.75	70	5,903.0	49,073.8	324,307.0	312.56	4,473.69	39,627.97
16	57,927	1,504,876	30,745,505	88.937	14,095.68	609,378.19	71	5,418.5	43,170.8	275,233.2	311.67	4,161.13	35,154.28
17	56,151	1,446,949	29,240,629	93.983	14,006.74	595,282.51	72	4,949.0	37,752.3	232,062.4	309.52	3,849.46	30,993.15
18	54,422	1,390,798	27,793,680	98.089	13,912.76	581,275.77	73	4,495.4	32,803.3	194,310.1	306.12	3,539.94	27,143.69
19	52,738	1,336,376	26,402,882	101.87	13,814.67	567,363.01	74	4,058.3	28,307.9	161,506.8	300.91	3,233.82	23,603.75
20	51,100	1,283,638	25,066,506	104.82	13,712.80	553,548.34	75	3,639.2	24,249.6	133,198.9	293.73	2,932.91	20,369.93
21	49,507	1,232,538	23,782,868	106.99	13,607.98	539,835.54	76	3,239.5	20,610.4	108,949.3	284.35	2,639.18	17,437.02
22	47,958	1,183,031	22,550,330	108.43	13,500.99	526,227.56	77	2,860.8	17,370.9	88,338.9	272.68	2,354.83	14,797.84
23	46,453	1,135,073	21,367,299	107.24	13,392.56	512,726.57	78	2,504.8	14,510.1	70,968.0	258.93	2,082.15	12,443.01
24	44,993	1,088,620	20,232,226	105.07	13,285.32	499,334.01	79	2,172.9	12,005.3	56,457.9	243.21	1,823.22	10,360.86
25	43,577	1,043,627	19,143,606	102.94	13,180.25	486,048.69	80	1,866.4	9,832.4	44,452.6	226.00	1,580.01	8,537.64
26	42,205	1,000,050	18,099,979	100.39	13,077.31	472,868.44	81	1,586.0	7,966.0	34,620.2	207.37	1,354.01	6,957.63
27	40,875	957,845	17,099,929	99.653	12,976.92	459,791.13	82	1,332.5	6,380.0	26,654.2	188.09	1,146.64	5,603.62
28	39,585	916,970	16,142,084	99.297	12,877.27	446,814.21	83	1,105.6	5,047.5	20,274.2	168.16	958.55	4,456.98
29	38,333	877,385	15,225,114	99.701	12,777.97	433,936.94	84	905.20	3,941.92	15,226.75	148.19	790.39	3,498.43
30	37,117	839,052	14,347,729	100.40	12,678.27	421,158.97	85	730.64	3,036.72	11,284.83	128.37	642.20	2,708.04
31	35,935	801,935	13,508,677	101.74	12,577.87	408,480.70	86	581.00	2,306.08	8,248.11	109.42	513.83	2,065.84
32	34,787	766,000	12,706,742	103.31	12,476.13	395,902.83	87	454.65	1,725.08	5,942.03	91.472	404.407	1,552.013
33	33,670	731,213	11,940,742	105.42	12,372.82	383,426.70	88	349.94	1,270.43	4,216.95	75.051	312.935	1,147.606
34	32,584	697,543	11,209,529	108.04	12,267.40	371,053.88	89	264.69	920.49	2,946.52	60.348	237.884	834.671
35	31,527	664,959	10,511,986	111.10	12,159.36	358,786.48	90	196.64	655.80	2,026.03	47.524	177.536	596.787
36	30,498	633,432	9,847,027	114.56	12,048.26	346,627.12	91	143.39	459.16	1,370.23	36.648	130.012	419.251
37	29,495	602,934	9,213,595	118.38	11,933.70	334,578.86	92	102.56	315.77	911.07	27.645	93.364	289.239
38	28,517	573,439	8,610,661	122.83	11,815.32	322,645.16	93	71.929	213.213	595.296	20.391	65.719	195.875
39	27,564	544,922	8,037,222	127.53	11,692.49	310,829.84	94	49.443	141.284	382.083	14.712	45.328	130.156
40	26,634	517,358	7,492,300	132.74	11,564.96	299,137.35	95	33.291	91.841	240.799	10.366	30.616	84.828
41	25,725	490,724	6,974,942	138.41	11,432.22	287,572.39	96	21.955	58.550	148.958	7.1413	20.2500	54.2117
42	24,837	464,999	6,484,218	144.48	11,293.81	276,140.17	97	14.175	36.595	90.408	4.8025	13.1087	33.9617
43	23,970	440,162	6,019,219	151.17	11,149.33	264,846.36	98	8.9592	22.4204	53.8130	3.1567	8.3062	20.8530
44	23,120	416,192	5,579,057	158.66	10,998.16	253,697.03	99	5.5416	13.4612	31.3926	2.0230	5.1495	12.5468
45	22,288	393,072	5,162,865	165.85	10,839.50	242,698.87	100	3.3572	7.9196	17.9314	1.2690	3.1265	7.3973
46	21,473	370,784	4,769,793	173.48	10,673.65	231,859.37	101	1.9904	4.5624	10.0118	.77689	1.85751	4.27084
47	20,674	349,311	4,399,009	181.50	10,500.17	221,185.72	102	1.1555	2.5720	5.4494	.46475	1.08062	2.41333
48	19,891	328,637	4,049,698	189.37	10,318.67	210,685.55	103	.65712	1.41650	2.87735	.27184	.61587	1.33271
49	19,122	308,746	3,721,061	197.54	10,129.30	200,366.88	104	.36615	.75938	1.46085	.15539	.34403	.71684
50	18,367	289,624	3,412,315	205.96	9,931.76	190,237.58	105	.20009	.39323	.70147	.086935	.188640	.372807
51	17,626	271,257	3,122,691	214.37	9,725.80	180,305.82	106	.10733	.19314	.30824	.047681	.101705	.184167
52	16,899	253,631	2,851,434	223.15	9,511.43	170,580.02	107	.056523	.085814	.115105	.025586	.054024	.082462
53	16,183	236,732	2,597,803	232.06	9,288.28	161,068.59	108	.029291	.029291	.029291	.028438	.028438	.028438
54	15,480	220,549	2,361,071	240.84	9,056.22	151,780.31							

$N_x = D_x + D_{x+1} + \dots$ $S_x = N_x + N_{x+1} + \dots$

TABLE 18.—UNITED STATES WHITE MALES: 1939–1941—COMMUTATION COLUMNS AT 3½ PERCENT INTEREST

x	D_x	N_x	S_x	C_x	M_x	R_x	x	D_x	N_x	S_x	C_x	M_x	R_x
0	100,000	2,453,468	53,997,179	4,649.3	17,032.6	627,477.7	55	11,330	150,201	1,525,974	190.09	6,251.12	98,597.98
1	91,969	2,353,468	51,543,711	433.15	12,383.32	610,445.09	56	10,757	138,871	1,375,773	195.62	6,061.03	92,346.86
2	88,426	2,261,499	49,190,243	225.49	11,950.17	598,061.77	57	10,198	128,114	1,236,902	200.29	5,865.41	86,285.83
3	85,210	2,173,073	46,928,744	155.99	11,724.68	586,111.60	58	9,652.6	117,915.7	1,108,788.0	204.69	5,665.12	80,420.42
4	82,173	2,087,863	44,755,671	122.09	11,568.69	574,386.92	59	9,121.5	108,263.1	990,872.3	208.55	5,460.43	74,755.30
5	79,272	2,005,690	42,667,808	105.76	11,446.60	562,818.23	60	8,604.5	99,141.6	882,609.2	211.80	5,251.88	69,294.87
6	76,485	1,926,418	40,662,118	91.175	11,340.84	551,371.63	61	8,101.7	90,537.1	783,467.6	214.83	5,040.08	64,042.99
7	73,808	1,849,933	38,735,700	82.016	11,249.66	540,030.79	62	7,612.9	82,435.4	692,930.5	217.07	4,825.25	59,002.91
8	71,230	1,776,125	36,885,767	72.639	11,167.65	528,781.13	63	7,138.4	74,822.5	610,495.1	219.13	4,608.18	54,177.66
9	68,748	1,704,895	35,109,642	68.056	11,095.01	517,613.48	64	6,677.9	67,684.1	535,672.6	220.70	4,389.05	49,569.48
10	66,356	1,636,147	33,404,747	63.700	11,026.95	506,518.47	65	6,231.4	61,006.2	467,988.5	221.80	4,168.35	45,180.43
11	64,048	1,569,791	31,768,600	62.869	10,963.25	495,491.52	66	5,798.8	54,774.8	406,982.3	222.68	3,946.55	41,012.08
12	61,819	1,505,743	30,198,809	63.301	10,900.38	484,528.27	67	5,380 1	48,976.0	352,207.5	223.16	3,723.87	37,065.53
13	59,665	1,443,924	28,693,066	65.485	10,837.08	473,627.89	68	4,975.0	43,595.9	303,231.5	223.15	3,500.71	33,341.66
14	57,582	1,384,259	27,249,142	71.030	10,771.60	462,790.81	69	4,583.6	38,620.9	259,635.6	222.72	3,277.56	29,840.95
15	55,564	1,326,677	25,864,883	76.702	10,700.57	452,019.21	70	4,205.9	34,037.3	221,014.7	221.62	3,054.84	26,563.39
16	53,608	1,271,113	24,538,206	81.909	10,623.86	441,318.64	71	3,842.0	29,831.4	186,977.4	219.92	2,833.22	23,508.55
17	51,714	1,217,505	23,267,093	86.138	10,541.96	430,694.78	72	3,492.2	25,989.4	157,146.0	217.35	2,613.30	20,675.33
18	49,879	1,165,791	22,049,588	89.467	10,455.82	420,152.82	73	3,156.7	22,497.2	131,156.6	213.92	2,395.95	18,062.03
19	48,102	1,115,912	20,883,797	92.472	10,366.35	409,697.00	74	2,836.1	19,340.5	108,659.4	209.27	2,182.03	15,666.08
20	46,383	1,067,810	19,767,885	94.686	10,273.88	399,330.65	75	2,530.9	16,504.4	89,318.9	203.29	1,972.76	13,484.05
21	44,720	1,021,427	18,700,075	96.176	10,179.19	389,056 77	76	2,242.0	13,973.5	72,814.5	195.85	1,769.47	11,511.29
22	43,112	976,707	17,678,648	97.003	10,083.02	378,877.58	77	1,970.3	11,731.5	58,841.0	186.90	1,573.62	9,741.82
23	41,557	933,595	16,701,941	95.475	9,986.01	368,794.56	78	1,716.8	9,761.2	47,109.5	176.62	1,386.72	8,168.20
24	40,056	892,038	15,768,346	93.092	9,890.54	358,808.55	79	1,482.1	8,044.4	37,348.3	165.10	1,210.10	6,781.48
25	38,608	851,982	14,876,308	90.762	9,797.45	348,918.01	80	1,266.9	6,562.3	29,303.9	152.67	1,045.00	5,571.38
26	37,212	813,374	14,024,326	88.088	9,706.68	339,120.56	81	1,071.4	5,295.4	22,741.6	139.41	892.33	4,526.38
27	35,866	776,162	13,210,952	87.017	9,618.60	329,413.88	82	895.77	4,224.03	17,446.19	125.83	752.92	3,634.05
28	34,566	740,296	12,434,790	86.287	9,531.58	319,795.28	83	739.64	3,328.26	13,222.16	111.96	627.09	2,881.13
29	33,310	705,730	11,694,494	86.219	9,445.29	310,263.70	84	602.67	2,588.62	9,893.90	98.185	515.133	2,254.036
30	32,098	672,420	10,988,764	86.402	9,359.07	300,818.41	85	484.11	1,985.95	7,305.28	84.642	416.948	1,738.903
31	30,926	640,322	10,316,344	87.139	9,272.67	291,459.34	86	383.09	1,501.84	5,319.33	71.801	332.306	1,321.955
32	29,793	609,396	9,676,022	88.048	9,185.53	282,186.67	87	298.34	1,118.75	3,817.49	59.733	260.505	989.649
33	28,698	579,603	9,066,626	89.417	9,097.48	273,001.14	88	228.52	820.41	2,698.74	48.773	200.772	729.144
34	27,638	550,905	8,487,023	91.193	9,008.07	263,903.66	89	172.01	591.89	1,878.33	39.028	151.999	528.372
35	26,612	523,267	7,936,118	93.326	8,916.87	254,895.59	90	127.17	419.88	1,286.44	30.586	112.971	376.373
36	25,619	496,655	7,412,851	95.771	8,823 55	245,978.72	91	92.283	292.707	866.562	23.473	82.385	263.402
37	24,657	471,036	6,916,196	98.485	8,727.78	237,155.17	92	65.690	200.424	573.855	17.621	58.912	181.017
38	23,724	446,379	6,445,160	101.69	8,629.29	228,427.39	93	45.847	134.734	373.431	12.934	41.291	122.105
39	22,820	422,655	5,998,781	105 07	8,527.60	219,798.10	94	31.363	88.887	238.697	9.2871	28.3567	80.8144
40	21,943	399,835	5,576,126	108.84	8,422.53	211,270.50	95	21.015	57.524	149.810	6.5118	19.0696	52.4577
41	21,093	377,892	5,176,291	112.94	8,313.69	202,847.97	96	13.792	36.509	92.286	4.4645	12.5578	33.3881
42	20,266	356,799	4,798,399	117.32	8,200.75	194,534.28	97	8.8615	22.7165	55.7771	2.9879	8.0933	20.8303
43	19,464	336,533	4,441,600	122.16	8,083.43	186,333.53	98	5.5740	13.8550	33.0606	1.9544	5.1054	12.7370
44	18,683	317,069	4,105,067	127.60	7,961.27	178,250.10	99	3.4310	8.2810	19.2056	1.2465	3.1510	7.6316
45	17,924	298,386	3,787,998	132.73	7,833 67	170,288.83	100	2.0685	4.8500	10.9246	.77812	1.90451	4.48057
46	17,185	280,462	3,489,612	138.17	7,700.94	162,455.16	101	1.2205	2.7815	6.0746	.47407	1.12639	2.57606
47	16,466	263,277	3,209,150	143.85	7,562.77	154,754.22	102	.70511	1.56103	3.29307	.28222	.65232	1.44967
48	15,765	246,811	2,945,873	149.37	7,418.92	147,191.45	103	.39905	.85592	1.73204	.16428	.37010	.79735
49	15,083	231,046	2,699,062	155.06	7,269.55	139,772.53	104	.22127	.45687	.87612	.093452	.205823	.427250
50	14,418	215,963	2,468,016	160.89	7,114.40	132,502.98	105	.12034	.23560	.41925	.052031	.112371	.211427
51	13,769	201,545	2,252,053	166.65	6,953.60	125,388.49	106	.064237	.115265	.183655	.028399	.060340	.109056
52	13,137	187,776	2,050,508	172.64	6,786.95	118,434.89	107	.033666	.051028	.068390	.015166	.031941	.048716
53	12,520	174,639	1,862,732	178.66	6,614.31	111,647.94	108	.017362	.017362	.017362	.016775	.016775	.016775
54	11,918	162,119	1,688,093	184.53	6,435.65	105,033.63							

$N_x = D_x + D_{x+1} + \ldots$ $S_x = N_x + N_{x+1} + \ldots$

TABLE 19.—UNITED STATES WHITE MALES: 1939–1941—COMMUTATION COLUMNS AT 4 PERCENT INTEREST

x	D_x	N_x	S_x	C_x	M_x	R_x	x	D_x	N_x	S_x	C_x	M_x	R_x
0	100,000	2,225,161	45,718,970	4,626.9	14,416.8	466,737.7	55	8,692.2	110,323.8	1,091,275.9	145.13	4,448.97	68,351.50
1	91,527	2,125,161	43,493,809	428.99	9,789.85	452,320.88	56	8,212.8	101,631.6	980,952.1	148.63	4,303.84	63,902.53
2	87,578	2,033,634	41,368,648	222.25	9,360.86	442,531.03	57	7,748.3	93,418.8	879,320.5	151.45	4,155.21	59,598.69
3	83,987	1,946,056	39,335,014	153.01	9,138.61	433,170.17	58	7,298.8	85,670.5	785,901.7	154.03	4,003.76	55,443.48
4	80,604	1,862,069	37,388,958	119.18	8,985.60	424,031.56	59	6,864.0	78,371.7	700,231.2	156.18	3,849.73	51,439.72
5	77,384	1,781,465	35,526,889	102.74	8,866.42	415,045.96	60	6,443.9	71,507.7	621,859.5	157.86	3,693.55	47,589.99
6	74,305	1,704,081	33,745,424	88.150	8,763.68	406,179.54	61	6,038.2	65,063.8	550,351.8	159.34	3,535.69	43,896.44
7	71,359	1,629,776	32,041,343	78.915	8,675.53	397,415.86	62	5,646.6	59,025.6	485,288.0	160.23	3,376.35	40,360.75
8	68,536	1,558,417	30,411,567	69.556	8,596.62	388,740.33	63	5,269.2	53,379.0	426,262.4	160.97	3,216.12	36,984.40
9	65,830	1,489,881	28,853,150	64.854	8,527.06	380,143.71	64	4,905.5	48,109.8	372,883.4	161.34	3,055.15	33,768.28
10	63,233	1,424,051	27,363,269	60.411	8,462.21	371,616.65	65	4,555.5	43,204.3	324,773.6	161.37	2,893.81	30,713.13
11	60,741	1,360,818	25,939,218	59.337	8,401.80	363,154.44	66	4,218.9	38,648.8	281,569.3	161.24	2,732.44	27,819.32
12	58,345	1,300,077	24,578,400	59.457	8,342.46	354,752.64	67	3,895.4	34,429.9	242,920.5	160.80	2,571.20	25,086.88
13	56,042	1,241,732	23,278,323	61.212	8,283.00	346,410.18	68	3,584.8	30,534.5	208,490.6	160.02	2,410.40	22,515.68
14	53,825	1,185,690	22,036,591	66.076	8,221.79	338,127.18	69	3,286.9	26,949.7	177,956.1	158.94	2,250.38	20,105.28
15	51,689	1,131,865	20,850,901	71.010	8,155.72	329,905.39	70	3,001.6	23,662.8	151,006.4	157.40	2,091.44	17,854.90
16	49,630	1,080,176	19,719,036	75.466	8,084.71	321,749.67	71	2,728.7	20,661.2	127,343.6	155.44	1,934.04	15,763.46
17	47,646	1,030,546	18,638,860	78.980	8,009.24	313,664.96	72	2,468.3	17,932.5	106,682.4	152.89	1,778.60	13,829.42
18	45,734	982,900	17,608,314	81.638	7,930.26	305,655.72	73	2,220.5	15,464.2	88,749.9	149.75	1,625.71	12,050.82
19	43,894	937,166	16,625,414	83.975	7,848.62	297,725.46	74	1,985.3	13,243.7	73,285.7	145.79	1,475.96	10,425.11
20	42,121	893,272	15,688,248	85.573	7,764.65	289,876.84	75	1,763.2	11,258.4	60,042.0	140.94	1,330.17	8,949.15
21	40,416	851,151	14,794,976	86.501	7,679.07	282,112.19	76	1,554.4	9,495.2	48,783.6	135.13	1,189.23	7,618.98
22	38,775	810,735	13,943,825	86.825	7,592.57	274,433.12	77	1,359.5	7,940.8	39,288.4	128.34	1,054.10	6,429.75
23	37,197	771,960	13,133,090	85.046	7,505.75	266,840.55	78	1,178.9	6,581.3	31,347.6	120.70	925.76	5,375.65
24	35,681	734,763	12,361,130	82.526	7,420.70	259,334.80	79	1,012.8	5,402.4	24,766.3	112.28	805.06	4,449.89
25	34,226	699,082	11,626,367	80.073	7,338.18	251,914.10	80	861.61	4,389.59	19,363.88	103.33	692.78	3,644.83
26	32,830	664,856	10,927,285	77.340	7,258.10	244,575.92	81	725.14	3,527.98	14,974.29	93.900	589.451	2,952.048
27	31,489	632,026	10,262,429	76.033	7,180.76	237,317.82	82	603.35	2,802.84	11,446.31	84.349	495.551	2,362.597
28	30,202	600,537	9,630,403	75.032	7,104.73	230,137.06	83	495.80	2,199.49	8,643.47	74.689	411.202	1,867.046
29	28,966	570,335	9,029,866	74.613	7,029.70	223,032.33	84	402.04	1,703.69	6,443.98	65.184	336.513	1,455.844
30	27,777	541,369	8,459,531	74.412	6,955.08	216,002.63	85	321.39	1,301.65	4,740.29	55.923	271.329	1,119.331
31	26,634	513,592	7,918,162	74.685	6,880.67	209,047.55	86	253.11	980.26	3,438.64	47.211	215.406	848.002
32	25,535	486,958	7,404,570	75.102	6,805.99	202,166.88	87	196.16	727.15	2,458.38	39.087	168.195	632.596
33	24,478	461,423	6,917,612	75.903	6,730.89	195,360.89	88	149.53	530.99	1,731.23	31.761	129.108	464.401
34	23,461	436,945	6,456,189	77.038	6,654.98	188,630.00	89	112.02	381.46	1,200.24	25.294	97.347	335.293
35	22,481	413,484	6,019,244	78.461	6,577.94	181,975.02	90	82.417	269.438	818.777	19.727	72.053	237.946
36	21,538	391,003	5,605,760	80.130	6,499.48	175,397.08	91	59.520	187.021	549.339	15.066	52.326	165.893
37	20,630	369,465	5,214,757	82.004	6,419.35	168,897.60	92	42.164	127.501	362.318	11.256	37.260	113.567
38	19,754	348,835	4,845,292	84.265	6,337.35	162,478.25	93	29.286	85.337	234.817	8.2225	26.0042	76.3066
39	18,910	329,081	4,496,457	86.648	6,253.08	156,140.90	94	19.937	56.051	149.480	5.8755	17.7817	50.3024
40	18,096	310,171	4,167,376	89.324	6,166.44	149,887.82	95	13.295	36.114	93.429	4.0999	11.9062	32.5207
41	17,311	292,075	3,857,205	92.243	6,077.11	143,721.38	96	8.6839	22.8188	57.3150	2.7974	7.8063	20.6145
42	16,553	274,764	3,565,130	95.362	5,984.87	137,644.27	97	5.5525	14.1349	34.4962	1.8632	5.0089	12.8082
43	15,821	258,211	3,290,366	98.816	5,889.51	131,659.40	98	3.4758	8.5824	20.3613	1.2129	3.1457	7.7993
44	15,113	242,390	3,032,155	102.72	5,790.69	125,769.89	99	2.1292	5.1066	11.7789	.76983	1.93282	4.65359
45	14,429	227,277	2,789,765	106.34	5,687.97	119,979.20	100	1.2775	2.9774	6.6723	.47825	1.16299	2.72077
46	13,768	212,848	2,562,488	110.16	5,581.63	114,291.23	101	.75012	1.69989	3.69490	.28997	.68474	1.55778
47	13,128	199,080	2,349,640	114.15	5,471.47	108,709.60	102	.43130	.94977	1.99501	.17180	.39477	.87304
48	12,509	185,952	2,150,560	117.95	5,357.32	103,238.13	103	.24291	.51847	1.04524	.099520	.222969	.478266
49	11,910	173,443	1,964,608	121.86	5,239.37	97,880.81	104	.13405	.27556	.52677	.056342	.123449	.255297
50	11,330	161,533	1,791,165	125.83	5,117.51	92,641.44	105	.072550	.141511	.251208	.031218	.067107	.131848
51	10,769	150,203	1,629,632	129.71	4,991.68	87,523.93	106	.038542	.068961	.109697	.016957	.035889	.064741
52	10,225	139,434	1,479,429	133.72	4,861.97	82,532.25	107	.020102	.030419	.040736	.0090119	.0189320	.0288521
53	9,697.8	129,208.7	1,339,995.5	137.72	4,728.25	77,670.28	108	.010317	.010317	.010317	.0099201	.0099201	.0099201
54	9,187.1	119,510.9	1,210,786.8	141.56	4,590.53	72,942.03							

$N_x = D_x + D_{x+1} + \ldots \qquad S_x = N_x + N_{x+1} + \ldots$

TABLE 20.—UNITED STATES WHITE MALES: 1939–1941—IMMEDIATE WHOLE LIFE ANNUITY, SINGLE AND ANNUAL NET PREMIUMS AT 2 PERCENT INTEREST

[Present value at each age of a life annuity of one per annum, first payment to be made at the end of 1 year; present value of a whole life assurance of one unit, and the annual payment of an equivalent whole life annuity-due]

AGE	IMMEDIATE LIFE ANNUITY	SINGLE PREMIUM	ANNUAL PREMIUM
x	a_x	A_x	P_x
0	33.5881	0.32180	0.00930
1	34.9916	.29428	.00818
2	34.8663	.29674	.00827
3	34.6578	.30083	.00844
4	34.4181	.30553	.00863
5	34.1604	.31058	.00883
6	33.8920	.31584	.00905
7	33.6125	.32132	.00928
8	33.3242	.32698	.00953
9	33.0268	.33281	.00978
10	32.7219	.33879	.01005
11	32.4093	.34491	.01032
12	32.0909	.35115	.01061
13	31.7676	.35749	.01091
14	31.4399	.36392	.01122
15	31.1099	.37040	.01154
16	30.7775	.37692	.01186
17	30.4424	.38348	.01220
18	30.1050	.39010	.01254
19	29.7643	.39678	.01290
20	29.4198	.40352	.01327
21	29.0720	.41035	.01365
22	28.7195	.41726	.01404
23	28.3622	.42427	.01445
24	27.9986	.43140	.01488
25	27.6275	.43868	.01532
26	27.2487	.44610	.01579
27	26.8618	.45369	.01628
28	26.4681	.46141	.01680
29	26.0675	.46927	.01734
30	25.6603	.47725	.01790
31	25.2465	.48536	.01849
32	24.8265	.49359	.01911
33	24.4010	.50194	.01976
34	23.9696	.51040	.02044
35	23.5327	.51896	.02115
36	23.0910	.52763	.02190
37	22.6443	.53639	.02269
38	22.1929	.54524	.02351
39	21.7377	.55417	.02437
40	21.2786	.56317	.02528
41	20.8158	.57223	.02623
42	20.3505	.58135	.02723
43	19.8829	.59053	.02828
44	19.4131	.59974	.02938
45	18.9426	.60898	.03054
46	18.4702	.61822	.03175
47	17.9978	.62749	.03303
48	17.5252	.63675	.03437
49	17.0530	.64603	.03578
50	16.5809	.65527	.03727
51	16.1105	.66451	.03884
52	15.6409	.67371	.04048
53	15.1740	.68288	.04222
54	14.7094	.69198	.04405
55	14.2480	0.70103	0.04598
56	13.7895	.71000	.04801
57	13.3349	.71890	.05015
58	12.8841	.72776	.05242
59	12.4368	.73654	.05482
60	11.9930	.74525	.05736
61	11.5526	.75387	.06006
62	11.1161	.76243	.06293
63	10.6835	.77093	.06598
64	10.2548	.77934	.06925
65	9.8301	.78765	.07273
66	9.4103	.79589	.07645
67	8.9957	.80400	.08044
68	8.5873	.81203	.08470
69	8.1853	.81989	.08926
70	7.7912	.82763	.09414
71	7.4054	.83519	.09936
72	7.0292	.84257	.10494
73	6.6634	.84973	.11088
74	6.3094	.85668	.11720
75	5.9677	.86338	.12391
76	5.6390	.86983	.13102
77	5.3234	.87601	.13853
78	5.0210	.88193	.14648
79	4.7317	.88761	.15486
80	4.4553	.89303	.16370
81	4.1920	.89820	.17300
82	3.9412	.90310	.18277
83	3.7040	.90777	.19298
84	3.4799	.91214	.20361
85	3.2694	.91627	.21461
86	3.0716	.92016	.22599
87	2.8872	.92380	.23765
88	2.7147	.92716	.24959
89	2.5540	.93030	.26176
90	2.4046	.93323	.27411
91	2.2657	.93597	.28661
92	2.1367	.93849	.29919
93	2.0171	.94083	.31183
94	1.9059	.94298	.32450
95	1.8033	.94504	.33712
96	1.7077	.94691	.34971
97	1.6194	.94863	.36215
98	1.5372	.95024	.37452
99	1.4612	.95174	.38670
100	1.3886	.95315	.39905

Table 21.—United States White Males: 1939–1941—Immediate Whole Life Annuity, Single and Annual Net Premiums at 2½ Percent Interest

[Present value at each age of a life annuity of one per annum, first payment to be made at the end of 1 year; present value of a whole life assurance of one unit, and the annual payment of an equivalent whole life annuity-due]

Age	Immediate life annuity	Single premium	Annual premium	Age	Immediate life annuity	Single premium	Annual premium
x	a_x	A_x	P_x	x	a_x	A_x	P_x
0	29.5650	0.25451	0.00833	55	13.5293	0.64565	0.04444
1	30.8362	.22351	.00702	56	13.1120	.65579	.04647
2	30 7617	.22532	.00709	57	12.6976	.66589	.04861
3	30 6141	.22892	.00724	58	12 2856	.67598	.05088
4	30 4591	.23319	.00742	59	11.8750	.68597	.05328
5	30.2482	.23785	.00761	60	11.4669	.69592	.05582
6	30.0474	.24275	.00782	61	11.0611	.70584	.05852
7	29.8366	.24789	.00804	62	10.6572	.71566	.06139
8	29.6176	.25323	.00827	63	10.2556	.72544	.06445
9	29.3901	.25878	.00852	64	9.8571	.73517	.06771
10	29.1558	.26449	.00877	65	9.4613	.74482	.07120
11	28.9143	.27038	.00904	66	9.0691	.75441	.07492
12	28.6674	.27640	.00932	67	8.6803	.76387	.07891
13	28.4153	.28255	.00961	68	8.2967	.77326	.08318
14	28.1588	.28881	.00990	69	7.9181	.78248	.08774
15	27.8996	.29513	.01021	70	7.5458	.79156	.09263
16	27.6383	.30151	.01053	71	7.1806	.80048	.09785
17	27.3738	.30795	.01085	72	6.8236	.80918	.10343
18	27.1069	.31447	.01119	73	6.4758	.81766	.10937
19	26.8361	.32107	.01153	74	6.1383	.82589	.11570
20	26.5617	.32776	.01189	75	5.8121	.83386	.12241
21	26.2835	.33455	.01226	76	5.4975	.84153	.12951
22	26.0009	.34145	.01265	77	5.1951	.84891	.13703
23	25.7131	.34847	.01304	78	4.9046	.85598	.14497
24	25.4183	.35564	.01346	79	4.6263	.86277	.15335
25	25.1166	.36300	.01390	80	4.3599	.86926	.16218
26	24.8078	.37055	.01436	81	4.1057	.87547	.17147
27	24.4900	.37829	.01484	82	3.8632	.88137	.18123
28	24.1659	.38621	.01535	83	3.6336	.88701	.19143
29	23 8338	.39429	.01588	84	3.4162	.89227	.20204
30	23.4953	.40255	.01643	85	3.2121	.89731	.21303
31	23.1503	.41098	.01702	86	3.0196	.90197	.22439
32	22.7984	.[illegible]956	.01763	87	2.8400	.90635	.23603
33	22.4397	.42829	.01827	88	2.6719	.91045	.24795
34	22.0753	.43719	.01895	89	2.5153	.91427	.26008
35	21.7048	.44623	.01965	90	2.3694	.91781	.27240
36	21.3284	.45541	.02040	91	2.2335	.92111	.28486
37	20.9463	.46472	.02118	92	2.1074	.92419	.29741
38	20.5592	.47416	.02199	93	1.9903	.92703	.31001
39	20.1675	.48374	.02285	94	1.8815	.92971	.32265
40	19.7702	.49340	.02376	95	1.7807	.93217	.33522
41	19.3689	.50319	.02470	96	1.6870	.93445	.34777
42	18.9639	.51307	.02570	97	1.6004	.93659	.36017
43	18.5556	.52305	.02675	98	1.5197	.93853	.37248
44	18.1432	.53308	.02785	99	1.4450	.94037	.38461
45	17.7298	.54319	.02900	100	1.3736	.94212	.39691
46	17.3133	.55335	.03022				
47	16.8949	.56355	.03149				
48	16.4753	.57378	.03283				
49	16.0543	.58404	.03425				
50	15.6327	.59433	.03573				
51	15.2105	.60461	.03730				
52	14.7891	.61492	.03895				
53	14 3674	.62518	.04068				
54	13.9476	.63544	.04251				

TABLE 22.—UNITED STATES WHITE MALES: 1939–1941—IMMEDIATE WHOLE LIFE ANNUITY, SINGLE AND ANNUAL NET PREMIUMS AT 3 PERCENT INTEREST

[Present value at each age of a life annuity of one per annum, first payment to be made at the end of 1 year; present value of a whole life assurance of one unit; and the annual payment of an equivalent whole life annuity-due]

AGE	IMMEDIATE LIFE ANNUITY	SINGLE PREMIUM	ANNUAL PREMIUM	AGE	IMMEDIATE LIFE ANNUITY	SINGLE PREMIUM	ANNUAL PREMIUM
x	a_x	A_x	P_x	x	a_x	A_x	P_x
0	26.2661	0.20584	0.00755	55	12.8673	0.59612	0.04299
1	27.4216	.17218	.00606	56	12.4875	.60718	.04502
2	27.3829	.17332	.00611	57	12.1091	.61822	.04716
3	27.2789	.17634	.00624	58	11.7305	.62920	.04942
4	27.1505	.18008	.00640	59	11.3538	.64019	.05182
5	27.0080	.18423	.00658	60	10.9775	.65112	.05436
6	26.8569	.18864	.00677	61	10.6027	.66206	.05706
7	26.6965	.19330	.00698	62	10.2288	.67294	.05993
8	26.5294	.19818	.00720	63	9.8561	.68380	.06299
9	26.3540	.20328	.00743	64	9.4850	.69462	.06625
10	26.1725	.20857	.00768	65	9.1155	.70538	.06973
11	25.9845	.21404	.00793	66	8.7482	.71608	.07346
12	25.7912	.21967	.00820	67	8.3835	.72670	.07744
13	25.5933	.22544	.00848	68	8.0223	.73721	.08171
14	25.3911	.23133	.00877	69	7.6653	.74761	.08628
15	25.1862	.23730	.00906	70	7.3134	.75787	.09116
16	24.9788	.24334	.00937	71	6.9673	.76795	.09639
17	24.7689	.24945	.00968	72	6.6283	.77783	.10197
18	24.5558	.25565	.01000	73	6.2971	.78746	.10791
19	24.3399	.26195	.01034	74	5.9753	.79684	.11424
20	24.1201	.26835	.01068	75	5.6634	.80592	.12095
21	23.8962	.27487	.01104	76	5.3622	.81469	.12805
22	23.6681	.28152	.01141	77	5.0720	.82314	.13556
23	23.4349	.28830	.01180	78	4.7929	.83126	.14350
24	23.1953	.29528	.01220	79	4.5250	.83907	.15187
25	22.9490	.30246	.01263	80	4.2681	.84655	.16069
26	22.6951	.30985	.01308	81	4.0227	.85373	.16997
27	22.4335	.31748	.01355	82	3.7880	.86052	.17972
28	22.1646	.32531	.01404	83	3.5654	.86700	.18991
29	21.8885	.33634	.01456	84	3.3548	.87317	.20051
30	21.6056	.34158	.01511	85	3.1562	.87896	.21148
31	21.3163	.35002	.01568	86	2.9692	.88439	.22282
32	21.0197	.35864	.01629	87	2.7943	.88949	.23443
33	20.7170	.36747	.01692	88	2.6304	.89425	.24632
34	20.4075	.37649	.01759	89	2.4776	.89873	.25843
35	20.0917	.38568	.01829	90	2.3350	.90285	.27072
36	19.7696	.39505	.01902	91	2.2022	.90670	.28315
37	19.4419	.40460	.01979	92	2.0789	.91034	29567
38	19.1087	.41433	.02060	93	1.9642	.91366	.30823
39	18.7693	.42419	.02146	94	1.8575	.91677	.32083
40	18.4247	.43422	.02235	95	1.7587	.91965	.33336
41	18.0758	.44440	.02330	96	1.6668	.92234	.34586
42	17.7220	.45472	.02429	97	1.5817	.92478	.35821
43	17.3630	.46514	.02533	98	1.5025	.92711	.37048
44	17.0014	.47570	.02643	99	1.4291	.92924	.38254
45	16.6360	.48634	.02758	100	1.3590	.93128	.39478
46	16.2675	.49707	.02879				
47	15.8961	.50789	.03006				
48	15.5219	.51876	.03140				
49	15.1461	.52972	.03281				
50	14.7687	.54074	.03429				
51	14.3896	.55179	.03585				
52	14.0086	.56284	.03750				
53	13.6284	.57395	.03924				
54	13.2474	.58503	.04106				

TABLE 23.—UNITED STATES WHITE MALES: 1939–1941—IMMEDIATE WHOLE LIFE ANNUITY, SINGLE AND ANNUAL NET PREMIUMS AT 3½ PERCENT INTEREST

[Present value at each age of a life annuity of one per annum, first payment to be made at the end of 1 year; present value of a whole life assurance of one unit, and the annual payment of an equivalent whole life annuity-due]

AGE	IMMEDIATE LIFE ANNUITY	SINGLE PREMIUM	ANNUAL PREMIUM	AGE	IMMEDIATE LIFE ANNUITY	SINGLE PREMIUM	ANNUAL PREMIUM
x	a_x	A_x	P_x	x	a_x	A_x	P_x
0	23.5347	0.17033	0.00694	55	12.2569	0.55173	0.04162
1	24.5898	.13465	.00526	56	11.9098	.56345	.04365
2	24.5750	.13514	.00528	57	11.5626	.57515	.04578
3	24.5026	.13760	.00540	58	11.2160	.58690	.04804
4	24.4081	.14078	.00554	59	10.8690	.59863	.05044
5	24.3014	.14440	.00571	60	10.5221	.61036	.05297
6	24.1869	.14828	.00589	61	10.1751	.62210	.05567
7	24.0641	.15242	.00608	62	9.8284	.63383	.05853
8	23.9351	.15678	.00629	63	9.4817	.64555	.06159
9	23.7992	.16139	.00651	64	9.1355	.65725	.06485
10	23.6571	.16618	.00674	65	8.7901	.66893	.06833
11	23.5096	.17117	.00698	66	8.4459	.68058	.07205
12	23.3573	.17633	.00724	67	8.1032	.69216	.07603
13	23.2005	.18163	.00751	68	7.7630	.70366	.08030
14	23.0398	.18707	.00778	69	7.4259	.71506	.08486
15	22.8766	.19258	.00807	70	7.0928	.72632	.08975
16	22.7113	.19818	.00836	71	6.7645	.73743	.09497
17	22.5430	.20385	.00866	72	6.4421	.74832	.10055
18	22.3724	.20962	.00897	73	6.1268	.75900	.10650
19	22.1989	.21551	.00929	74	5.8194	.76938	.11282
20	22.0216	.22150	.00962	75	5.5212	.77947	.11953
21	21.8405	.22762	.00997	76	5.2326	.78924	.12663
22	21.6551	.23388	.01032	77	4.9542	.79867	.13414
23	21.4654	.24030	.01070	78	4.6857	.80774	.14206
24	21.2698	.24692	.01109	79	4.4277	.81648	.15043
25	21.0675	.25377	.01150	80	4.1798	.82485	.15924
26	20.8578	.26085	.01193	81	3.9425	.83286	.16851
27	20.6406	.26818	.01239	82	3.7155	.84053	.17825
28	20.4169	.27575	.01288	83	3.4998	.84783	.18841
29	20.1867	.28356	.01338	84	3.2953	.85475	.19900
30	19.9490	.29158	.01392	85	3.1023	.86127	.20995
31	19.7050	.29983	.01448	86	2.9203	.86744	.22127
32	19.4543	.30831	.01507	87	2.7499	.87318	.23285
33	19.1966	.31701	.01570	88	2.5901	.87858	.24472
34	18.9329	.32593	.01635	89	2.4410	.88366	.25680
35	18.6628	.33507	.01704	90	2.3017	.88835	.26906
36	18.3862	.34441	.01777	91	2.1718	.89274	.28146
37	18.1035	.35397	.01853	92	2.0511	.89682	.29394
38	17.8155	.36374	.01933	93	1.9388	.90063	.30646
39	17.5213	.37369	.02018	94	1.8341	.90415	.31902
40	17.2215	.38384	.02107	95	1.7373	.90743	.33151
41	16.9155	.39414	.02200	96	1.6471	.91051	.34396
42	16.6058	.40466	.02298	97	1.5635	.91331	.35627
43	16.2900	.41530	.02402	98	1.4856	.91593	.36849
44	15.9710	.42612	.02511	99	1.4136	.91839	.38051
45	15.6473	.43705	.02625	100	1.3447	.92072	.39268
46	15.3202	.44812	.02746				
47	14.9891	.45930	.02873				
48	14.6556	.47059	.03006				
49	14.3183	.48197	.03146				
50	13.9787	.49344	.03294				
51	13.6376	.50502	.03450				
52	13.2937	.51663	.03614				
53	12.9488	.52830	.03787				
54	12.6029	.53999	.03970				

TABLE 24.—UNITED STATES WHITE MALES: 1939-1941—IMMEDIATE WHOLE LIFE ANNUITY, SINGLE AND ANNUAL NET PREMIUMS AT 4 PERCENT INTEREST

[Present value at each age of a life annuity of one per annum, first payment to be made at the end of 1 year; present value of a whole life assurance of one unit, and the annual payment of an equivalent whole life annuity-due]

AGE	IMMEDIATE LIFE ANNUITY	SINGLE PREMIUM	ANNUAL PREMIUM	AGE	IMMEDIATE LIFE ANNUITY	SINGLE PREMIUM	ANNUAL PREMIUM
x	a_x	A_x	P_x	x	a_x	A_x	P_x
0	21.2516	0.14417	0.00648	55	11.6923	0.51183	0.04033
1	22 2190	.10696	.00461	56	11.3748	.52404	.04235
2	22.2208	.10689	.00460	57	11.0567	.53627	.04448
3	22.1709	.10881	.00470	58	10.7376	.54855	.04673
4	22.1014	.11148	.00483	59	10.4178	.56086	.04912
5	22.0211	.11458	.00498	60	10.0970	.57319	.05165
6	21.9336	.11794	.00514	61	9.7754	.58555	.05434
7	21.8391	.12158	.00532	62	9.4633	.59794	.05720
8	21.7387	.12543	.00552	63	9.1304	.61036	.06025
9	21.6322	.12953	.00572	64	8.8073	.62280	.06350
10	21.5207	.13383	.00594	65	8.4840	.63523	.06698
11	21.4036	.13832	.00617	66	8.1609	.64767	.07070
12	21.2826	.14299	.00642	67	7.8386	.66006	.07468
13	21.1572	.14780	.00667	68	7.5178	.67239	.07894
14	21.0286	.15275	.00693	69	7.1991	.68465	.08350
15	20.8976	.15778	.00721	70	6.8834	.69678	.08839
16	20.7646	.16290	.00748	71	6.5718	.70878	.09361
17	20.6292	.16810	.00777	72	6.2651	.72058	.09918
18	20.4917	.17340	.00807	73	5.9643	.73214	.10513
19	20.3507	.17881	.00837	74	5.6709	.74344	.11145
20	20.2073	.18434	.00869	75	5.3852	.75441	.11815
21	20.0598	.19000	.00902	76	5.1086	.76507	.12525
22	19.9087	.19581	.00937	77	4.8410	.77536	.13274
23	19.7533	.20178	.00972	78	4.5826	.78527	.14067
24	19.5926	.20797	.01010	79	4.3341	.79489	.14902
25	19.4255	.21440	.01050	80	4.0946	.80405	.15782
26	19.2515	.22108	.01092	81	3.8652	.81288	.16708
27	19.0713	.22804	.01136	82	3.6455	.82133	.17680
28	18.8840	.23524	.01183	83	3.4362	.82037	.18695
29	18.6898	.24269	.01233	84	3.2376	.83701	.19752
30	18.4898	.25039	.01285	85	3.0501	.84424	.20845
31	18.2833	.25834	.01340	86	2 8729	.85104	.21974
32	18.0702	.26654	.01398	87	2.7069	.85744	.23131
33	17.8505	.27498	.01459	88	2.5511	.86343	.24315
34	17.6243	.28366	.01523	89	2.4053	.86901	.25520
35	17.3926	.29260	.01591	90	2.2692	.87425	.26742
36	17.1541	.30177	.01662	91	2.1422	.87913	.27979
37	16.9091	.31117	.01737	92	2.0239	.88369	.29223
38	16.6590	.32081	.01817	93	1.9139	.88794	.30472
39	16.4025	.33068	.01900	94	1.8114	.89189	.31724
40	16.1403	.34076	.01988	95	1.7163	.89554	.32968
41	15.8722	.35105	.02081	96	1.6277	.89894	.34210
42	15.5990	.36156	.02178	97	1.5457	.90210	.35436
43	15.3208	.37226	.02281	98	1.4692	.90503	.36653
44	15.0385	.38316	.02389	99	1.3984	.90777	.37849
45	14.7514	.39420	.02503	100	1.3306	.91036	.39061
46	14.4596	.40541	.02622				
47	14.1645	.41678	.02748				
48	13.8655	.42828	.02881				
49	13.5628	.43991	.03021				
50	13.2571	.45168	.03168				
51	12.9477	.46352	.03323				
52	12.6365	.47550	.03487				
53	12.3235	.48756	.03659				
54	12.0086	.49967	.03841				

TABLE 25.—UNITED STATES WHITE FEMALES: 1939–1941—ELEMENTARY VALUES

[In the interest of internal consistency within the actuarial tables, certain of these values have been altered very slightly from those appearing in table 6, p. 36. For explanation, see text, p. 137]

AGE	OF 100,000 BORN ALIVE		PROBABILITY OF SURVIVING 1 YEAR AT EACH AGE	PROBABILITY OF DYING IN EACH YEAR OF AGE	FORCE OF MORTALITY AT EACH AGE	AGE	OF 100,000 BORN ALIVE		PROBABILITY OF SURVIVING 1 YEAR AT EACH AGE	PROBABILITY OF DYING IN EACH YEAR OF AGE	FORCE OF MORTALITY AT EACH AGE
	Number surviving to each age	Number dying in each year of age					Number surviving to each age	Number dying in each year of age			
x	l_x	d_x	p_x	q_x	μ_x	x	l_x	d_x	p_x	q_x	μ_x
0	100,000	3,789	0.96211	0.03789	8.06964	55	81,520	919	0.98873	0.01127	0.01089
1	96,211	415	.99569	.00431	.00743	56	80,601	987	.98775	.01225	.01181
2	95,796	211	.99780	.00220	.00260	57	79,614	1,059	.98670	.01330	.01284
3	95,585	154	.99839	.00161	.00176	58	78,555	1,136	.98554	.01446	.01396
4	95,431	122	.99872	.00128	.00142	59	77,419	1,219	.98425	.01575	.01520
5	95,309	106	.99889	.00111	.00118	60	76,200	1,306	.98286	.01714	.01656
6	95,203	91	.99904	.00096	.00103	61	74,894	1,399	.98132	.01868	.01805
7	95,112	80	.99916	.00084	.00090	62	73,495	1,495	.97966	.02034	.01968
8	95,032	74	.99922	.00078	.00080	63	72,000	1,596	.97783	.02217	.02146
9	94,958	68	.99928	.00072	.00074	64	70,404	1,703	.97581	.02419	.02342
10	94,890	66	.99930	.00070	.00070	65	68,701	1,816	.97357	.02643	.02559
11	94,824	66	.99930	.00070	.00069	66	66,885	1,935	.97107	.02893	.02802
12	94,758	69	.99927	.00073	.00071	67	64,950	2,061	.96827	.03173	.03075
13	94,689	73	.99923	.00077	.00074	68	62,889	2,194	.96511	.03489	.03382
14	94,616	82	.99913	.00087	.00081	69	60,695	2,332	.96158	.03842	.03727
15	94,534	91	.99904	.00096	.00091	70	58,363	2,470	.95768	.04232	.04114
16	94,443	101	.99893	.00107	.00102	71	55,893	2,610	.95330	.04670	.04545
17	94,342	111	.99882	.00118	.00112	72	53,283	2,744	.94850	.05150	.05026
18	94,231	119	.99874	.00126	.00122	73	50,539	2,870	.94321	.05679	.05558
19	94,112	128	.99864	.00136	.00131	74	47,669	2,984	.93740	.06260	.06146
20	93,984	136	.99855	.00145	.00141	75	44,685	3,078	.93112	.06888	.06791
21	93,848	145	.99845	.00155	.00150	76	41,607	3,149	.92432	.07568	.07494
22	93,703	152	.99838	.00162	.00159	77	38,458	3,192	.91700	.08300	.08257
23	93,551	159	.99830	.00170	.00166	78	35,266	3,203	.90918	.09082	.09082
24	93,392	164	.99824	.00176	.00173	79	32,063	3,181	.90079	.00921	.09973
25	93,228	169	.99819	.00181	.00179	80	28,882	3,125	.89180	.10820	.10936
26	93,059	175	.99812	.00188	.00185	81	25,757	3,034	.88221	.11779	.11978
27	92,884	181	.99805	.00195	.00191	82	22,723	2,911	.87189	.12811	.13105
28	92,703	188	.99797	.00203	.00199	83	19,812	2,755	.86094	.13906	.14324
29	92,515	195	.99789	.00211	.00207	84	17,057	2,570	.84933	.15067	.15638
30	92,320	204	.99779	.00221	.00216	85	14,487	2,361	.83703	.16297	.17044
31	92,116	212	.99770	.00230	.00225	86	12,126	2,131	.82426	.17574	.18541
32	91,904	220	.99761	.00239	.00235	87	9,905	1,880	.81101	.18899	.20124
33	91,684	231	.99748	.00252	.00246	88	8,106	1,644	.79719	.20281	.21792
34	91,453	242	.99735	.00265	.00258	89	6,462	1,401	.78319	.21681	.23542
35	91,211	253	.99723	.00277	.00271	90	5,061	1,171	.76862	.23138	.25373
36	90,958	266	.99708	.00292	.00285	91	3,890	958	.75373	.24627	.27283
37	90,692	280	.99691	.00309	.00301	92	2,932	766	.73874	.26126	.29271
38	90,412	295	.99673	.00327	.00318	93	2,166	600	.72299	.27701	.31334
39	90,117	312	.99654	.00346	.00336	94	1,566	457	.70817	.29183	.33472
40	89,805	330	.99633	.00367	.00357	95	1,109	341.8	.69179	.30821	.35681
41	89,475	352	.99607	.00393	.00381	96	767.2	248.4	.67623	.32377	.37960
42	89,123	374	.99580	.00420	.00407	97	518.8	176.2	.66037	.33963	.40305
43	88,749	400	.99549	.00451	.00436	98	342.6	121.9	.64419	.35581	.42714
44	88,349	429	.99514	.00486	.00460	99	220.7	82.0	.62845	.37155	.45182
45	87,920	460	.99477	.00523	.00505	100	138.7	53.71	.61276	.38724	.47706
46	87,460	493	.99436	.00564	.00544	101	84.99	34.27	.59678	.40322	.50281
47	86,967	528	.99393	.00607	.00587	102	50.72	21.24	.58123	.41877	.52902
48	86,439	566	.99345	.00655	.00633	103	29.48	12.80	.56581	.43419	.55562
49	85,873	606	.99294	.00706	.00682	104	16.68	7.499	.55042	.44958	.58254
50	85,267	650	.99238	.00762	.00736	105	9.181	4.265	.53545	.46455	.60970
51	84,617	695	.99179	.00821	.00794	106	4.916	2.355	.52095	.47905	.63702
52	83,922	746	.99111	.00889	.00858	107	2.561	1.264	.50644	.49356	.66440
53	83,176	799	.99039	.00961	.00928	108	1.297	.6583	.49244	.50756	.69171
54	82,377	857	.98960	.01040	.01004	109	.6387	.6387	.00000	1.00000	------------

TABLE 26.—UNITED STATES WHITE FEMALES: 1939–1941—COMMUTATION COLUMNS AT 2 PERCENT INTEREST

x	D_x	N_x	S_x	C_x	M_x	R_x	x	D_x	N_x	S_x	C_x	M_x	R_x
0	100,000	3,605,671	101,903,354	3,714.7	29,300.6	1,607,561.7	55	27,432	463,398	5,485,065	303.18	18,345.56	355,847.47
1	94,325	3,505,671	98,297,683	398.89	25.585.85	1,578,261.13	56	26,591	435,966	5,021,667	319.23	18,042.38	337,501.91
2	92,076	3,411,346	94,792,012	198.83	25,186.96	1,552,675.28	57	25,750	409,375	4,585,701	335.80	17,723.15	319,459.53
3	90,072	3,319,270	91,380,666	142.27	24,088.13	1,527,488.32	58	24,909	383,625	4,176,326	353 16	17,387.35	301,736.38
4	88,163	3,229,198	88,061,396	110.50	24,845.86	1,502,500.19	59	24,068	358,716	3,792,701	371.53	17,034.19	284,349.03
5	86,324	3,141,035	84,832,198	94.125	24,735.36	1,477,654.33	60	23,224	334,648	3,433,985	390.24	16,662.66	267,314.84
6	84,538	3,054,711	81,691,163	79 221	24,641.24	1,452,918.97	61	22,379	311,424	3,099,337	409.83	16,272.42	250,652.18
7	82,801	2,970,173	78,636,452	68.279	24,562.02	1,428,277.73	62	21,530	289,045	2,787,913	429.37	15,862.59	234,379.76
8	81,109	2,887,372	75,666,279	61.920	24,493.74	1,403,715.71	63	20,670	267,515	2,498,868	449.39	15,433.22	218,517.17
9	79,457	2,806,263	72,778,907	55.784	24,431.82	1,379,221 97	64	19,824	246,836	2,231,353	470.11	14,983.83	203,083.95
10	77,843	2,726,806	69,972.644	53.081	24,376.03	1,354,790.15	65	18,965	227,012	1,984,517	491.48	14,513.72	188,100.12
11	76,263	2,648,963	67,245,838	52 041	24,322.95	1,330,414.12	66	18,102	208.047	1,757,505	513.42	14,022 24	173,586.40
12	74,716	2,572,700	64,596,875	53 339	24.270.91	1,306,091.17	67	17,233	189,045	1,549,458	536.13	13,508.82	159,564.16
13	73,198	2,497,984	62,024,175	55.325	24.217.57	1,281,820 26	68	16,359	172,712	1,359,513	559.53	12,972.69	146,055.34
14	71,707	2,424,786	59,526,191	60.927	24,162.25	1,257,602.69	69	15,479	156,353	1,186,801	583.06	12,413.16	133,082 65
15	70,240	2,353.079	57,101,405	66.289	24,101.32	1,233,440.44	70	14,592	140,874	1,030,448	605.46	11,830.10	120,669.49
16	68,797	2,282,839	54,748,326	72.130	24,035.03	1,209,339.12	71	13,701	126,282	889,574	627.23	11,224.64	108,839.39
17	67,376	2,214,042	52,465,487	77.718	23,962.90	1,185,304.09	72	12,805	112,581	763,292	646.50	10,597.41	97,614.75
18	65,977	2,146,666	50,251,445	81.685	23.885.18	1,161,341.19	73	11,907	99,776	650,711	662.93	9,950.91	87,017.34
19	64,601	2,080,689	48,104,779	86.140	23,803.50	1,137,456.01	74	11,011	87,869	550,935	675.75	9,287.08	77,066.43
20	63,249	2,016,088	46,024,090	89.730	23,717.36	1,113,652.51	75	10,119	76,858	463,066	683.37	8,612.23	67,778.45
21	61,919	1,952,839	44,008.002	93.792	23,627.63	1,089,935.15	76	9,237.5	66,739.0	386,208.2	685.42	7,928.86	59,166.22
22	60,611	1,890,920	42,055,163	96.392	23,533.84	1,066,307.52	77	8,370.9	57,501.5	319,469.2	681.16	7,243.44	51,237.36
23	59,326	1,830,309	40,164,243	98.854	23,437.44	1,042,773.68	78	7,525.6	49,130.6	261,967.7	670.11	6,562.28	43,993 92
24	58,064	1,770,983	38,333,934	99.963	23,338.59	1,019,336.24	79	6,708.0	41,605.0	212,837.1	652.45	5,892.17	37,431.64
25	56,825	1,712,919	36,562,951	100.99	23,238.63	995,997.65	80	5,924.0	34,897.0	171,232.1	628.40	5,239.72	31,539.47
26	55,610	1.656,094	34,850,032	102.53	23,137.64	972,759.02	81	5,179.4	28,973.0	136,335.1	598.14	4,611.32	26,209 75
27	54,417	1,600,484	33,193,938	103.96	23,035.11	949,621.38	82	4,479.7	23,793.6	107,362.1	562.64	4,013.18	21,688.43
28	53,246	1,546,067	31,593,454.	105.87	22,931.15	926,586.27	83	3,829.3	19,313.9	83,568.5	522.04	3,450.54	17,675.25
29	52,096	1,492,821	30,047,387	107.65	22,825.28	903,655.12	84	3,232.1	15,484.6	64.254.6	477.44	2,028.50	14,224.71
30	50,967	1,440,725	28,554,566	110.41	22,717.63	880,829.84	85	2,691.3	12,252.5	48,770 0	430.01	2,451.06	11,296.21
31	49,857	1,389,758	27,113,841	112.49	22,607.22	858.112.21	86	2,208.5	9,561.2	36,517.5	380.51	2,021.05	8,845.15
32	48,767	1,339,901	25,724,083	114.45	22.494.73	835,504.99	87	1,784.7	7,352.7	26,956.3	330 69	1,640.54	6,824.10
33	47,697	1,291,134	24,384,182	117.82	22.380.28	813,010.26	88	1,419.0	5,568.0	19,603 6	282 15	1,309.85	5,183.56
34	46,644	1,243,437	23,093,048	121.01	22,262 46	790,629.98	89	1,109 1	4.140.0	14,035.6	235.73	1,027.70	3.873.71
35	45,608	1,196,793	21,849,611	124.03	22,141.45	768,367.52	90	851.57	3,030.85	9,886.58	193.17	791.97	2,846.01
36	44,590	1,151,185	20,652,818	127.84	22,017.42	746,226.07	91	641.70	2,188.28	6,846.73	154.94	598.80	2,054.04
37	43,588	1,106,505	19,501,633	131.93	21,889.58	724,208.65	92	474.19	1,546.58	4,658.45	121.45	443.86	1,455.24
38	42,601	1,063,007	18,395,038	136.27	21,757.65	702,319.07	93	343 43	1,072 39	3,111.87	93 269	322.408	1,011.385
39	41,629	1,020,406	17,332,031	141.30	21,621.38	680,561.42	94	243.43	728.96	2,030.48	60.647	229.139	688.077
40	40,672	978,777	16,311,625	146.52	21,480.08	658,940.04	95	169.01	485.53	1,310 52	51 060	159 492	459.838
41	39,728	938,105	15,332,848	153.23	21,333.56	637,459.96	96	114.63	316.52	824.99	36 386	108 423	300.346
42	38,796	898,377	14,394,743	159.61	21,180.33	616,126.40	97	75.995	201.890	508.466	25.304	72.037	191.923
43	37,875	859,581	13,496,366	167.36	21,020.72	594,946.07	98	49.201	125.895	306.576	17.163	46.733	119.886
44	36,965	821,706	12,636,785	175.97	20,853.36	573,925.35	99	31.073	76.694	180.681	11.319	29 570	73.153
45	36,065	784,741	11,815,079	184.99	20,677.39	553,071.99	100	19.145	45.621	103.987	7.2684	18.2506	43.5829
46	35,172	748,676	11,030,338	194.37	20,492.40	532,394.60	101	11.501	26.476	58.366	4 5467	10.9822	25 3323
47	34,288	713,504	10,281,662	204.09	20,298.03	511,902.20	102	6.7202	14 9755	31 8898	2.7627	6.4355	14.3501
48	33,412	679,216	9,568,158	214.49	20,093.94	491,604.17	103	3.8345	8 2463	16 9143	1.6323	3.6728	7.9146
49	32,542	645,804	8,888,942	225.15	19,879.45	471,510.23	104	2.1271	4 4118	8.6680	.93753	2.04055	4 24178
50	31,670	613,262	8,243,138	236.76	19,654.30	451,630.78	105	1.1478	2 2847	4 2562	.52276	1.10302	2.20123
51	30,821	581,583	7,629,876	248.19	19,417.54	431,976.48	106	.60255	1.13687	1 97153	.28299	.58026	1.09821
52	29,969	550,762	7.048,293	261.17	19,169.35	412,558.94	107	.30775	.53432	.83466	.14891	.29727	.51795
53	29,120	520,793	6,497,531	274.24	18,908.18	393,389.59	108	.15280	.22657	.30034	.076034	.148357	.220680
54	28,275	491,673	5,976,738	288.38	18,633.94	374,481.41	109	.073770	.073770	.073770	.072323	.072323	.072323

$N_x = D_x + D_{x+1} + \ldots$ $S_x = N_x + N_{x+1} + \ldots$

TABLE 27.—UNITED STATES WHITE FEMALES: 1939–1941—COMMUTATION COLUMNS AT 2½ PERCENT INTEREST

x	D_x	N_x	S_x	C_x	M_x	R_x	x	D_x	N_x	S_x	C_x	M_x	R_x
0	100,000	3,171,576	83,187,057	3,696.6	22,644.5	1,142,624.3	55	20,963	336,119	3,866,678	230.56	12,764 89	241,809.78
1	93,864	3,071,576	80,015,481	395.00	18,947.90	1,119,979.77	56	20,221	315,156	3,530,559	241.58	12,534.33	229,044.89
2	91,180	2,977,712	76,943,905	196.93	18,552.90	1,101,031.87	57	19,486	294,935	3,215,403	252.88	12,292.75	216,510.56
3	88,760	2,886,532	73,966,193	139.52	18,356.97	1,082,478.97	58	18,758	275,449	2,920,468	264.65	12,039.87	204,217.81
4	86,456	2,797,772	71,079,661	107.83	18,217.45	1,064,122.00	59	18,036	256,691	2,645,019	277.06	11,775.22	192,177.94
5	84,239	2,711,316	68,281,889	91.403	18,109.62	1,045,904.55	60	17,319	238,655	2,388,328	289.59	11,498 16	180,402.72
6	82,093	2,627,077	65,570,573	76.555	18,018 21	1,027,794.93	61	16,607	221,336	2,149,673	302.65	11,208.57	168,904.56
7	80,014	2,544,984	62,943,496	65.660	17,941.66	1,009,776.72	62	15,899	204,729	1,928,337	315.53	10,905.92	157,695.99
8	77,997	2,464,970	60,398,512	59.254	17,876.00	991,835.06	63	15,196	188,830	1,723,608	328.63	10,590.39	146,790.07
9	76,036	2,386,973	57,933,542	53.121	17,816.75	973,959.06	64	14,497	173,634	1,534,778	342.11	10,261.76	136,199.68
10	74,128	2,310,937	55,546,569	50.302	17,763.62	956,142.31	65	13,801	159,137	1,361,144	355.91	9,919.65	125,937.92
11	72,270	2,236,809	53,235,632	49.075	17,713 32	938,378.69	66	13,109	145,336	1,202,007	369.98	9,563.74	116,018.27
12	70,458	2,164,539	50,998,823	50.054	17,664.25	920,665.37	67	12,419	132,227	1,056,671	384.46	9,193.76	106,454.53
13	68,689	2,094,081	48,834,284	51.664	17,614.19	903,001.12	68	11,731	119,808	924,444	399.29	8,800.30	97,260.77
14	66,962	2,025,392	46,740,203	56.618	17,562.53	885,386.93	69	11,046	108,077	804,636	414.05	8,410.01	88,451.47
15	65,272	1,958,430	44,714,811	61.300	17,505.91	867,824.40	70	10,363	97,031	696,559	427.86	7,995.96	80,041.46
16	63,619	1,893,158	42,756,381	66.377	17,444.61	850,318.49	71	9,682.0	86,668.2	599,528.2	441.08	7,568.10	72,045.50
17	62,001	1,829,539	40,863,223	71.169	17,378.23	832,873.88	72	9,004.7	76,986.2	512,860.0	452.42	7,127.02	64,477.40
18	60,418	1,767.538	39,033,684	74.438	17,307.07	815,495.65	73	8,332.7	67,981.5	435,873.8	461.65	6,674.60	57,350.38
19	58,870	1,707,120	37,266,146	78.115	17,232.63	798,188.58	74	7,667.8	59,648.8	367,892.3	468.28	6,212.95	50,675.78
20	57,356	1,648,250	35,559,026	80.973	17,154.51	780,955.95	75	7,012.5	51,981.0	308,243.5	471.25	5,744.67	44,462.83
21	55,876	1,590,894	33,910,776	84.225	17,073.54	763,801.44	76	6,370.2	44,968.5	256,262.5	470.37	5,273.42	38,718.16
22	54,429	1,535,018	32,319,882	86.138	16,989.31	746,727.90	77	5,744.5	38,598.3	211,294.0	465.16	4,803.05	33,444.74
23	53,015	1,480,589	30,784,864	87.907	16,903.18	729,738.59	78	5,139.2	32,853.8	172,695.7	455.38	4,337.89	28,641.69
24	51,634	1,427,574	29,304,275	88.460	16,815.27	712,835.41	79	4,558.5	27,714.6	139,841.9	441.22	3,882.51	24,303.80
25	50,286	1,375,940	27,876,701	88.934	16,726.81	696,020.14	80	4,006.1	23,156.1	112,127.3	422.88	3,441.29	20,421.29
26	48,971	1,325,654	26,500,761	89.845	16,637.88	679,293.33	81	3,485.5	19,150.0	88,971.2	400.55	3,018.41	16,980.00
27	47,687	1,276,683	25,175,107	90.659	16,548.03	662,655.45	82	2,999.9	15,664.5	69,821.2	374.94	2,617.86	13,961.59
28	46,433	1,228,996	23,898,424	91.868	16,457.37	646,107.42	83	2,551.8	12,664.6	54,156.7	346.19	2,242.92	11,343.73
29	45,208	1,182.563	22,669,428	92.965	16,365.50	629,650.05	84	2,143.4	10,112.8	41,492.1	315.07	1,896.73	9,100.81
30	44,013	1,137,355	21,486,865	94.883	16,272.54	613,284.55	85	1,776.0	7,969.4	31,379.3	282.39	1,581.66	7,204.08
31	42,845	1,093,342	20,349,510	96.199	16,177.66	597,012.01	86	1,450.3	6,193.4	23,409.9	248.66	1,299.27	5,622.42
32	41,703	1,050,497	19,256,168	97.395	16,081.46	580,834.35	87	1,166.3	4,743.1	17,216.5	215.05	1,050.61	4,323.15
33	40,589	1,008,794	18,205,671	99.770	15,984.06	564,752.89	88	922.80	3,576.78	12,473.35	182.59	835.56	3,272.54
34	39,499	968,205	17,196,877	101.97	15,884.29	548,768.83	89	717.70	2,653.98	8,896.57	151.81	652 97	2,436.98
35	38,434	928,706	16,228,672	104.01	15,782.32	532,884.54	90	548.39	1,936.28	6,242.59	123.79	501.16	1,784.01
36	37,392	890,272	15,299,966	106.68	15,678.31	517,102.22	91	411.22	1,387.89	4,306.31	98.803	377.372	1,282.853
37	36,374	852,880	14,409,694	109.56	15,571.63	501,423.91	92	302.39	976.67	2,918.42	77.074	278 569	905.481
38	35,377	816,506	13,556,814	112.61	15,462.07	485,852.28	93	217.94	674.28	1,941.75	58.899	201.495	626.912
39	34,401	781,129	12,740,308	116.20	15,349.46	470,390.21	94	153.73	456.34	1,267.47	43.767	142.596	425.417
40	33,446	746,728	11,959,179	119.90	15,233.26	455,040.75	95	106.21	302.61	811.13	31.936	98.829	282.821
41	32,510	713,282	11,212,451	124.78	15,113.36	439,807.49	96	71.683	196.396	508.521	22.643	66.893	183.992
42	31,593	680,772	10,499,169	129.34	14,988.58	424,694.13	97	47.292	124.713	312.125	15.670	44.250	117.099
43	30,693	649,179	9,818,397	134.96	14,859.24	409,705.55	98	30.468	77.421	187.412	10.576	28.580	72.849
44	29,809	618,486	9,169,218	141.22	14,724.28	394,846.31	99	19.149	46.953	109.991	6.9411	18.0035	44.2694
45	28,941	588,677	8,550,732	147.73	14,583.06	380,122.03	100	11.741	27.804	63.038	4.4355	11.0624	26.2659
46	28,087	559,736	7,962,055	154.46	14,435.33	365,538.97	101	7.0187	16.0629	35.2340	2.7611	6.6269	15.2035
47	27,248	531,649	7,402,319	161.39	14,280.87	351,103.64	102	4.0864	9.0442	19.1711	1.6695	3.8658	8.5766
48	26,422	504,401	6,870,670	168.79	14,119.48	336,822.77	103	2.3172	4.9578	10.1269	.98159	2.19631	4.71084
49	25,609	477,979	6,366,269	176.31	13,950.69	322,703.29	104	1.2791	2.6406	5.1691	.56104	1.21472	2.51453
50	24,808	452,370	5,888,290	184.50	13,774.38	308,752.60	105	.68689	1.36148	2.52853	.31131	.65368	1.29981
51	24,018	427,562	5,435,920	192.46	13,589.88	294,978.22	106	.35882	.67459	1.16705	.16770	.34237	.64613
52	23,240	403,544	5,008,358	201.55	13,397.42	281,388.34	107	.18237	.31577	.49246	.087815	.174669	.303758
53	22,472	380,304	4,604,814	210.60	13,195.87	267,990.92	108	.090108	.133399	.176690	.044619	.086854	.129089
54	21,713	357,832	4,224,510	220.38	12,985.27	254,795.05	109	.043291	.043291	.043291	.042235	.042235	.042235

$N_x = D_x + D_{x+1} + \ldots$ $S_x = N_x + N_{x+1} + \ldots$

TABLE 28.—UNITED STATES WHITE FEMALES: 1939–1941—COMMUTATION COLUMNS AT 3 PERCENT INTEREST

x	D_x	N_x	S_x	C_x	M_x	R_x	x	D_x	N_x	S_x	C_x	M_x	R_x
0	100,000	2,818,139	68,639,305	3,678.6	17,918.1	818,932.5	55	16,040	244,568	2,735,022	175.56	8,917.09	164,907.31
1	93,409	2,718,139	65,821,166	391.18	14,239.53	801,014.39	56	15,398	228,528	2,490,454	183.06	8,741.53	155,990.22
2	90,297	2,624,730	63,103,027	193.09	13,848.35	786,774.86	57	14,766	213,130	2,261,926	190.69	8,558.47	147,248.69
3	87,474	2,534,433	60,478,297	136.83	13,655.26	772,926.51	58	14,145	198,364	2,048,796	198.60	8,367.78	138,690.22
4	84,789	2,446,959	57,943,864	105.24	13,518.43	750,271.25	59	13,535	184,219	1,850,432	206.90	8,169.18	130,322.44
5	82,214	2,362,170	55,496,905	88.773	13,413.19	745,752.82	60	12,934	170,684	1,666,213	215.21	7,962.28	122,153.26
6	79,731	2,279,956	53,134,735	73.991	13,324.42	732,339.63	61	12,342	157,750	1,495,529	223.83	7,747.07	114,190.98
7	77,335	2,200,225	50,854,779	63.153	13,250.43	719,015.21	62	11,758	145,408	1,337,779	232.22	7,523.24	106,443.91
8	75,019	2,122,890	48,654,554	56.715	13,187.27	705,764.78	63	11,184	133,650	1,192,371	240.69	7,291.02	98,920.67
9	72,777	2,047,871	46,531,664	50.598	13,130.56	692,577.51	64	10,617	122,466	1,058,721	249.34	7,050.33	91,629.65
10	70,607	1,975,094	44,483,793	47.680	13,079.96	679,446.95	65	10,059	111,849	936,255	258.14	6,800.99	84,579.32
11	68,503	1,904,487	42,508,699	46.291	13,032.28	666,366.99	66	9,507.6	101,790.2	824,405.7	267.05	6,542.85	77,778.33
12	66,461	1,835,984	40,604,212	46.986	12,985.99	653,334.71	67	8,963.7	92,282.6	722,615.5	276.15	6,275.80	71,235.48
13	64,479	1,769,523	38,768,228	48.262	12,939.00	640,348.72	68	8,426.4	83,318.9	630,332.9	285.41	5,999.65	64,959.68
14	62,552	1,705,044	36,998,705	52.633	12,890.74	627,409.72	69	7,895.6	74,892.5	547,014.0	294.53	5,714.24	58,960.03
15	60,678	1,642,492	35,293,661	56.708	12,838.11	614,518.98	70	7,371.1	66,996.9	472,121.5	302.87	5,419.71	53,245.79
16	58,854	1,581,814	33,651,169	61.107	12,781.40	601,680.87	71	6,853.5	59,625.8	405,124.6	310.71	5,116.84	47,826.08
17	57,078	1,522,960	32,069,355	65.201	12,720.29	588,899.47	72	6,343.2	52,772.3	345,498.8	317.15	4,806.13	42,709.24
18	55,351	1,465,882	30,546,395	67.864	12,655.09	576,179.18	73	5,841.3	46,429.1	292,726.5	322.05	4,488.98	37,903.11
19	53,671	1,410,531	29,080,513	70.870	12,587.23	563,524.09	74	5,349.1	40,587.8	246,297.4	325.09	4,166.93	33,414.13
20	52,037	1,356,860	27,669,982	73.107	12,516.36	550,936.86	75	4,868.2	35,238.7	205,709.6	325.57	3,841.84	29,247.20
21	50,448	1,304,823	26,313,122	75.674	12,443.25	538,420.50	76	4,400.9	30,370.5	170,470.9	323.37	3,516.27	25,405.36
22	48,903	1,254,375	25,008,299	77.017	12,367.58	525,977.25	77	3,949.3	25,969.6	140,100.4	318.24	3,192.90	21,889.09
23	47,402	1,205,472	23,753,924	78.217	12,290.56	513,609.67	78	3,516.0	22,020.3	114,130.8	310.04	2,874.66	18,696.19
24	45,943	1,158,070	22,548,452	78.327	12,212.34	501,319.11	79	3,103.6	18,504.3	92,110.5	298.94	2,564.62	15,821.53
25	44,526	1,112,127	21,390,382	78.364	12,134.02	489,106.77	80	2,714.2	15,400.7	73,606.2	285.12	2,265.68	13,256.91
26	43,151	1,067,601	20,278,255	78.783	12,055.65	476,972.75	81	2,350.1	12,686.5	58,205.5	268.76	1,980.56	10,991.23
27	41,815	1,024,450	19,210,654	79.111	11,976.87	464,917.10	82	2,012.8	10,336.4	45,519.0	250.35	1,711.80	9,010.67
28	40,518	982,635	18,186,204	79.777	11,897.76	452,940.23	83	1,703.9	8,323.6	35,182.6	230.04	1,461.45	7,298.87
29	39,258	942,117	17,203,569	80.337	11,817.98	441,042.47	84	1,424.2	6,619.7	26,859.0	208.34	1,231.41	5,837.42
30	38,035	902,859	16,261,452	81.597	11,737.65	429,224.49	85	1,174.4	5,195.5	20,239.3	185.82	1,023.07	4,606.01
31	36,845	864,824	15,358,593	82.327	11,656.05	417,486.84	86	954.37	4,021.10	15,043.76	162.83	837.25	3,582.94
32	35,690	827,979	14,493,769	82.946	11,573.72	405,830.79	87	763.74	3,066.73	11,022.66	140.14	674.42	2,745.69
33	34,567	792,289	13,665,790	84.556	11,490.78	394,257.07	88	601.35	2,302.99	7,955.93	118.41	534.28	2,071.27
34	33,476	757,722	12,873,501	86.003	11,406.22	382,766.29	89	465.43	1,701.64	5,652.94	97.969	415.866	1,536.991
35	32,415	724,246	12,115,779	87.293	11,320.22	371,360.07	90	353.90	1,236.21	3,951.30	79.500	317.897	1,121.125
36	31,383	691,831	11,391,533	89.105	11,232.92	360,039.85	91	264.10	882.31	2,715.09	63.145	238.397	803.228
37	30,380	660,448	10,699,702	91.063	11,143.82	348,806.93	92	193.26	618.21	1,832.78	49.019	175.252	564.831
38	29,404	630,068	10,039,254	93.147	11,052.76	337,663.11	93	138.61	424.95	1,214.57	37.278	126.233	389.579
39	28,455	600,664	9,400,186	95.646	10,959.61	326,610.35	94	97.295	286.344	789.622	27.566	88.955	263.346
40	27,530	572,209	8,808,522	98.217	10,863.96	315,650.74	95	66.895	189.049	503.278	20.017	61.389	174.391
41	26,630	544,679	8,236,313	101.71	10,765 75	304,786.78	96	44.930	122.154	314.229	14.123	41.372	113.002
42	25,753	518,049	7,691,634	104.92	10,664 04	294,021.03	97	29.498	77.224	192.075	9.7265	27.2485	71.6298
43	24,898	492,296	7,173,585	108.95	10,559.12	283,356.99	98	18.912	47.726	114.851	6.5331	17.5220	44.3813
44	24,064	467,398	6,681,289	113.44	10,450.17	272,797.87	99	11.828	28.814	67.125	4.2667	10.9889	26.8593
45	23,249	443,334	6,213,891	118.10	10,336.73	262,347.70	100	7.2170	16.9864	38.3106	2.7133	6.7222	15.8704
46	22,454	420,085	5,770,557	122.88	10,218.63	252,010.97	101	4.2935	9.7694	21.3242	1.6808	4.0089	9.1482
47	21,677	397,631	5,350,472	127.78	10,095.75	241,792.34	102	2.4876	5.4759	11.5548	1.0114	2.3281	5 1393
48	20,918	375,954	4,952,841	132.98	9,967.97	231,696.59	103	1.4038	2.0883	6.0789	.59175	1.31671	2.81119
49	20,176	355,036	4,576,887	138.23	9,834.99	221,728.62	104	.77112	1.58452	3.09056	.33658	.72496	1.49448
50	19,450	334,860	4,221,851	143.95	9,696.76	211,893.63	105	.41208	.81340	1.50604	.18585	.38838	.76952
51	18,740	315,410	3,886,991	149.43	9,552.81	202,196.87	106	.21422	.40132	.69264	.099634	.202534	.381144
52	18,044	296,670	3,571,581	155.73	9,403.38	192,644.06	107	.10835	.18710	.29132	.051919	.102900	.178610
53	17,363	278,626	3,274,911	161.93	9,247.65	183,240.68	108	.053275	.078746	.104217	.026252	.050981	.075710
51	16,695	261,263	2,996,285	168.63	9,085.72	173,903.03	109	.025471	.025471	.025471	.024729	.024729	.024729

$N_x = D_x + D_{x+1} + \ldots$ $S_x = N_x + N_{x+1} + \ldots$

TABLE 29.—UNITED STATES WHITE FEMALES: 1939–1941—COMMUTATION COLUMNS AT 3½ PERCENT INTEREST

x	D_x	N_x	S_x	C_x	M_x	R_x	x	D_x	N_x	S_x	C_x	M_x	R_x
0	100,000	2,527,377	57,226,448	3,660.9	14,533.2	592,184.4	55	12,290	178,500	1,941,011	133.86	6,253.56	112,862.73
1	92,957	2,427,377	54,699,071	387.41	10,872.35	577,651.23	56	11,740	166,210	1,762,511	138.90	6,119.70	106,609.17
2	89,427	2,334,420	52,271,694	190.31	10,484.94	566,778.88	57	11,204	154,470	1,596,301	144.00	5,980.80	100,489.47
3	86,212	2,244,993	49,937,274	134.20	10,294.63	556,293.94	58	10,682	143,266	1,441,831	149.24	5,836.80	94,508.67
4	83,163	2,158,781	47,692,281	102.72	10,160.43	545,999.31	59	10,171	132,584	1,298,565	154.73	5,687.56	88,671.87
5	80,248	2,075,618	45,533,500	86.231	10,057.71	535,838.88	60	9,672.4	122,413.3	1,165,981.5	160.17	5,532.83	82,984.31
6	77,448	1,995,370	43,457,882	71.525	9,971.48	525,781.17	61	9,185.1	112,740.9	1,043,568.2	165.77	5,372.66	77,451.48
7	74,757	1,917,922	41,462,512	60.753	9,899.95	515,809.69	62	8,708.8	103,555.8	930,827.3	171.16	5,206.89	72,078.82
8	72,168	1,843,165	39,544,590	54.296	9,839.20	505,909.74	63	8,243.1	94,847.0	827,271.5	176.54	5,035.73	66,871.93
9	69,674	1,770,997	37,701,425	48.206	9,784.90	496,070.54	64	7,787.8	86,603.9	732,424.5	182.01	4,859.19	61,836.20
10	67,269	1,701,323	35,930,428	45.206	9,736.70	486,285.64	65	7,342.4	78,816.1	645,820.6	187.52	4,677.18	56,977.01
11	64,949	1,634,054	34,229,105	43.678	9,691.49	476,548.94	66	6,906.6	71,473.7	567,004.5	193.05	4,489.66	52,299.83
12	62,709	1,569,105	32,595,051	44.119	9,647.81	466,857.45	67	6,480.0	64,567.1	495,530.8	198.67	4,296.61	47,810.17
13	60,545	1,506,396	31,025,946	45.098	9,603.69	457,209.64	68	6,062.2	58,087.1	430,963.7	204.34	4,097.94	43,513.56
14	58,452	1,445,851	29,519,550	48.945	9,558.59	447,605.95	69	5,652.9	52,024.9	372,876.6	209.85	3,893.60	39,415.62
15	56,426	1,387,399	28,073,699	52.480	9,509.65	438,047.36	70	5,251.9	46,372.0	320,851.7	214.75	3,683.75	35,522.02
16	54,466	1,330,973	26,686,300	56.278	9,457.17	428,537.71	71	4,859.5	41,120.1	274,479.7	219.25	3,469.00	31,838.27
17	52,568	1,276,507	25,355,327	59.758	9,400.89	419,080.54	72	4,475.9	36,260.6	233,359.6	222.71	3,249.75	28,369.27
18	50,730	1,223,939	24,078,820	61.899	9,341.13	409,679.65	73	4,101.9	31,784.7	197,099.0	225.06	3,027.04	25,119.52
19	48,953	1,173,209	22,854,881	64.328	9,279.23	400,338.52	74	3,738.1	27,682.8	165,314.3	226.09	2,801.98	22,092.48
20	47,233	1,124,256	21,681,672	66.038	9,214.91	391,059.29	75	3,385.6	23,944.7	137,631.5	225.32	2,575.89	19,290.50
21	45,570	1,077,023	20,557,416	68.027	9,148.87	381,844.38	76	3,045.8	20,559.1	113,686.8	222.72	2,350.57	16,714.61
22	43,961	1,031,453	19,480,393	68.899	9,080.84	372,695.51	77	2,720.1	17,513.3	93,127.7	218.13	2,127.85	14,364.04
23	42,405	987,492	18,448,940	69.635	9,011.94	363,614.67	78	2,410.0	14,793.2	75,614.4	211.48	1,909.72	12,236.19
24	40,902	945,087	17,461,448	69.396	8,942.31	354,602.73	79	2,117.0	12,383.2	60,821.2	202.93	1,698.24	10,326.47
25	39,449	904,185	16,516,361	69.094	8,872.91	345,660.42	80	1,842.5	10,266.2	48,438.0	192.61	1,495.31	8,628.23
26	38,046	864,736	15,612,176	69.127	8,803.82	336,787.51	81	1,587.5	8,423.7	38,171.8	180.68	1,302.70	7,132.92
27	36,690	826,690	14,747,440	69.079	8,734.69	327,983.69	82	1,353.2	6,836.2	29,748.1	167.49	1,122.02	5,830.22
28	35,381	790,000	13,920,750	69.325	8,665.61	319,249.00	83	1,139.9	5,483.0	22,911.9	153.16	954.53	4,708.20
29	34,115	754,619	13,130,750	69.474	8,596.29	310,583.39	84	948.23	4,343.05	17,428.88	138.04	801.37	3,753.67
30	32,892	720,504	12,376,131	70.223	8,526.81	301,987.10	85	778.12	3,394.82	13,085.83	122.53	663.33	2,952.30
31	31,709	687,612	11,655,627	70.509	8,456.59	293,460.29	86	629.29	2,616.70	9,691.01	106.85	540.80	2,288.97
32	30,566	655,903	10,968,015	70.695	8,386.08	285,003.70	87	501.16	1,987.41	7,074.31	91.513	433.948	1,748.166
33	29,462	625,337	10,312,112	71.720	8,315.39	276,617.62	88	392.70	1,486.25	5,086.90	76.950	342.435	1,314.218
34	28,394	595,875	9,686,775	72.594	8,243.67	268,302.23	89	302.47	1,093.55	3,600.65	63.359	265.485	971.783
35	27,361	567,481	9,090,900	73.328	8,171.07	260,058.56	90	228.88	791.08	2,507.10	51.166	202.126	706.298
36	26,363	540,120	8,523,419	74.488	8,097.74	251,887.49	91	169.97	562.20	1,716.02	40.444	150.960	504.172
37	25,897	513,757	7,983,299	75.757	8,023.26	243,789.75	92	123.78	392.23	1,153.82	31.245	110.516	353.212
38	24,462	488,360	7,469,542	77.117	7,947.50	235,766.49	93	88.350	268.452	761.590	23.646	79.271	242.696
39	23,558	463,898	6,981,182	78.803	7,870.38	227,818.99	94	61.716	180.102	493.138	17.401	55.625	163.425
40	22,682	440,340	6,517,284	80.530	7,791.58	219,948.61	95	42.228	118.386	313.036	12.575	38.224	107.800
41	21,835	417,658	6,076,944	82.994	7,711.05	212,157.03	96	28.225	76.158	194.650	8.8295	25.6495	69.5756
42	21,013	395,823	5,659,286	85.199	7,628.05	204,445.98	97	18.441	47.933	118.492	6.0513	16.8200	43.9261
43	20,218	374,810	5,263,463	88.041	7,542.86	196,817.93	98	11.766	29.492	70.559	4.0449	10.7687	27.1061
44	19,446	354,592	4,888,653	91.231	7,454.81	189,275.07	99	7.3233	17.7262	41.0670	2.6289	6.7238	16.3374
45	18,697	335,146	4,534,061	94.515	7,363.58	181,820.26	100	4.4467	10.4029	23.3408	1.6637	4.0949	9.6136
46	17,970	316,449	4,198,915	97.870	7,269.07	174,456.68	101	2.6326	5.9562	12.9379	1.0256	2.4312	5.5187
47	17,265	298,479	3,882,466	101.27	7,171.20	167,187.61	102	1.5180	3.3236	6.9817	.61418	1.40558	3.08754
48	16,580	281,214	3,583,987	104.89	7,069.93	160,016.41	103	.85245	1.80564	3.65812	.35761	.79140	1.68196
49	15,914	264,634	3,302,773	108.51	6,965.04	152,946.48	104	.46601	.95319	1.85248	.20243	.43379	.89056
50	15,267	248,720	3,038,139	112.45	6,856.53	145,981.44	105	.24783	.48718	.89929	.11124	.23136	.45677
51	14,639	233,453	2,789,419	116.17	6,744.08	139,124.91	106	.12821	.23935	.41211	.059344	.120120	.225414
52	14,027	218,814	2,555,966	120.48	6,627.91	132,380.83	107	.064535	.111137	.172763	.030774	.060776	.105294
53	13,433	204,787	2,337,152	124.67	6,507.43	125,752.92	108	.031578	.046602	.061626	.015486	.030002	.044518
54	12,854	191,354	2,132,365	129.20	6,382.76	119,245.49	109	.015024	.015024	.015024	.014516	.014516	.014516

$N_x = D_x + D_{x+1} + \ldots$ $S_x = N_x + N_{x+1} + \ldots$

TABLE 30.—UNITED STATES WHITE FEMALES: 1939–1941—COMMUTATION COLUMNS AT 4 PERCENT INTEREST

x	D_x	N_x	S_x	C_x	M_x	R_x	x	D_x	N_x	S_x	C_x	M_x	R_x
0	100,000	2,285,754	48,189,353	3,643.3	12,086.5	432,318.5	55	9,428.2	130,668.6	1,382,001.3	102.20	4,402.53	77,514.99
1	92,511	2,185,754	45,903,599	383.69	8,443.15	420,231.99	56	8,963.4	121,240.4	1,251,332.7	105.54	4,300.33	73,112.46
2	88,569	2,093,243	43,717,845	187.58	8,059.46	411,788.84	57	8,513.1	112,277.0	1,130,092.3	108.88	4,194.79	68,812.13
3	84,975	2,004,674	41,624,602	131.64	7,871.88	403,729.38	58	8,076.8	103,763.9	1,017,815.3	112.31	4,085.91	64,617.34
4	81,575	1,919,699	39,619,928	100.28	7,740.24	395,857.50	59	7,653.9	95,687.1	914,051.4	115.88	3,973.60	60,531.43
5	78,337	1,838,124	37,700,229	83.773	7,639.96	388,117.26	60	7,243.6	88,033.2	818,364.3	119.37	3,857.72	56,557.83
6	75,240	1,759,787	35,862,105	69.153	7,556.19	380,477.30	61	6,845.6	80,789.6	730,331.1	122.96	3,738.35	52,700.11
7	72,277	1,684,547	34,102,318	58.455	7,487.04	372,921.11	62	6,459.4	73,944.0	649,541.5	126.34	3,615.39	48,961.76
8	69,439	1,612,270	32,417,771	51.991	7,428.58	365,434.07	63	6,084.6	67,484.6	575,597.5	129.69	3,489.05	45,346.37
9	66,716	1,542,831	30,805,501	45.938	7,376.59	358,005.49	64	5,720.9	61,400.0	508,112.9	133.06	3,359.36	41,857.32
10	64,104	1,476,115	29,262,670	42.872	7,330.65	350,628.90	65	5,367.8	55,679.1	446,712.9	136.43	3,226.30	38,497.96
11	61,596	1,412,011	27,786,555	41.223	7,287.78	343,298.25	66	5,024.9	50,311.3	391,033.8	139.78	3,089.87	35,271.66
12	59,186	1,350,415	26,374,544	41.440	7,246.56	336,010.47	67	4,691.9	45,286.4	340,722.5	143.16	2,950.09	32,181.79
13	56,868	1,291,229	25,024,129	42.156	7,205.12	328,763.91	68	4,368.3	40,594.5	295,436.1	146.53	2,806.93	29,231.70
14	54,638	1,234,361	23,732,900	45.532	7,162.96	321,558.79	69	4,053.7	36,226.2	254,841.6	149.76	2,660.40	26,424.77
15	52,491	1,179,723	22,498,539	48.586	7,117.43	314,395.83	70	3,748.0	32,172.5	218,615.4	152.52	2,510.64	23,764.37
16	50,424	1,127,232	21,318,816	51.851	7,068.84	307,278.40	71	3,451.4	28,424.5	186,442.9	154.97	2,358.12	21,253.73
17	48,433	1,076,808	20,191,584	54.793	7,016.99	300,209.56	72	3,163.6	24,973.1	158,018.4	156.66	2,203.15	18,895.61
18	46,515	1,028,375	19,114,776	56.482	6,962.20	293,192.57	73	2,885.3	21,809.5	133,045.3	157.55	2,046.49	16,692.46
19	44,670	981,860	18,086,401	58.418	6,905.72	286,230.37	74	2,616.8	18,924.2	111,235.8	157.51	1,888.94	14,645.97
20	42,893	937,190	17,104,541	59.681	6,847.30	279,324.65	75	2,358.6	16,307.4	92,311.6	156.22	1,731.43	12,757.03
21	41,184	894,297	16,167,351	61.183	6,787.62	272,477.35	76	2,111.7	13,948.8	76,004.2	153.68	1,575.21	11,025.60
22	39,538	853,113	15,273,054	61.670	6,726.44	265,689.73	77	1,876.8	11,837.1	62,055.4	149.78	1,421.53	9,450.39
23	37,956	813,575	14,419,941	62.029	6,664.77	258,963.29	78	1,654.8	9,960.3	50,218.3	144.52	1,271.75	8,028.86
24	36,434	775,619	13,606,366	61.519	6,602.74	252,298.52	79	1,446.7	8,305.5	40,258.0	138.01	1,127.23	6,757.11
25	34,971	739,185	12,830,747	60.956	6,541.22	245,695.78	80	1,253.0	6,858.8	31,952.5	130.36	989.22	5,629.88
26	33,565	704,214	12,091,562	60.693	6,480.26	239,154.56	81	1,074.5	5,605.8	25,093.7	121.70	858.86	4,640.66
27	32,214	670,649	11,387,348	60.359	6,419.57	232,674.30	82	911.45	4,531.33	19,487.89	112.27	737.16	3,781.80
28	30,914	638,435	10,716,699	60.282	6,359.21	226,254.73	83	764.12	3,619.88	14,956.56	102.17	624.89	3,044.64
29	29,665	607,521	10,078,264	60.122	6,298.93	219,895.52	84	632.56	2,855.76	11,336.68	91.643	522.725	2,419.748
30	28,464	577,856	9,470,743	60.478	6,238.81	213,596.59	85	516.59	2,223.20	8,480.92	80.952	431.082	1,897.023
31	27,309	549,392	8,892,887	60.432	6,178.33	207,357.78	86	415.77	1,706.61	6,257.72	70.256	350.130	1,465.941
32	26,198	522,083	8,343,495	60.301	6,117.90	201,179.45	87	329.52	1,290.84	4,551.11	59.882	279.874	1,115.811
33	25,130	495,885	7,821,412	60.881	6,057.59	195,061.55	88	256.96	961.32	3,260.27	50.111	219.992	835.937
34	24,103	470,755	7,325,527	61.327	5,996.71	189,003.96	89	196.97	704.36	2,298.95	41.062	169.881	615.945
35	23,114	446,652	6,854,772	61.648	5,935.39	183,007.25	90	148.33	507.39	1,594.59	33.001	128.819	446.064
36	22,164	423,538	6,408,120	62.323	5,873.74	177,071.86	91	109.63	359.06	1,087.20	25.960	95.818	317.245
37	21,249	401,374	5,984,582	63.080	5,811.42	171,198.12	92	79.451	249.431	728.138	19.959	69.858	221.427
38	20,369	380,125	5,583,208	63.903	5,748.34	165,386.70	93	56.436	169.980	478.707	15.032	49.899	151.569
39	19,521	359,756	5,203,083	64.986	5,684.43	159,638.36	94	39.234	113.544	308.727	11.009	34.867	101.670
40	18,705	340,235	4,843,327	66.092	5,619.45	153,953.93	95	26.716	74.310	195.183	7.9172	23.8576	66.8030
41	17,920	321,530	4,503,092	67.786	5,553.35	148,334.48	96	17.771	47.594	120.873	5.5325	15.9404	42.9454
42	17,163	303,610	4,181,562	69.253	5,485.57	142,781.13	97	11.555	29.823	73.279	3.7734	10.4079	27.0050
43	16,433	286,447	3,877,952	71.219	5,416.32	137,295.56	98	7.3370	18.2683	43.4557	2.5102	6.6345	16.5971
44	15,730	270,014	3,591,505	73.444	5,345.10	131,879.24	99	4.5447	10.9313	25.1874	1.6236	4.1243	9.9626
45	15,052	254,284	3,321,491	75.722	5,271.65	126,534.14	100	2.7463	6.3866	14.2561	1.0226	2.5007	5.8383
46	14,397	239,232	3,067,207	78.033	5,195.93	121,262.49	101	1.6181	3.6403	7.8695	.62736	1.47807	3.33764
47	13,765	224,835	2,827,975	80.359	5,117.90	116,066.56	102	.92849	2.02223	4.22915	.37387	.85071	1.85957
48	13,156	211,070	2,603,140	82.829	5,037.54	110,948.66	103	.51891	1.09374	2.20692	.21664	.47684	1.00886
49	12,567	197,914	2,392,070	85.272	4,954.71	105,911.12	104	.28231	.57483	1.11318	.12204	.26020	.53202
50	11,998	185,347	2,194,156	87.945	4,869.44	100,956.41	105	.14941	.29252	.53835	.066740	.138162	.271817
51	11,449	173,349	2,008,809	90.417	4,781.49	96,086.97	106	.076927	.143111	.245830	.035434	.071422	.133655
52	10,918	161,900	1,835,460	93.319	4,691.08	91,305.48	107	.038534	.066184	.102719	.018287	.035988	.062233
53	10,405	150,982	1,673,560	96.105	4,597.76	86,614.40	108	.018765	.027650	.036535	.0091578	.0177012	.0262446
54	9,908.4	140,577.0	1,522,578.3	99.117	4,501.65	82,016.64	109	.0088851	.0088851	.0088851	.0085434	.0085434	.0085434

$N_x = D_x + D_{x+1} + \ldots$ $S_x = N_x + N_{x+1} + \ldots$

TABLE 31.—UNITED STATES WHITE FEMALES: 1939–1941—IMMEDIATE WHOLE LIFE ANNUITY, SINGLE AND ANNUAL NET PREMIUMS AT 2 PERCENT INTEREST

[Present value at each age of a life annuity of one per annum, first payment to be made at the end of 1 year; present value of a whole life assurance of one unit, and the annual payment of an equivalent whole life annuity-due]

AGE	IMMEDIATE LIFE ANNUITY	SINGLE PREMIUM	ANNUAL PREMIUM
x	a_x	A_x	P_x
0	35.0567	0.29301	0.00813
1	36.1659	.27125	.00730
2	36.0492	.27355	.00738
3	35.8513	.27742	.00753
4	35.6276	.28182	.00769
5	35.3866	.28654	.00787
6	35.1342	.29148	.00807
7	34.8712	.29664	.00827
8	34.5987	.30199	.00848
9	34.3180	.30748	.00871
10	34.0296	.31314	.00894
11	33.7346	.31894	.00918
12	33.4331	.32484	.00943
13	33.1264	.33085	.00969
14	32.8152	.33696	.00996
15	32.5006	.34313	.01024
16	32.1822	.34936	.01053
17	31 8610	.35566	.01082
18	31.5366	.36202	.01113
19	31.2083	.36847	.01144
20	30.8754	.37498	.01176
21	30.5386	.38159	.01210
22	30.1976	.38828	.01245
23	29.8517	.39506	.01281
24	29.5005	.40195	.01318
25	29.1438	.40895	.01357
26	28 7805	.41607	.01397
27	28.4115	.42331	.01439
28	28.0363	.43066	.01483
29	27.6552	.43814	.01529
30	27.2678	.44573	.01577
31	26.8749	.45344	.01627
32	26.4756	.46127	.01679
33	26.0695	.46922	.01733
34	25.6580	.47728	.01790
35	25.2409	.48547	.01850
36	24.8171	.49377	.01913
37	24.3876	.50219	.01978
38	23.9526	.51073	.02047
39	23.5119	.51938	.02119
40	23.0651	.52813	.02195
41	22.6132	.53699	.02274
42	22.1564	.54594	.02358
43	21.6952	.55500	.02445
44	21.2293	.56414	.02538
45	20.7591	.57334	.02635
46	20.2861	.58263	.02737
47	19.8091	.59199	.02845
48	19.3285	.60140	.02958
49	18.8452	.61089	.03078
50	18.3586	.62042	.03205
51	17.8697	.63001	.03339
52	17.3777	.63964	.03481
53	16.8844	.64932	.03631
54	16.3890	.65903	.03790
55	15.8926	0.66876	0.03959
56	15.3952	.67851	.04138
57	14.8981	.68828	.04329
58	14.4011	.69803	.04532
59	13.9043	.70775	.04749
60	13.4096	.71748	.04979
61	12.9159	.72713	.05225
62	12.4252	.73677	.05488
63	11.9366	.74632	.05769
64	11.4514	.75584	.06070
65	10.9701	.76529	.06393
66	10.4930	.77462	.06740
67	10.0222	.78389	.07112
68	9.5576	.79300	.07511
69	9.1010	.80194	.07939
70	8.6542	.81073	.08398
71	8.2170	.81926	.08889
72	7.7920	.82760	.09413
73	7.3796	.83572	.09973
74	6.9801	.84352	.10570
75	6.5954	.85109	.11205
76	6.2248	.85833	.11880
77	5.8692	.86531	.12597
78	5.5285	.87199	.13357
79	5.2023	.87838	.14162
80	4.8908	.88449	.15015
81	4.5939	.89032	.15916
82	4.3114	.89586	.16867
83	4.0437	.90109	.17866
84	3.7909	.90607	.18912
85	3.5526	.91073	.20005
86	3.3293	.91512	.21138
87	3.1199	.91922	.22312
88	2.9239	.92308	.23525
89	2.7408	.92661	.24770
90	2.5697	.93001	.26053
91	2.4101	.93315	.27364
92	2.2615	.93604	.28699
93	2.1226	.93879	.30064
94	1.9945	.94129	.31434
95	1.8728	.94368	.32849
96	1.7612	.94585	.34255
97	1.6566	.94792	.35681
98	1.5588	.94984	.37121
99	1.4682	.95163	.38556
100	1.3829	.95328	.40005

Table 32.—United States White Females: 1939–1941—Immediate Whole Life Annuity, Single and Annual Net Premiums at 2½ Percent Interest

[Present value at each age of a life annuity of one per annum, first payment to be made at the end of 1 year; present value of a whole life assurance of one unit, and the annual payment of an equivalent whole life annuity-due]

Age	Immediate life annuity	Single premium	Annual premium
x	a_x	A_x	P_x
0	30.7158	0.22644	0.00714
1	31.7237	.20187	.00617
2	31.6575	.20348	.00623
3	31.5206	.20682	.00636
4	31.3606	.21071	.00651
5	31.1860	.21498	.00668
6	31.0012	.21949	.00686
7	30.8067	.22423	.00705
8	30.6034	.22919	.00725
9	30.3927	.23432	.00746
10	30.1750	.23963	.00769
11	29.9507	.24510	.00792
12	29.7210	.25071	.00816
13	29.4864	.25643	.00841
14	29.2469	.26228	.00867
15	29.0041	.26820	.00894
16	28.7577	.27420	.00921
17	28.5082	.28029	.00950
18	28.2552	.28646	.00979
19	27.9981	.29272	.01009
20	27.7372	.29909	.01041
21	27.4719	.30556	.01073
22	27.2022	.31214	.01107
23	26.9277	.31884	.01142
24	26.6479	.32566	.01178
25	26.3623	.33263	.01216
26	26.0702	.33975	.01255
27	25.7721	.34701	.01296
28	25.4682	.35443	.01339
29	25.1583	.36200	.01384
30	24.8413	.36972	.01431
31	24.5185	.37759	.01480
32	24.1900	.38562	.01531
33	23.8539	.39380	.01584
34	23.5121	.40214	.01641
35	23.1637	.41063	.01699
36	22.8092	.41930	.01761
37	22.4475	.42810	.01826
38	22.0801	.43707	.01894
39	21.7066	.44619	.01965
40	21.3264	.45546	.02040
41	20.9404	.46488	.02119
42	20.5482	.47443	.02202
43	20.1507	.48412	.02289
44	19.7483	.49395	.02381
45	19.3406	.50389	.02477
46	18.9287	.51395	.02579
47	18.5115	.52411	.02686
48	18.0902	.53438	.02799
49	17.6645	.54476	.02919
50	17.2348	.55524	.03045
51	16.8017	.56582	.03178
52	16.3642	.57648	.03320
53	15.9235	.58721	.03470
54	15.4801	.59804	.03629
55	15.0339	0.60892	0.03798
56	14.5856	.61987	.03977
57	14.1357	.63085	.04168
58	13.6843	.64185	.04371
59	13.2321	.65287	.04587
60	12.7800	.66390	.04818
61	12.3279	.67493	.05064
62	11.8768	.68595	.05327
63	11.4263	.69692	.05608
64	10.9772	.70785	.05910
65	10.5308	.71876	.06233
66	10.0867	.72956	.06580
67	9.6472	.74030	.06953
68	9.2129	.75094	.07353
69	8.7843	.76136	.07781
70	8.3632	.77159	.08241
71	7.9515	.78167	.08732
72	7.5496	.79148	.09258
73	7.1584	.80101	.09818
74	6.7791	.81027	.10416
75	6.4126	.81920	.11051
76	6.0592	.82783	.11727
77	5.7192	.83611	.12444
78	5.3928	.84408	.13204
79	5.0798	.85171	.14009
80	4.7802	.85901	.14861
81	4.4942	.86599	.15762
82	4.2217	.87265	.16712
83	3.9630	.87896	.17710
84	3.7181	.88492	.18756
85	3.4873	.89057	.19847
86	3.2704	.89586	.20978
87	3.0668	.90081	.22150
88	2.8760	.90546	.23361
89	2.6979	.90981	.24603
90	2.5308	.91388	.25883
91	2.3751	.91769	.27190
92	2.2298	.92122	.28522
93	2.0939	.92454	.29883
94	1.9685	.92757	.31248
95	1.8491	.93051	.32659
96	1.7398	.93318	.34060
97	1.6371	.93568	.35481
98	1.5411	.93803	.36915
99	1.4520	.94018	.38344
100	1.3681	.94220	.39787

TABLE 33.—UNITED STATES WHITE FEMALES: 1939–1941—IMMEDIATE WHOLE LIFE ANNUITY, SINGLE AND ANNUAL NET PREMIUMS AT 3 PERCENT INTEREST

[Present value at each age of a life annuity of one per annum, first payment to be made at the end of 1 year; present value of a whole life assurance of one unit, and the annual payment of an equivalent whole life annuity-due]

AGE	IMMEDIATE LIFE ANNUITY	SINGLE PREMIUM	ANNUAL PREMIUM	AGE	IMMEDIATE LIFE ANNUITY	SINGLE PREMIUM	ANNUAL PREMIUM
x	a_x	A_x	P_x	x	a_x	A_x	P_x
0	27.1814	0.17918	0.00636	55	14.2474	0.55593	0.03646
1	28.0993	.15244	.00524	56	13.8414	.56771	.03825
2	28.0677	.15336	.00528	57	13.4338	.57961	.04016
3	27.9736	.15611	.00539	58	13.0236	.59157	.04218
4	27.8594	.15944	.00552	59	12.6106	.60356	.04434
5	27.7320	.16315	.00568	60	12.1965	.61561	.04665
6	27.5956	.16712	.00584	61	11.7816	.62770	.04911
7	27.4506	.17134	.00602	62	11.3667	.63984	.05174
8	27.2980	.17579	.00621	63	10.9501	.65192	.05455
9	27.1390	.18042	.00641	64	10.5349	.66406	.05757
10	26.9731	.18525	.00662	65	10.1193	.67611	.06081
11	26.8015	.19024	.00684	66	9.7062	.68817	.06428
12	26.6250	.19539	.00707	67	9.2951	.70013	.06801
13	26.4434	.20067	.00731	68	8.8878	.71201	.07201
14	26.2580	.20608	.00756	69	8.4853	.72372	.07630
15	26.0690	.21158	.00782	70	8.0891	.73526	.08089
16	25.8769	.21717	.00808	71	7.7001	.74660	.08582
17	25.6821	.22286	.00835	72	7.3195	.75768	.09107
18	25.4834	.22863	.00863	73	6.9484	.76849	.09668
19	25.2811	.23453	.00892	74	6.5878	.77900	.10266
20	25.0749	.24053	.00922	75	6.2385	.78917	.10902
21	24.8647	.24665	.00954	76	5.9010	.79899	.11578
22	24.6503	.25290	.00986	77	5.5757	.80847	.12295
23	24.4308	.25928	.01020	78	5.2629	.81759	.13055
24	24.2067	.26582	.01055	79	4.9622	.82634	.13860
25	23.9770	.27252	.01091	80	4.6741	.83475	.14712
26	23.7410	.27938	.01129	81	4.3983	.84276	.15612
27	23.4996	.28643	.01169	82	4.1353	.85046	.16561
28	23.2518	.29364	.01211	83	3.8850	.85771	.17558
29	22.9981	.30103	.01254	84	3.6480	.86463	.18602
30	22.7376	.30860	.01300	85	3.4240	.87114	.19691
31	22.4720	.31635	.01348	86	3.2134	.87728	.20821
32	22.1992	.32428	.01398	87	3.0154	.88305	.21992
33	21.9204	.33242	.01450	88	2.8297	.88847	.23199
34	21.6348	.34073	.01505	89	2.6561	.89351	.24439
35	21.3429	.34923	.01563	90	2.4931	.89827	.25715
36	21.0448	.35793	.01624	91	2.3408	.90268	.27020
37	20.7396	.36681	.01687	92	2.1989	.90682	.28348
38	20.4280	.37589	.01754	93	2.0658	.91071	.29705
39	20.1093	.38516	.01825	94	1.9430	.91428	.31066
40	19.7849	.39462	.01899	95	1.8261	.91769	.32473
41	19.4536	.40427	.01977	96	1.7188	.92081	.33869
42	19.1161	.41409	.02059	97	1.6179	.92374	.35285
43	18.7725	.42410	.02145	98	1.5236	.92650	.36714
44	18.4231	.43427	.02236	99	1.4361	.92906	.38137
45	18.0689	.44461	.02332	100	1.3537	.93144	.39574
46	17.7087	.45509	.02433				
47	17.3435	.46574	.02539				
48	16.9728	.47653	.02651				
49	16.5969	.48746	.02770				
50	16.2165	.49855	.02896				
51	15.8308	.50976	.03029				
52	15.4415	.52114	.03170				
53	15.0471	.53261	.03319				
54	14.6492	.54422	.03478				

TABLE 34.—UNITED STATES WHITE FEMALES: 1939–1941—IMMEDIATE WHOLE LIFE ANNUITY, SINGLE AND ANNUAL NET PREMIUMS AT 3½ PERCENT INTEREST

[Present value at each age of a life annuity of one per annum, first payment to be made at the end of 1 year; present value of a whole life assurance of one unit, and the annual payment of an equivalent whole life annuity-due]

AGE	IMMEDIATE LIFE ANNUITY	SINGLE PREMIUM	ANNUAL PREMIUM	AGE	IMMEDIATE LIFE ANNUITY	SINGLE PREMIUM	ANNUAL PREMIUM
x	a_x	A_x	P_x	x	a_x	A_x	P_x
0	24.2738	0.14533	0.00575	55	13.5240	0.50883	0.03503
1	25.1129	.11696	.00448	56	13.1576	.52127	.03682
2	25.1042	.11725	.00449	57	12.7870	.53381	.03872
3	25.0404	.11941	.00459	58	12.4119	.54641	.04074
4	24.9584	.12217	.00471	59	12.0355	.55919	.04290
5	24.8650	.12533	.00485	60	11.6559	.57202	.04520
6	24.7640	.12875	.00500	61	11.2743	.58493	.04765
7	24.6554	.13243	.00516	62	10.8909	.59789	.05028
8	24.5399	.13634	.00534	63	10.5062	.61090	.05309
9	24.4183	.14044	.00553	64	10.1205	.62395	.05611
10	24.2913	.14474	.00572	65	9.7344	.63701	.05934
11	24.1590	.14922	.00593	66	9.3486	.65005	.06282
12	24.0220	.15385	.00615	67	8.9641	.66306	.06654
13	23.8806	.15862	.00638	68	8.5819	.67598	.07055
14	23.7357	.16353	.00661	69	8.2032	.68878	.07484
15	23.5879	.16853	.00685	70	7.8296	.70141	.07944
16	23.4368	.17363	.00711	71	7.4618	.71386	.08436
17	23.2830	.17883	.00736	72	7.1013	.72606	.08962
18	23.1265	.18413	.00763	73	6.7488	.73796	.09524
19	22.9660	.18955	.00791	74	6.4056	.74957	.10122
20	22.8023	.19509	.00820	75	6.0725	.76084	.10758
21	22.6345	.20077	.00849	76	5.7500	.77174	.11433
22	22.4629	.20657	.00880	77	5.4385	.78227	.12150
23	22.2872	.21252	.00913	78	5.1383	.79241	.12909
24	22.1061	.21863	.00946	79	4.8494	.80219	.13714
25	21.9204	.22492	.00981	80	4.5719	.81157	.14565
26	21.7287	.23140	.01018	81	4.3063	.82060	.15465
27	21.5318	.23807	.01057	82	4.0519	.82916	.16413
28	21.3284	.24492	.01097	83	3.8100	.83738	.17409
29	21.1199	.25198	.01139	84	3.5802	.84512	.18452
30	20.9051	.25924	.01183	85	3.3628	.85248	.19539
31	20.6851	.26669	.01230	86	3.1582	.85938	.20667
32	20.4586	.27436	.01279	87	2.9656	.86589	.21835
33	20.2252	.28224	.01330	88	2.7847	.87200	.23040
34	19.9859	.29033	.01383	89	2.6154	.87772	.24277
35	19.7405	.29864	.01440	90	2.4563	.88311	.25551
36	19.4878	.30716	.01499	91	2.3076	.88816	.26852
37	19.2290	.31591	.01562	92	2.1688	.89284	.28176
38	18.9640	.32489	.01627	93	2.0385	.89724	.29529
39	18.6917	.33409	.01697	94	1.9182	.90131	.30885
40	18.4136	.34351	.01769	95	1.8035	.90518	.32288
41	18.1279	.35315	.01846	96	1.6982	.90875	.33679
42	17.8371	.36302	.01927	97	1.5993	.91210	.35091
43	17.5384	.37308	.02012	98	1.5066	.91524	.36514
44	17.2347	.38336	.02102	99	1.4205	.91814	.37931
45	16.9251	.39384	.02197	100	1.3395	.92089	.39363
46	16.6098	.40451	.02297				
47	16.2881	.41536	.02403				
48	15.9610	.42641	.02514				
49	15.6290	.43767	.02632				
50	15.2913	.44911	.02757				
51	14.9473	.46069	.02889				
52	14.5995	.47251	.03029				
53	14.2451	.48444	.03178				
54	13.8867	.49656	.03336				

Table 35.—United States White Females: 1939–1941—Immediate Whole Life Annuity, Single and Annual Net Premiums at 4 Percent Interest

[Present value at each age of a life annuity of one per annum, first payment to be made at the end of 1 year; present value of a whole life assurance of one unit, and the annual payment of an equivalent whole life annuity-due]

Age	Immediate life annuity	Single premium	Annual premium	Age	Immediate life annuity	Single premium	Annual premium
x	a_x	A_x	P_x	x	a_x	A_x	P_x
0	21.8575	0.12086	0.00529	55	12.8593	0.46695	0.03369
1	22.6270	.09127	.00386	56	12.5262	.47977	.03547
2	22.6340	.09100	.00385	57	12.1887	.49275	.03736
3	22.5913	.09264	.00393	58	11.8472	.50588	.03938
4	22.5329	.09488	.00403	59	11.5017	.51916	.04153
5	22.4643	.09753	.00416	60	11.1532	.53257	.04382
6	22.3890	.10043	.00429	61	10.8017	.54610	.04627
7	22.3068	.10359	.00444	62	10.4475	.55971	.04889
8	22.2185	.10698	.00461	63	10.0910	.57342	.05170
9	22.1254	.11057	.00478	64	9.7326	.58721	.05471
10	22.0269	.11436	.00497	65	9.3728	.60105	.05794
11	21.9237	.11832	.00516	66	9.0124	.61491	.06142
12	21.8165	.12244	.00537	67	8.6520	.62876	.06514
13	21.7057	.12670	.00558	68	8.2030	.64257	.06915
14	21.5916	.13110	.00580	69	7.9366	.65629	.07344
15	21.4748	.13559	.00603	70	7.5839	.66986	.07804
16	21.3551	.14019	.00627	71	7.2356	.68324	.08296
17	21.2329	.14488	.00652	72	6.8939	.69641	.08822
18	21.1085	.14968	.00677	73	6.5588	.70928	.09383
19	20.9803	.15459	.00703	74	6.2318	.72185	.09982
20	20.8495	.15964	.00731	75	5.9140	.73409	.10617
21	20.7147	.16481	.00759	76	5.6055	.74594	.11293
22	20.5770	.17013	.00788	77	5.3071	.75742	.12009
23	20.4347	.17559	.00819	78	5.0190	.76852	.12768
24	20.2883	.18122	.00851	79	4.7410	.77917	.13572
25	20.1371	.18705	.00885	80	4.4739	.78948	.14423
26	19.9806	.19307	.00920	81	4.2172	.79031	.15321
27	19.8186	.19928	.00957	82	3.9716	.80878	.16268
28	19.6520	.20571	.00996	83	3.7373	.81779	.17263
29	19.4794	.21234	.01037	84	3.5146	.82636	.18304
30	19.3013	.21918	.01080	85	3.3036	.83448	.19390
31	19.1176	.22624	.01125	86	3.1047	.84212	.20516
32	18.9284	.23353	.01172	87	2.9173	.84934	.21682
33	18.7328	.24105	.01222	88	2.7411	.85613	.22884
34	18.5310	.24880	.01274	89	2.5760	.86247	.24118
35	18.3239	.25679	.01329	90	2.4207	.86846	.25389
36	18.1093	.26501	.01387	91	2.2752	.87401	.26686
37	17.8891	.27349	.01448	92	2.1394	.87926	.28007
38	17.6619	.28221	.01512	93	2.0119	.88417	.29356
39	17.4292	.29120	.01580	94	1.8940	.88869	.30708
40	17.1895	.30043	.01652	95	1.7815	.89301	.32106
41	16.9425	.30990	.01727	96	1.6782	.89699	.33492
42	16.6898	.31962	.01807	97	1.5810	.90073	.34899
43	16.4312	.32960	.01891	98	1.4899	.90425	.36317
44	16.1655	.33980	.01980	99	1.4053	.90750	.37729
45	15.8937	.35023	.02073	100	1.3255	.91057	.39155
46	15.6168	.36000	.02172				
47	15.3338	.37181	.02276				
48	15.0436	.38291	.02387				
49	14.7487	.39426	.02503				
50	14.4482	.40585	.02627				
51	14.1410	.41763	.02758				
52	13.8287	.42966	.02898				
53	13.5105	.44188	.03045				
54	13.1877	.45433	.03202				

Table 36.—United States Total Whites: 1939–1941, Makeham Constants

Constant	Value	Common logarithm
c	1.0924931	+0.03841870
g	.9989073	−.000474834
s	.9989391	−.000461004
k	95,664.45	+4.98075057
A	.0010615	−2.97408
B	.00009672	−4.01448

$l_x = ks^xg^{c^x}$ $\mu_x = A + Bc^x$

TABLE 37.—UNITED STATES TOTAL WHITES: 1939–1941, MAKEHAMIZED—TABLE OF UNIFORM SENIORITY

[Showing the addition to be made to the age of the younger of two lives in order to obtain the equivalent equal age: law of uniform seniority applicable only when both lives are age 17 or older]

DIFFERENCE OF AGE	Addition to younger age	DIFFERENCE OF AGE	Addition to younger age	DIFFERENCE OF AGE	Addition to younger age	DIFFERENCE OF AGE	Addition to younger age	DIFFERENCE OF AGE	Addition to younger age	DIFFERENCE OF AGE	Addition to younger age
1	0.511	16	10.622	31	23.870	46	38.356	56	48.244	66	58.197
2	1.044	17	11.433	32	24.812	47	39.340	57	49.237	67	59.195
3	1.599	18	12.258	33	25.759	48	40.325	58	50.231	68	60.192
4	2.176	19	13.095	34	26.710	49	41.312	59	51.225	69	61.190
5	2.774	20	13.944	35	27.664	50	42.299	60	52.220	70	62.187
6	3.393	21	14.803	36	28.623	51	43.288	61	53.216	71	63.186
7	4.033	22	15.674	37	29.585	52	44.277	62	54.211	72	64.184
8	4.693	23	16.553	38	30.550	53	45.268	63	55.207	73	65.182
9	5.373	24	17.442	39	31.518	54	46.259	64	56.204	74	66.181
10	6.071	25	18.339	40	32.488	55	47.251	65	57.200	75	67.179
11	6.788	26	19.245	41	33.461						
12	7.523	27	20.157	42	34.436						
13	8.274	28	21.076	43	35.414						
14	9.041	29	22.002	44	36.393						
15	9.824	30	22.933	45	37.374						

TABLE 38.—UNITED STATES TOTAL WHITES: 1939–1941, MAKEHAMIZED—ELEMENTARY VALUES

AGE	OF 1,000,000 BORN ALIVE		PROBABILITY OF SURVIVING 1 YEAR AT EACH AGE	PROBABILITY OF DYING IN EACH YEAR OF AGE	FORCE OF MORTALITY AT EACH AGE	AGE	OF 1,000,000 BORN ALIVE		PROBABILITY OF SURVIVING 1 YEAR AT EACH AGE	PROBABILITY OF DYING IN EACH YEAR OF AGE	FORCE OF MORTALITY AT EACH AGE
	Number surviving to each age	Number dying in each year of age					Number surviving to each age	Number dying in each year of age			
x	l_x	d_x	p_x	q_x	μ_x	x	l_x	d_x	p_x	q_x	μ_x
0	1,000,000	43,148	0.95685	0.04315	9.20886	55	783,063	11,026	0.98592	0.01408	0.01361
1	956,852	4,402	.99540	.00460	.00769	56	772,037	11,793	.98472	.01528	.01477
2	952,450	2,312	.99757	.00243	.00282	57	760,244	12,605	.98342	.01658	.01604
3	950,138	1,669	.99824	.00176	.00194	58	747,639	13,460	.98200	.01800	.01742
4	948,469	1,340	.99859	.00141	.00154	59	734,179	14,358	.98044	.01956	.01894
5	947,129	1,176	.99876	.00124	.00131	60	719,821	15,296	.97875	.02125	.02059
6	945,953	1,044	.99890	.00110	.00117	61	704,525	16,271	.97691	.02309	.02239
7	944,909	942	.99900	.00100	.00105	62	688,254	17,282	.97489	.02511	.02437
8	943,967	869	.99908	.00092	.00095	63	670,972	18,320	.97270	.02730	.02652
9	943,098	822	.99913	.00087	.00089	64	652,652	19,382	.97030	.02970	.02888
10	942,276	803	.99915	.00085	.00085	65	633,270	20,456	.96770	.03230	.03145
11	941,473	878	.99907	.00093	.00088	66	612,814	21,536	.96486	.03514	.03426
12	940,595	958	.99898	.00102	.00098	67	591,278	22,608	.96176	.03824	.03733
13	939,637	1,043	.99889	.00111	.00106	68	568,670	23,659	.95840	.04160	.04069
14	938,594	1,130	.99880	.00120	.00116	69	545,011	24,672	.95473	.04527	.04435
15	937,464	1,222	.99870	.00130	.00125	70	520,339	25,631	.95074	.04926	.04836
16	936,242	1,316	.99859	.00141	.00135	71	494,708	26,515	.94640	.05360	.05273
17	934,926	1,417	.99848	.00152	.00150	72	468,193	27,302	.94169	.05831	.05751
18	933,509	1,454	.99844	.00156	.00154	73	440,891	27,970	.93656	.06344	.06273
19	932,055	1,494	.99840	.00160	.00158	74	412,921	28,496	.93099	.06901	.06844
20	930,561	1,538	.99835	.00165	.00163	75	384,425	28,854	.92494	.07506	.07467
21	929,023	1,587	.99829	.00171	.00168	76	355,571	29,020	.91838	.08162	.08148
22	927,436	1,640	.99823	.00177	.00174	77	326,551	28,975	.91127	.08873	.08891
23	925,796	1,697	.99817	.00183	.00180	78	297,576	28,699	.90356	.09644	.09704
24	924,099	1,761	.99809	.00191	.00187	79	268,877	28,175	.89521	.10479	.10592
25	922,338	1,828	.99802	.00198	.00194	80	240,702	27,397	.88618	.11382	.11562
26	920,510	1,904	.99793	.00207	.00203	81	213,305	26,361	.87642	.12358	.12621
27	918,606	1,985	.99784	.00216	.00212	82	186,944	25,074	.86587	.13413	.13779
28	916,621	2,075	.99774	.00226	.00221	83	161,870	23,552	.85450	.14550	.15043
29	914,546	2,171	.99763	.00237	.00232	84	138,318	21,821	.84224	.15776	.16425
30	912,375	2,276	.99751	.00249	.00244	85	116,497	19,914.0	.82906	.17094	.17934
31	910,099	2,392	.99737	.00263	.00256	86	96,583.0	17,878.8	.81489	.18511	.19583
32	907,707	2,517	.99723	.00277	.00270	87	78,704.2	15,766.0	.79968	.20032	.21385
33	905,190	2,653	.99707	.00293	.00285	88	62,938.2	13,632.9	.78339	.21661	.23353
34	902,537	2,801	.99690	.00310	.00302	89	49,305.3	11,538.5	.76598	.23402	.25503
35	899,736	2,962	.99671	.00329	.00320	90	37,766.8	9,540.2	.74739	.25261	.27852
36	896,774	3,137	.99650	.00350	.00340	91	28,226.6	7,688.7	.72761	.27239	.30418
37	893,637	3,328	.99628	.00372	.00361	92	20,537.9	6,026.1	.70659	.29341	.33222
38	890,309	3,534	.99603	.00397	.00385	93	14,511.8	4,581.15	.68432	.31568	.36285
39	886,775	3,758	.99576	.00424	.00411	94	9,930.65	3,368.60	.66079	.33921	.39631
40	883,017	4,002	.99547	.00453	.00439	95	6,562.05	2,388.54	.63601	.36399	.43287
41	879,015	4,264	.99515	.00485	.00470	96	4,173.51	1,627.70	.60999	.39001	.47281
42	874,751	4,551	.99480	.00520	.00503	97	2,545.81	1,062.13	.58279	.41721	.51644
43	870,200	4,859	.99442	.00558	.00540	98	1,483.68	661.048	.55445	.44555	.56411
44	865,341	5,193	.99400	.00600	.00580	99	822.632	390.691	.52507	.47493	.61619
45	860,148	5,554	.99354	.00646	.00624	100	431.941	218.240	.49475	.50525	.67309
46	854,594	5,943	.99305	.00695	.00672	101	213.701	114.6256	.46362	.53638	.73525
47	848,651	6,363	.99250	.00750	.00724	102	99.0754	56.2906	.43184	.56816	.80315
48	842,288	6,815	.99191	.00809	.00782	103	42.7848	25.6876	.39961	.60039	.87734
49	835,473	7,302	.99126	.00874	.00844	104	17.0972	10.82013	.36714	.63286	.95839
50	828,171	7,823	.99055	.00945	.00912	105	6.27707	6.27707	.00000	1.00000	1.04694
51	820,348	8,382	.98978	.01022	.00986						
52	811,966	8,981	.98894	.01106	.01068						
53	802,985	9,620	.98802	.01198	.01157						
54	793,365	10,302	.98701	.01299	.01255						

TABLE 39.—UNITED STATES TOTAL WHITES: 1939–1941, MAKEHAMIZED—IMMEDIATE LIFE ANNUITIES AT 2 PERCENT INTEREST

[Single and joint lives—Equal ages]

AGE	ONE LIFE	TWO LIVES	THREE LIVES	FOUR LIVES	AGE	ONE LIFE	TWO LIVES	THREE LIVES	FOUR LIVES
x	a_x	a_{xx}	a_{xxx}	a_{xxxx}	x	a_x	a_{xx}	a_{xxx}	a_{xxxx}
0	34.2889	30.0985	26.9733	24.4185	55	15.0219	11.2216	9.1518	7.7877
1	35.5519	32.5317	30.4051	28.7126	56	14.5412	10.7753	8.7405	7.4070
2	35.4305	32.4898	30.4452	28.8320	57	14.0621	10.3345	8.3367	7.0350
3	35.2271	32.3011	30.2813	28.6959	58	13.5851	9.8996	7.9408	6.6720
4	34.9948	32.0631	30.0503	28.4764	59	13.1109	9.4712	7.5533	6.3184
5	34.7452	31.7970	29.7816	28.2107	60	12.6398	9.0499	7.1746	5.9745
6	34.4842	31.5136	29.4907	27.9182	61	12.1726	8.6361	6.8052	5.6408
7	34.2127	31.2150	29.1803	27.6027	62	11.7096	8.2302	6.4454	5.3173
8	33.9318	30.9028	28.8531	27.2673	63	11.2514	7.8328	6.0955	5.0044
9	33.6423	30.5790	28.5116	26.9153	64	10.7986	7.4443	5.7558	4.7022
10	33.3451	30.2450	28.1580	23.5495	65	10.3516	7.0651	5.4266	4.4109
11	33.0410	29.9026	27.7947	26.1730	66	9.9111	6.6955	5.1081	4.1306
12	32.7333	29.5576	27.4300	25.7963	67	9.4775	6.3359	4.8006	3.8613
13	32.4220	29.2103	27.0643	25.4197	68	9.0515	5.9868	4.5042	3.6033
14	32.1072	28.8607	26.6977	25.0435	69	8.6332	5.6482	4.2189	3.3563
15	31.7888	28.5089	26.3303	24.6677	70	8.2234	5.3204	3.9449	3.1204
16	31.4669	28.1551	25.9622	24.2927	71	7.8225	5.0037	3.6822	2.8954
17	31.1414	27.7991	25.5934	23.9184	72	7.4308	4.6982	3.4307	2.6813
18	30.8125	27.4412	25.2243	23.5452	73	7.0488	4.4041	3.1905	2.4780
19	30.4778	27.0775	24.8494	23.1664	74	6.6768	4.1214	2.9615	2.2852
20	30.1372	26.7078	24.4687	22.7818	75	6.3151	3.8501	2.7434	2.1027
21	29.7908	26.3322	24.0822	22.3917	76	5.9641	3.5903	2.5363	1.9304
22	29.4386	25.9508	23.6902	21.9962	77	5.6240	3.3420	2.3399	1.7678
23	29.0806	25.5637	23.2926	21.5956	78	5.2951	3.1050	2.1539	1.6149
24	28.7167	25.1708	22.8896	21.1897	79	4.9775	2.8792	1.9783	1.4713
25	28.3470	24.7724	22.4814	20.7791	80	4.6713	2.6646	1.8126	1.3367
26	27.9713	24.3683	22.0679	20.3635	81	4.3767	2.4609	1.6567	1.2107
27	27.5899	23.9588	21.6495	19.9435	82	4.0937	2.2679	1.5102	1.0931
28	27.2026	23.5439	21.2263	19.5192	83	3.8224	2.0854	1.3728	.9836
29	26.8096	23.1239	20.7985	19.0909	84	3.5628	1.9132	1.2443	.8818
30	26.4109	22.6988	20.3663	18.6587	85	3.3147	1.7510	1.1243	.7875
31	26.0065	22.2687	19.9299	18.2230	86	3.0781	1.5984	1.0125	.7002
32	25.5965	21.8339	19.4895	17.7841	87	2.8529	1.4552	.9085	.6197
33	25.1810	21.3946	19.0456	17.3424	88	2.6389	1.3212	.8120	.5457
34	24.7602	20.9510	18.5984	16.8982	89	2.4359	1.1958	.7228	.4779
35	24.3340	20.5033	18.1481	16.4518	90	2.2437	1.0789	.6404	.4159
36	23.9027	20.0518	17.6951	16.0036	91	2.0621	.9701	.5646	.3596
37	23.4663	19.5966	17.2397	15.5541	92	1.8908	.8690	.4951	.3086
38	23.0251	19.1383	16.7824	15.1037	93	1.7294	.7753	.4316	.2626
39	22.5792	18.6769	16.3236	14.6529	94	1.5778	.6888	.3738	.2215
40	22.1288	18.2130	15.8635	14.2020	95	1.4355	.6090	.3214	.1849
41	21.6741	17.7468	15.4028	13.7516	96	1.3022	.5358	.2741	.1526
42	21.2154	17.2786	14.9417	13.3021	97	1.1775	.4687	.2317	.1244
43	20.7529	16.8090	14.4809	12.8543	98	1.0609	.4074	.1939	.1000
44	20.2868	16.3383	14.0208	12.4083	99	.9516	.3518	.1606	.0791
45	19.8174	15.8669	13.5618	11.9649	100	.8486	.3014	.1313	.0615
46	19.3452	15.3953	13.1045	11.5246	101	.7495	.2561	.1059	.0469
47	18.8702	14.9239	12.6493	11.0878	102	.6489	.2152	.0841	.0350
48	18.3930	14.4532	12.1969	10.6552	103	.5328	.1772	.0656	.0254
49	17.9139	13.9837	11.7478	10.2273	104	.3599	.1321	.0485	.0178
50	17.4333	13.5160	11.3025	9.8047					
51	16.9516	13.0506	10.8616	9.3877					
52	16.4691	12.5878	10.4254	8.9770					
53	15.9864	12.1284	9.9948	8.5732					
54	15.5038	11.6728	9.5700	8.1765					

TABLE 40.—UNITED STATES TOTAL WHITES: 1939–1941, MAKEHAMIZED—IMMEDIATE LIFE ANNUITIES AT 2½ PERCENT INTEREST

[Single and joint lives—Equal ages]

AGE	ONE LIFE	TWO LIVES	THREE LIVES	FOUR LIVES	AGE	ONE LIFE	TWO LIVES	THREE LIVES	FOUR LIVES
x	a_x	a_{xx}	a_{xxx}	a_{xxxx}	x	a_x	a_{xx}	a_{xxx}	a_{xxxx}
0	30.1153	26.7270	24.1211	21.9525	55	14.2384	10.7574	8.8279	7.5434
1	31.2602	28.9216	27.2220	25.8429	56	13.8028	10.3435	8.4418	7.1832
2	31.1897	28.9193	27.2912	25.9820	57	13.3673	9.9336	8.0618	6.8304
3	31.0473	28.7867	27.1781	25.8918	58	12.9325	9.5281	7.6884	6.4853
4	30.8795	28.6103	27.0049	25.7264	59	12.4988	9.1277	7.3220	6.1485
5	30.6962	28.4086	26.7977	25.5191	60	12.0668	8.7329	6.9632	5.8203
6	30.5028	28.1913	26.5702	25.2874	61	11.6371	8.3441	6.6123	5.5011
7	30.2999	27.9599	26.3248	25.0343	62	11.2100	7.9619	6.2698	5.1910
8	30.0884	27.7161	26.0638	24.7627	63	10.7862	7.5868	5.9360	4.8905
9	29.8690	27.4614	25.7894	24.4755	64	10.3662	7.2191	5.6113	4.5997
10	29.6424	27.1971	25.5033	24.1750	65	9.9505	6.8595	5.2960	4.3189
11	29.4094	26.9246	25.2078	23.8640	66	9.5397	6.5082	4.9903	4.0483
12	29.1728	26.6492	24.9105	23.5521	67	9.1344	6.1657	4.6946	3.7879
13	28.9326	26.3712	24.6114	23.2395	68	8.7350	5.8323	4.4090	3.5378
14	28.6888	26.0906	24.3109	22.9265	69	8.3420	5.5084	4.1337	3.2981
15	28.4415	25.8074	24.0088	22.6132	70	7.9560	5.1942	3.8687	3.0687
16	28.1906	25.5217	23.7056	22.2998	71	7.5774	4.8901	3.6143	2.8498
17	27.9360	25.2334	23.4010	21.9862	72	7.2067	4.5961	3.3704	2.6411
18	27.6779	24.9428	23.0954	21.6731	73	6.8443	4.3126	3.1370	2.4425
19	27.4141	24.6462	22.7837	21.3538	74	6.4906	4.0395	2.9141	2.2540
20	27.1446	24.3436	22.4660	21.0286	75	6.1460	3.7771	2.7016	2.0754
21	26.8692	24.0348	22.1422	20.6974	76	5.8109	3.5253	2.4995	1.9065
22	26.5881	23.7201	21.8124	20.3604	77	5.4854	3.2842	2.3075	1.7471
23	26.3011	23.3993	21.4768	20.0177	78	5.1700	3.0538	2.1256	1.5968
24	26.0081	23.0724	21.1352	19.6692	79	4.8649	2.8340	1.9535	1.4556
25	25.7092	22.7397	20.7879	19.3154	80	4.5702	2.6247	1.7910	1.3231
26	25.4043	22.4008	20.4348	18.9560	81	4.2862	2.4258	1.6379	1.1990
27	25.0934	22.0561	20.0762	18.5915	82	4.0128	2.2372	1.4939	1.0831
28	24.7764	21.7055	19.7121	18.2219	83	3.7503	2.0585	1.3587	.9750
29	24.4534	21.3492	19.3427	17.8475	84	3.4986	1.8897	1.2321	.8744
30	24.1244	20.9872	18.9681	17.4685	85	3.2578	1.7305	1.1138	.7812
31	23.7893	20.6196	18.5886	17.0849	86	3.0277	1.5806	1.0034	.6948
32	23.4483	20.2466	18.2043	16.6974	87	2.8084	1.4398	.9007	.6152
33	23.1014	19.8684	17.8155	16.3060	88	2.5097	1.3078	.8054	.5418
34	22.7485	19.4850	17.4224	15.9110	89	2.4014	1.1843	.7172	.4746
35	22.3898	19.0967	17.0253	15.5128	90	2.2135	1.0690	.6356	.4132
36	22.0254	18.7036	16.6244	15.1118	91	2.0357	.9616	.5606	.3573
37	21.6553	18.3061	16.2201	14.7082	92	1.8677	.8617	.4917	.3067
38	21.2796	17.9043	15.8127	14.3026	93	1.7094	.7692	.4288	.2611
39	20.8985	17.4984	15.4026	13.8952	94	1.5604	.6835	.3714	.2202
40	20.5121	17.0888	14.9901	13.4866	95	1.4204	.6046	.3194	.1839
41	20.1207	16.6760	14.5757	13.0773	96	1.2892	.5320	.2724	.1518
42	19.7242	16.2599	14.1596	12.6675	97	1.1663	.4655	.2304	.1237
43	19.3231	15.8412	13.7425	12.2579	98	1.0513	.4048	.1928	.0995
44	18.9174	15.4201	13.3247	11.8490	99	.9434	.3496	.1597	.0787
45	18.5073	14.9970	12.9066	11.4412	100	.8417	.2996	.1306	.0612
46	18.0933	14.5724	12.4889	11.0351	101	.7[illegible]38	.2546	.1054	.0466
47	17.6756	14.1466	12.0720	10.6311	102	.6444	.2140	.0837	.0348
48	17.2543	13.7202	11.6563	10.2299	103	.5295	.1763	.0653	.0253
49	16.8299	13.2935	11.2425	9.8320	104	.3582	.1315	.0483	.0177
50	16.4027	12.8672	10.8311	9.4379					
51	15.9731	12.4416	10.4225	9.0482					
52	15.5415	12.0173	10.0173	8.6634					
53	15.1082	11.5949	9.6162	8.2839					
54	14.6737	11.1747	9.2195	7.9104					

TABLE 41.—UNITED STATES TOTAL WHITES: 1939–1941, MAKEHAMIZED—IMMEDIATE LIFE ANNUITIES AT 3 PERCENT INTEREST

[Single and joint lives—Equal ages]

AGE	ONE LIFE	TWO LIVES	THREE LIVES	FOUR LIVES	AGE	ONE LIFE	TWO LIVES	THREE LIVES	FOUR LIVES
x	a_x	a_{xx}	a_{xxx}	a_{xxxx}	x	a_x	a_{xx}	a_{xxx}	a_{xxxx}
0	26.7047	23.9239	21.7244	19.8639	55	13.5186	10.3239	8.5226	7.3117
1	27.7461	25.9141	24.5417	23.4075	56	13.1230	9.9595	8.1598	6.9706
2	27.7106	25.9388	24.6301	23.5586	57	12.7264	9.5578	7.8018	6.6357
3	27.6114	25.8471	24.5546	23.5023	58	12.3292	9.1793	7.4492	6.3075
4	27.4898	25.7163	24.4250	23.3783	59	11.9319	8.8045	7.1025	5.9864
5	27.3545	25.5628	24.2647	23.2162	60	11.5350	8.4341	6.7622	5.6729
6	27.2102	25.3952	24.0860	23.0318	61	11.1390	8.0684	6.4286	5.3672
7	27.0575	25.2149	23.8909	22.8278	62	10.7443	7.7080	6.1022	5.0698
8	26.8970	25.0232	23.6813	22.6066	63	10.3517	7.3535	5.7836	4.7811
9	26.7294	24.8214	23.4593	22.3707	64	9.9616	7.0053	5.4729	4.5012
10	26.5553	24.6107	23.2263	22.1223	65	9.5745	6.6639	5.1707	4.2304
11	26.3753	24.3923	22.9844	21.8639	66	9.1909	6.3297	4.8772	3.9688
12	26.1919	24.1710	22.7403	21.6040	67	8.8114	6.0031	4.5926	3.7168
13	26.0052	23.9469	22.4942	21.3430	68	8.4366	5.6846	4.3173	3.4744
14	25.8151	23.7201	22.2464	21.0812	69	8.0669	5.3745	4.0515	3.2417
15	25.6216	23.4907	21.9967	20.8185	70	7.7029	5.0732	3.7952	3.0187
16	25.4247	23.2586	21.7454	20.5552	71	7.3450	4.7809	3.5487	2.8054
17	25.2243	23.0238	21.4925	20.2913	72	6.9938	4.4979	3.3120	2.6019
18	25.0205	22.7866	21.2383	20.0272	73	6.6497	4.2243	3.0851	2.4080
19	24.8113	22.5435	20.9779	19.7571	74	6.3131	3.9605	2.8681	2.2237
20	24.5966	22.2944	20.7115	19.4808	75	5.9845	3.7065	2.6610	2.0488
21	24.3765	22.0393	20.4390	19.1984	76	5.6643	3.4624	2.4637	1.8832
22	24.1507	21.7783	20.1604	18.9101	77	5.3527	3.2283	2.2760	1.7267
23	23.9193	21.5112	19.8758	18.6158	78	5.0501	3.0042	2.0980	1.5791
24	23.6822	21.2380	19.5850	18.3155	79	4.7568	2.7901	1.9293	1.4403
25	23.4392	20.9587	19.2883	18.0094	80	4.4730	2.5860	1.7699	1.3098
26	23.1903	20.6733	18.9856	17.6975	81	4.1989	2.3917	1.6195	1.1875
27	22.9355	20.3819	18.6770	17.3800	82	3.9347	2.2072	1.4779	1.0732
28	22.6747	20.0843	18.3626	17.0570	83	3.6806	2.0322	1.3449	.9665
29	22.4080	19.7808	18.0425	16.7287	84	3.4365	1.8667	1.2202	.8671
30	22.1352	19.4714	17.7167	16.3951	85	3.2026	1.7105	1.1035	.7749
31	21.8562	19.1559	17.3855	16.0565	86	2.9788	1.5632	.9946	.6895
32	21.5712	18.8347	17.0490	15.7132	87	2.7651	1.4247	.8932	.6107
33	21.2802	18.5078	16.7074	15.3654	88	2.5615	1.2948	.7989	.5380
34	20.9830	18.1753	16.3608	15.0133	89	2.3679	1.1730	.7116	.4714
35	20.6798	17.8373	16.0095	14.6571	90	2.1840	1.0593	.6309	.4105
36	20.3705	17.4939	15.6537	14.2973	91	2.0099	.9533	.5566	.3551
37	20.0553	17.1455	15.2937	13.9341	92	1.8452	.8546	.4884	.3048
38	19.7341	16.7921	14.9298	13.5679	93	1.6897	.7631	.4259	.2595
39	19.4072	16.4340	14.5623	13.1991	94	1.5433	.6784	.3690	.2189
40	19.0744	16.0714	14.1915	12.8279	95	1.4057	.6002	.3174	.1828
41	18.7362	15.7047	13.8178	12.4551	96	1.2764	.5283	.2708	.1510
42	18.3923	15.3339	13.4415	12.0807	97	1.1553	.4624	.2290	.1231
43	18.0432	14.9595	13.0631	11.7055	98	1.0418	.4021	.1918	.0989
44	17.6888	14.5819	12.6829	11.3297	99	.9354	.3474	.1588	.0783
45	17.3295	14.2012	12.3014	10.9540	100	.8349	.2978	.1299	.0609
46	16.9654	13.8180	11.9191	10.5788	101	.7381	.2531	.1048	.0464
47	16.5966	13.4325	11.5364	10.2045	102	.6399	.2128	.0833	.0346
48	16.2237	13.0453	11.1538	9.8319	103	.5263	.1753	.0649	.0252
49	15.8467	12.6568	10.7718	9.4613	104	.3564	.1309	.0480	.0176
50	15.4660	12.2674	10.3911	9.0935					
51	15.0819	11.8775	10.0119	8.7287					
52	14.6948	11.4878	9.6350	8.3676					
53	14.3049	11.0986	9.2608	8.0107					
54	13.9127	10.7105	8.8898	7.6586					

TABLE 42.—UNITED STATES TOTAL WHITES: 1939–1941, MAKEHAMIZED—IMMEDIATE LIFE ANNUITIES AT 4 PERCENT INTEREST

[Single and joint lives—Equal ages]

AGE	ONE LIFE	TWO LIVES	THREE LIVES	FOUR LIVES	AGE	ONE LIFE	TWO LIVES	THREE LIVES	FOUR LIVES
x	a_x	a_{xx}	a_{xxx}	a_{xxxx}	x	a_x	a_{xx}	a_{xxx}	a_{xxxx}
0	21.5428	19.5872	17.9647	16.5537	55	12.2451	9.5391	7.9629	6.8832
1	22.4148	21.2493	20.3264	19.5375	56	11.9168	9.2061	7.6413	6.5763
2	22.4191	21.3040	20.4339	19.6973	57	11.5857	8.8737	7.3226	6.2737
3	22.3726	21.2641	20.4068	19.6853	58	11.2523	8.5424	7.0072	5.9759
4	22.3085	21.1926	20.3353	19.6172	59	10.9169	8.2129	6.6957	5.6834
5	22.2336	21.1027	20.2386	19.5176	60	10.5800	7.8855	6.3886	5.3966
6	22.1517	21.0014	20.1268	19.3994	61	10.2421	7.5609	6.0863	5.1160
7	22.0633	20.8898	20.0013	19.2647	62	9.9037	7.2395	5.7894	4.8419
8	21.9687	20.7688	19.8637	19.1154	63	9.5651	6.9220	5.4983	4.5748
9	21.8685	20.6393	19.7154	18.9534	64	9.2269	6.6087	5.2134	4.3149
10	21.7630	20.5024	19.5577	18.7804	65	8.8897	6.3002	4.9352	4.0626
11	21.6529	20.3588	19.3921	18.5983	66	8.5539	5.9969	4.6639	3.8181
12	21.5400	20.2127	19.2243	18.4146	67	8.2201	5.6994	4.4001	3.5817
13	21.4244	20.0641	19.0545	18.2294	68	7.8887	5.4080	4.1438	3.3536
14	21.3062	19.9131	18.8828	18.0430	69	7.5604	5.1232	3.8955	3.1339
15	21.1852	19.7596	18.7092	17.8553	70	7.2357	4.8454	3.6554	2.9228
16	21.0613	19.6037	18.5339	17.6667	71	6.9150	4.5749	3.4236	2.7204
17	20.9346	19.4452	18.3567	17.4770	72	6.5989	4.3121	3.2004	2.5266
18	20.8050	19.2845	18.1781	17.2867	73	6.2878	4.0572	2.9859	2.3416
19	20.6710	19.1185	17.9938	17.0906	74	5.9822	3.8104	2.7800	2.1652
20	20.5323	18.9471	17.8038	16.8887	75	5.6827	3.5721	2.5830	1.9974
21	20.3890	18.7703	17.6081	16.6808	76	5.3896	3.3424	2.3948	1.8382
22	20.2408	18.5880	17.4066	16.4671	77	5.1033	3.1214	2.2153	1.6874
23	20.0877	18.4000	17.1992	16.2474	78	4.8242	2.9092	2.0446	1.5448
24	19.9296	18.2064	16.9859	16.0218	79	4.5527	2.7059	1.8826	1.4104
25	19.7[illegible]4	18.0071	16.7668	15.7903	80	4.2891	2.5115	1.7290	1.2839
26	19.5979	17.8018	16.5415	15.5528	81	4.0336	2.3260	1.5839	1.1652
27	19.4240	17.5906	16.3104	15.3094	82	3.7864	2.1494	1.4469	1.0539
28	19.2448	17.3736	16.0733	15.0601	83	3.5479	1.9815	1.3180	.9499
29	19.0600	17.1507	15.8302	14.8052	84	3.3181	1.8224	1.1969	.8529
30	18.8695	16.9217	15.5812	14.5445	85	3.0971	1.6717	1.0834	.7628
31	18.6734	16.6867	15.3264	14.2781	86	2.8851	1.5295	.9773	.6792
32	18.4715	16.4457	15.0657	14.0064	87	2.6822	1.3954	.8783	.6019
33	18.2638	16.1988	14.7995	13.7293	88	2.4882	1.2694	.7863	.5306
34	18.0501	15.9459	14.5275	13.4471	89	2.3033	1.1511	.7008	.4651
35	17.8306	15.6872	14.2502	13.1600	90	2.1273	1.0404	.6218	.4052
36	17.6050	15.4226	13.9675	12.8681	91	1.9601	.9370	.5488	.3507
37	17.3735	15.1523	13.6798	12.5717	92	1.8016	.8407	.4818	.3012
38	17.1360	14.8764	13.3871	12.2712	93	1.6517	.7512	.4204	.2565
39	16.8924	14.5950	13.0897	11.9667	94	1.5102	.6682	.3644	.2165
40	16.6429	14.3083	12.7878	11.6586	95	1.3770	.5916	.3136	.1808
41	16.3875	14.0165	12.4818	11.3473	96	1.2516	.5210	.2677	.1493
42	16.1260	13.7196	12.1718	11.0329	97	1.1339	.4562	.2264	.1218
43	15.8588	13.4180	11.8584	10.7162	98	1.0234	.3970	.1896	.0979
44	15.5857	13.1119	11.5416	10.3972	99	.9197	.3431	.1571	.0775
45	15.3070	12.8015	11.2220	10.0766	100	.8216	.2942	.1285	.0603
46	15.0227	12.4872	10.8999	9.7548	101	.7271	.2502	.1037	.0459
47	14.7330	12.1692	10.5757	9.4321	102	.6311	.2104	.0824	.0343
48	14.4381	11.8479	10.2499	9.1092	103	.5199	.1734	.0643	.0249
49	14.1382	11.5237	9.9229	8.7865	104	.3530	.1296	.0476	.0175
50	13.8333	11.1969	9.5952	8.4646					
51	13.5238	10.8679	9.2672	8.1437					
52	13.2100	10.5372	8.9395	7.8247					
53	12.8920	10.2051	8.6125	7.5079					
54	12.5703	9.8723	8.2868	7.1939					

PART IV

MATHEMATICAL THEORY AND USE OF THE ACTUARIAL TABLES

It is the purpose of part IV to explain and illustrate the use of the actuarial tables in part III, and to present enough of the underlying mathematical theory to enable the reader without actuarial training to grasp the general import of these tables and to understand some of their simpler applications. For the convenience of such readers, the synopsis of mathematical theory has been placed before the technical explanation of the arrangement and use of the tables.

The section dealing with the mathematical theory assumes only a knowledge of elementary algebra, and covers only the formulas for net values of the most simple types of life annuities, and net premiums for the most simple types of life assurance benefits, including some annuities and assurances involving two or more lives. No consideration is given to the important subject of policy values (reserves).

A. GENERAL MATHEMATICAL THEORY

Compound interest

If a sum P is invested at compound interest at the rate i (that is, $100i$ percent) compounded annually, the amount accumulated at the end of 1 year is $P(1+i)$. The amount at the end of the second year is $P(1+i)$ multiplied again by $(1+i)$: that is, $P(1+i)^2$. In general, the amount at the end of n years is $P(1+i)^n$.

The *present value,* on the basis of compound interest at the rate i, of a sum A due n years hence, is that amount which, if available now, would accumulate to exactly the sum A in n years by the addition of compound interest at the rate i. In other words, it is an amount P, such that $P(1+i)^n=A$. Solving for P gives:

$$P=A(1+i)^{-n}=Av^n$$

where the symbol v is used to stand for $(1+i)^{-1}$.

Pure endowment

A *pure endowment* on the life of a specified individual is an agreement to pay a stipulated sum on a designated future date, called the *maturity date,* provided the specified individual is then alive. If each of l_x individuals, all exactly at age x, purchases an n-year pure endowment of one unit, the total cost being shared equally at the time of issue, payments will be made at the end of the n years to l_{x+n} persons, and the total present value of these payments is $v^n l_{x+n}$. If ${}_nE_x$ denotes the net single premium for the pure endowment: that is, the amount which each of the l_x individuals will have to pay, then,

$$ {}_nE_x=\frac{v^n l_{x+n}}{l_x} \qquad (1)$$

Annuities

An *annuity* is a series of payments made at equal intervals and continuing during the existence of a given status. Unless otherwise specified, the payments are assumed to be equal in amount. An *annuity certain* is one in which the payments continue for a specified period of time, regardless of any other contingency. A *life annuity* is one in which each payment is contingent on the continued survival of a designated individual, called the *annuitant.* In a whole life annuity, the payments continue during the entire lifetime of the annuitant. Under a *temporary* life annuity, a maximum period of time is specified, beyond which the payments are not to continue, even though the annuitant be alive. The *value* or *present value* of an annuity is the sum of the values of all the individual payments, each discounted (or, in some cases, accumulated) at compound interest to a specified date, called the *valuation date.* In the case of a life annuity, valuation also implies the assumption that similar annuities have been issued to a large number of persons all at the same age and subject throughout the duration of all the annuities to exactly the rates of mortality of a specified life table; and further that the total fund is contributed (or shared) equally by all the annuitants alive on the valuation date. If the first payment is made exactly one payment interval after the valuation date, the annuity is called an *immediate* annuity. If the first payment is made at a later date, it is called a *deferred* annuity. If the first payment is made *on* the valuation date, it is called an *annuity-due.* If the *last* payment is made prior to the valuation date, it is called a *forborne* annuity. A concrete illustration of the forborne annuity is provided by the *tontine fund,* to which a group of individuals contribute regularly until the end of a specified period of years (or until prior death), the accumulated fund being then divided equally among the survivors on a designated date.

Temporary life annuity

Each payment of a life annuity can be regarded as a pure endowment; or, in other words, a life annuity can be regarded as the sum of a number of pure endowments. Thus, if $a_{x:\overline{n}|}$ denotes the present value of an n-year immediate temporary life annuity with payments of one unit, then

$$a_{x:\overline{n}|}={}_1E_x+{}_2E_x+{}_3E_x+\cdots+{}_nE_x$$

It follows from formula (1) that

$$a_{x:\overline{n}|}=\frac{1}{l_x}(vl_{x+1}+v^2l_{x+2}+v^3l_{x+3}+\cdots+v^nl_{x+n}) \quad (2)$$

This expression is called the *net* value of the annuity to indicate that it is based on interest and mortality only, ignoring expenses and business contingencies.

Commutation columns

The evaluation of temporary annuities by formula (2) for many different terms and ages would involve very extensive and laborious computations. Fortunately, the calculation can be very much simplified by employing the ingenious device known as *commutation columns*. Since the value of a fraction is not changed by multiplying both numerator and denominator by the same quantity, formula (2) is transformed by multiplying and dividing by v^x. This gives:

$$a_{x:\overline{n}|}=\frac{1}{v^xl_x}(v^{x+1}l_{x+1}+v^{x+2}l_{x+2}+\cdots+v^{x+n}l_{x+n})$$

Now, if the symbol D_x is used to represent v^xl_x, the equation may be written in the form:

$$a_{x:\overline{n}|}=\frac{1}{D_x}(D_{x+1}+D_{x+2}+\cdots+D_{x+n})$$

Finally, if the symbol N_x is defined by

$N_x=D_x+D_{x+1}+D_{x+2}+\cdots$ to end of life table,

it is possible to write:

$$a_{x:\overline{n}|}=\frac{N_{x+1}-N_{x+n+1}}{D_x}$$

This is, in fact, formula III of table P, page 87. Similarly, formula (1) on page 85 can be written in the form:

$${}_nE_x=\frac{D_{x+n}}{D_x}$$

It is clear that if values of D_x and N_x are tabulated for all ages, the net value of a pure endowment or temporary life annuity for any age and term can be calculated with very little effort. The functions D_x and N_x are members of the class of actuarial functions called commutation columns. Although very useful in actuarial calculations, commutation columns are mere mathematical abstractions—short cuts in computation having no real meaning in themselves.

Other types of annuities

The expression for the net value of an immediate whole life annuity of one per annum is similar to formula (2) except that the expression within the parentheses is not limited to n terms, but continues to the end of the life table. By the same process used in the case of the temporary life annuity, this expression reduces to the formula:

$$a_x=\frac{N_{x+1}}{D_x}$$

The net value of an n-year temporary life annuity of one per annum deferred m years is given by:

$${}_{m|}a_{x:\overline{n}|}=\frac{1}{l_x}(v^{m+1}l_{x+m+1}+v^{m+2}l_{x+m+2}+\cdots+v^{m+n}l_{x+m+n})$$

Expressed in terms of commutation symbols, this becomes:

$${}_{m|}a_{x:\overline{n}|}=\frac{N_{x+m+1}-N_{x+m+n+1}}{D_x}=\frac{D_{x+m}}{D_x}\,\frac{N_{x+m+1}-N_{x+m+n+1}}{D_{x+m}}={}_mE_xa_{x+m:\overline{n}|}$$

This is reasonable, since an m-year pure endowment of amount $a_{x+m:\overline{n}|}$ to an individual aged x at the time of issue, would enable the purchaser to use the proceeds at age $x+m$ to buy an n-year immediate temporary life annuity of one per annum commencing at that age. Therefore, an m-year pure endowment of amount $a_{x+m:\overline{n}|}$ can provide benefits identical with those provided by the deferred annuity represented by ${}_{m|}a_{x:\overline{n}|}$. Adaptation to the case of a deferred whole life annuity gives the analogous formula:

$${}_{m|}a_x=\frac{N_{x+m+1}}{D_x}={}_mE_x\,a_{x+m}$$

Table P provides a reference list of formulas in terms of commutation symbols for the present values of the more common types of annuities. In all the formulas in the table, it is assumed that the payments are of one unit each, and are made at intervals of 1 year. In connection with the formulas in this table, it will be noted that the value of an annuity-due (of one per annum) may be obtained by adding unity to the value of the corresponding immediate annuity in which the temporary period (if any) has been reduced by 1 year. Thus, in the case of whole life annuities,

$$\ddot{a}_x=1+a_x$$

while in the case of temporary life annuities,

$$\ddot{a}_{x:\overline{n}|}=1+a_{x:\overline{n-1}|}$$

The principles underlying the choice of symbols to represent the different annuity values are explained on pages 90 and 92.

TABLE P.—REFERENCE LIST OF FORMULAS FOR PRESENT VALUE OF SINGLE LIFE ANNUITIES

Reference number	DESCRIPTION OF ANNUITY	Age at time of first payment	Age at time of last payment (if annuitant does not die previously)	Symbol and formula[1] for value at age x
I	Immediate whole life annuity	$x+1$	None	$a_x=\frac{N_{x+1}}{D_x}$
II	Whole life annuity-due	x	None	$\ddot{a}_x=\frac{N_x}{D_x}=1+a_x$
III	Immediate temporary life annuity for term of n years	$x+1$	$x+n$	$a_{x:\overline{n}\vert}=\frac{N_{x+1}-N_{x+n+1}}{D_x}$
IV	Temporary life annuity-due for term of n years	x	$x+n-1$	$\ddot{a}_{x:\overline{n}\vert}=\frac{N_x-N_{x+n}}{D_x}=1+a_{x:\overline{n-1}\vert}$
V	Whole life annuity deferred m years	$x+m+1$	None	${}_{m\vert}a_x=\frac{N_{x+m+1}}{D_x}$
VI	Temporary life annuity for term of n years deferred m years	$x+m+1$	$x+m+n$	${}_{m\vert}a_{x:\overline{n}\vert}=\frac{N_{x+m+1}-N_{x+m+n+1}}{D_x}$
VII	Forborne life annuity for term of n years	$x-n$	$x-1$	${}_nu_{x-n}=\frac{N_{x-n}-N_x}{D_x}$

[1] On the basis of annual payments of one per annum.

Life assurances

A *whole life assurance* is an agreement to pay a specified sum upon the death of a designated individual, called the *insured*, regardless of when such death may occur. In a *term* or *temporary* assurance, the payment is made only if the death occurs within a specified period. In the case of a *deferred* assurance, payment is made only if the death occurs after the expiration of a specified period. An *endowment assurance* is an agreement to pay a specified sum either upon the death of the insured, if the death occurs within a stipulated period, or at the end of such period, if the insured is then alive. From a strictly mathematical point of view, an endowment assurance may be regarded as the combination of a term assurance and a pure endowment.

If l_x persons all exactly at age x purchase temporary life assurances of one unit for a period of n years, and if it be assumed that claims are paid on the birthday next succeeding the date of death, the payments made at the end of the first year would total d_x, and their present value would be vd_x. Similarly, the present value of the payments made at the end of the second year would be v^2d_{x+1}, and so on up to the end of the nth year, when payments would be made having a present value of v^nd_{x+n-1}. The net single premium for each assurance (denoted by $A^1_{x:\overline{n}|}$) is therefore given by:

$$A^1_{x:\overline{n}|}=\frac{1}{l_x}(vd_x+v^2d_{x+1}+v^3d_{x+2}+\cdots+v^nd_{x+n-1})$$

Upon multiplying and dividing by v_x, this becomes:

$$A^1_{x:\overline{n}|}=\frac{1}{v^xl_x}(v^{x+1}d_x+v^{x+2}d_{x+1}+\cdots+v^{x+n}d_{x+n-1})$$

y using C_x to denote $v^{x+1}d_x$, this can be written:

$$A^1_{x:\overline{n}|}=\frac{1}{D_x}(C_x+C_{x+1}+\cdots+C_{x+n-1})$$

Finally, after introducing the symbol M_x defined by:

$$M_x=C_x+C_{x+1}+C_{x+2}+\cdots \text{ to end of life table,}$$

the formula becomes:

$$A^1_{x:\overline{n}|}=\frac{M_x-M_{x+n}}{D_x}$$

By a similar process, it is easily found that, for a whole life assurance,

$$A_x=\frac{M_x}{D_x}$$

while, for a deferred life assurance,

$${}_{m|}A_x=\frac{M_{x+m}}{D_x}={}_mE_x\,A_{x+m}$$

The expression for the net annual premium for a whole life assurance (denoted by P_x) can be obtained by observing that the annual premiums constitute a whole life annuity-due. Therefore, $P_x\,\ddot{a}_x=A_x$; whence, solving for P_x,

$$P_x=\frac{A_x}{\ddot{a}_x}=\frac{M_x}{D_x}\div\frac{N_x}{D_x}=\frac{M_x}{N_x}$$

In the case of a limited payment whole life assurance, where the number of annual premiums (net premium denoted by ${}_mP_x$) is limited to a maximum of m years, the equation to be solved is ${}_mP_x\,\ddot{a}_{x:\overline{m}|}=A_x$, which gives:

$${}_mP_x=\frac{A_x}{\ddot{a}_{x:\overline{m}|}}=\frac{M_x}{N_x-N_{x+m}}$$

Formulas for net annual premiums for other types of assurance contracts are similarly obtained.

Table Q provides a reference list of formulas in terms of commutation symbols for single and annual net premiums for the more common forms of life insurance benefits. In all the formulas in the table, it is assumed that the sum insured is one unit, and is payable, in case

TABLE Q.—REFERENCE LIST OF FORMULAS FOR SINGLE AND ANNUAL NET PREMIUMS [1] FOR INSURANCE BENEFITS

Reference number	DESCRIPTION OF INSURANCE BENEFIT	Symbol and formula for single premium	Symbol and formula for annual premium [2]
I	Whole life assurance	$A_x=\frac{M_x}{D_x}$	$P_x=\frac{M_x}{N_x}$
II	m-payment [3] life assurance	None	${}_mP_x=\frac{M_x}{N_x-N_{x+m}}$
III	n-year term assurance	$A^{1}_{x:\overline{n}\vert}=\frac{M_x-M_{x+n}}{D_x}$	$P^{1}_{x:\overline{n}\vert}=\frac{M_x-M_{x+n}}{N_x-N_{x+n}}$
IV	m-payment [3] n-year term assurance	None	${}_mP^{1}_{x:\overline{n}\vert}=\frac{M_x-M_{x+n}}{N_x-N_{x+m}}$
V	n-year pure endowment	${}_nE_x$ or $A_{x:\overline{n}\vert}^{\ \ 1}=\frac{D_{x+n}}{D_x}$	$P_{x:\overline{n}\vert}^{\ \ 1}=\frac{D_{x+n}}{N_x-N_{x+n}}$
VI	n-year endowment assurance	$A_{x:\overline{n}\vert}=\frac{M_x-M_{x+n}+D_{x+n}}{D_x}$	$P_{x:\overline{n}\vert}=\frac{M_x-M_{x+n}+D_{x+n}}{N_x-N_{x+n}}$
VII	m-payment [3] n-year endowment assurance	None	${}_mP_{x:\overline{n}\vert}=\frac{M_x-M_{x+n}+D_{x+n}}{N_x-N_{x+m}}$
VIII	Whole life assurance deferred m years	${}_{m\vert}A_x=\frac{M_{x+m}}{D_x}$	Premium [4] $=\frac{M_{x+m}}{N_x}$

[1] On the basis of a sum insured of one unit payable on the contract anniversary next succeeding the date of death.
[2] Premiums assumed payable throughout the duration of the contract unless otherwise specified in column 2.
[3] This implies that payments by the insured continue until m payments have been made or until death if earlier.
[4] There is no accepted symbol for the annual premium. The formula given assumes that premium payments begin immediately.

of death, on the anniversary of the insurance contract next following the date of death. It is also assumed, in the case of annual premiums, that they are payable in advance: that is, the first premium is due at the time the contract is made; and the last premium is due, in the case of endowments, 1 year before the maturity date. The principles underlying the choice of symbols to represent the premiums for different types of assurances are explained on pages 90 and 92.

The actual practice of life insurance companies today is to pay the sum insured immediately upon receipt of proofs of death, and not to wait until the next contract anniversary. Nevertheless, it is customary to calculate net premiums for life insurance on the assumption stated in the preceding paragraph, and to include the adjustment for immediate payment of claims in the addition made to the net premium to provide for expenses and contingencies. If, however, it should be desired to include this adjustment in the net premium, this can be done approximately (on the assumption that dates of death are, on the average, evenly spaced over the contract year) by multiplying the net premium obtained from the formula by $(1+i)^{\frac{1}{2}-k}$, where i denotes the rate of interest and k represents the average period of time (expressed as a fraction of a year) required to obtain complete proofs of death. As just pointed out, net premiums obtained by these formulas do not include any allowance for expenses or contingencies, and therefore are not comparable with the premiums actually charged by life insurance companies. This is particularly true of "participating" policies, under which a refund, or so-called "dividend," is returned to the policyholder out of each year's premium.

Joint life annuities

A *joint life annuity* is one under which the payments continue so long as two or more designated persons are all alive. For example, a joint life annuity on the lives of three persons continues only so long as all three are alive; it terminates as soon as any one of them dies. Suppose there are l_xl_y distinct pairs of individuals, each pair consisting of one person at exactly age x and another person exactly age y, and that an n-year joint pure endowment of one unit is issued on each pair of lives. Such a contract provides for payment of the amount specified only in case *both* members of the pair are alive at the end of the n-year period. This will be true in $l_{x+n}l_{y+n}$ cases out of the total l_xl_y pairs of lives. Therefore, the net single premium (denoted by ${}_nE_{xy}$) for the joint pure endowment is given by:

$${}_nE_{xy}=\frac{v^nl_{x+n}l_{y+n}}{l_xl_y} \qquad (3)$$

As in the case of single life annuities,[1] a joint life annuity can be regarded as the sum of a number of joint pure endowments. Thus, if $a_{xy:\overline{n}\vert}$ denotes the net value of an n-year temporary joint life annuity on two lives aged x and y,

$$a_{xy:\overline{n}\vert}={}_1E_{xy}+{}_2E_{xy}+\cdots+{}_nE_{xy}$$

Therefore, substitution of formula (3) gives:

$$a_{xy:\overline{n}\vert}=\frac{1}{l_xl_y}(vl_{x+1}l_{y+1}+v^2l_{x+2}l_{y+2}+\cdots+v^nl_{x+n}l_{y+n}) \qquad (4)$$

Likewise, in the case of a temporary joint life annuity on three lives,

$$a_{xyz:\overline{n}\vert}=\frac{1}{l_xl_yl_z}(vl_{x+1}l_{y+1}l_{z+1}+v^2l_{x+2}l_{y+2}l_{z+2}+\cdots+v^nl_{x+n}l_{y+n}l_{z+n})$$

A similar expression can be written for any number of lives.

[1] See p. 86.

It is explained later[2] that when joint life annuities are calculated on the basis of a mortality table which follows Makeham's law, any group of ages on which a joint life annuity is based can be replaced by a group of equal ages. In other words, if a joint life annuity is based on m lives aged $x, y, z, \ldots (m)$, an age w can readily be found, such that

$$a_{xyz\ldots(m):\overline{n}|}=a_{www\ldots(m):\overline{n}|}$$

Therefore, it is sufficient, in such a case, to consider the formulas for joint life annuities when the ages are equal. When the two ages x and y are equal, formula (4) reduces to:

$$a_{xx:\overline{n}|}=\frac{1}{(l_x)^2}[v(l_{x+1})^2+v^2(l_{x+2})^2+v^3(l_{x+3})^2+\cdots+v^n(l_{x+n})^2]$$

By multiplying and dividing by v^x, writing D_{xx} for $v^x(l_x)^2=D_xl_x$, and taking $N_{xx}=D_{xx}+D_{x+1:x+1}+D_{x+2:x+2}+\cdots$ to the end of the life table, it is easily shown that

$$a_{xx:\overline{n}|}=\frac{N_{x+1:x+1}-N_{x+n+1:x+n+1}}{D_{xx}}$$

In the particular case of a joint whole life annuity, this reduces to

$$a_{xx}=\frac{N_{x+1:x+1}}{D_{xx}}.$$

while, for a joint pure endowment,

$${}_nE_{xx}=\frac{D_{x+n:x+n}}{D_{xx}}$$

and, for a deferred joint whole life annuity,

$${}_{n|}a_{xx}=\frac{N_{x+n+1:x+n+1}}{D_{xx}}={}_nE_{xx}\,a_{x+n:x+n}$$

Similar expressions hold for three or more lives, taking $D_{xxx}=D_{xx}l_x$, $D_{xxxx}=D_{xxx}l_x$, and so on.

Reversionary annuities and last survivor annuities

A *reversionary annuity* (or *survivorship annuity*) "to (x) after (y)" is an annuity to commence on the death of (y) and to continue thereafter so long as (x) is alive.[3] If (x) predeceases (y), no payments at all are made. If $a_{y|x}$ denotes the net value of a reversionary annuity of one per annum "to (x) after (y)," it is obvious that

$$a_{y|x}=a_x-a_{xy} \tag{5}$$

For, the value a_x provides an annuity during the entire lifetime of (x), and the deduction of a_{xy} eliminates the value of those payments made while (y) also is alive. Therefore, the remainder is the present value of only those payments which are made during the lifetime of (x) after the death of (y).

[2] See p. 94.

[3] The notation (x) denotes "a specified individual at age x."

By similar reasoning, it is easily seen that

$$a_{yz|x}=a_x-a_{xyz}$$

and

$$a_{z|xy}=a_{xy}-a_{xyz}$$

where $a_{yz|x}$ denotes the net value of an annuity of one per annum commencing at the death of either (y) or (z) (whichever occurs first), and continuing thereafter during the entire lifetime of (x); and $a_{z|xy}$ denotes an annuity of one per annum commencing at the death of (z) and continuing thereafter only so long as (x) and (y) are both alive.

A *last survivor* (or *joint and survivor*) *annuity* to (x) and (y) is one which begins now and continues so long as either (x) or (y) or both are alive. If $a_{\overline{xy}}$ denotes the present value of a last survivor annuity of one per annum on the lives of (x) and (y), it is clear that

$$a_{\overline{xy}}=a_y+a_{y|x}=a_x+a_y-a_{xy}$$

The last expression was obtained from the second by substituting formula (5) for $a_{y|x}$. Similarly, in the case of three lives,

$$a_{\overline{xyz}}=a_{\overline{yz}}+a_{y|x}-a_{y|xz}=a_x+a_y+a_z-a_{xy}-a_{xz}-a_{yz}+a_{xyz}$$

The reasoning which leads to the second member of this equation is as follows. If to a last survivor annuity on the lives of (y) and (z) is added a reversionary annuity to (x) after (y), the sum provides for making payments so long as (x) or (y) or (z) (or any combination of the three) is alive. However, it provides for duplicate payments under one particular set of conditions: namely, when (y) is dead and both (x) and (z) are alive. Hence, the subtraction of a reversionary annuity to (x) and (z) after (y) is exactly what is needed to eliminate the duplicate payments.

Formulas for more complicated benefits can be similarly obtained. For example, in the case of formulas VI, XXIII, and XXIV of table R (p. 91), the steps would be as follows:

Formula VI:

$$a_{\overline{xy}z}=a_{xz}+a_{x|yz}=a_{xz}+a_{yz}-a_{xyz}$$

Formula XXIII:

$$a_{\overline{yz}|x}=a_{y|x}-a_{y|xz}=a_x-a_{xy}-a_{xz}+a_{xyz}$$

Formula XXIV:

$$a_{z|\overline{xy}}=a_{\overline{xyz}}-a_z=a_x+a_y-a_{xy}-a_{xz}-a_{yz}+a_{xyz}$$

Relation between annuities and assurances

There is an important general relationship between the net values of annuities and net single premiums for assurances, which can be stated as follows.

If a *denotes the net value of an annuity-due of one per annum to continue in effect during the existence of a given status, and if A denotes the net single premium for an assurance providing for payment of one unit on the contract anniversary next following the termination of the given status, then*

$$A=1-da \tag{6}$$

where d denotes the rate of discount corresponding to the interest rate assumed.

The *rate of discount* may be defined as the annual amount of interest per unit of principal when interest is payable at the *beginning*, rather than the *end* of each year. It is given by the relations:

$$d=i/(1+i)=iv=1-v$$

This general proposition can be demonstrated as follows. If one unit is invested so as to earn interest at the rate i per annum, the amount i will be received at the end of each year. However, if arrangements could be made to receive the interest at the beginning of each year rather than at the end, the amount received each year would be the present value of i due 1 year hence: that is,[4] $iv=d$. Suppose that one unit is invested, under the latter arrangement, during the continuance of the given status, with the understanding that the unit invested will be withdrawn at the end of the year in which the given status terminates. Then it may be considered that an immediate down payment of one unit has purchased two distinct benefits, namely:

(1) an annuity-due of d per annum during the continuance of the given status, and

(2) the right to receive one unit at the end of the year in which the given status terminates.

It should be clearly understood that the unit originally invested does not become available for withdrawal until the *end* of the year in which the status terminates because the interest paid in advance at the beginning of that year is not fully earned until the end of the year. Now the present value of benefit (1) is, by hypothesis, $d\mathrm{a}$, while that of benefit (2) is A. Since the initial payment must be equal in value to the benefits purchased by it, it follows that

$$1=d\mathrm{a}+A$$

Upon transposing, this gives at once the equation (6).

A simple illustration is the case in which the given status is the survival of a specified life (x). In this case, formula (6) becomes

$$A_x=1-d\mathrm{a}_x=1-d(1+a_x)$$

Similarly, when the status is the joint existence of two lives (x) and (y)

$$A_{xy}=1-d(1+a_{xy})$$

If the given status is the survival of (x) during a period of n years only, the formula gives:

$$A_{x:\overline{n|}}=1-d\mathrm{a}_{x:\overline{n|}}=1-d(1+a_{x:\overline{n-1|}})$$

If the status in question is the survival of any one or more of three lives (x), (y), and (z), the relation is:

$$A_{\overline{xyz}}=1-d(1+a_{\overline{xyz}})$$

Other examples appear among the formulas of table R.

[4] See p. 85.

As a practical illustration, consider the following situation: A certain estate includes a property of value P, which yields an annual income I. Under the terms of the will, one of the heirs, (x), is to receive the income during his lifetime. After the death of (x), another heir, (y), if then alive, is to receive the income as long as he lives. At the death of the survivor of (x) and (y), the title to the property is to pass to a third heir, (z), or to the estate of (z) if he is not then alive. The problem is to determine the present value of the interests of (x), (y), and (z) in the property.

It is obvious that the value of (x)'s interest is Ia_x, and that the value of (y)'s interest is $Ia_{x|y}$. The value of the combined interest of (x) and (y) is $I(a_x+a_{x|y})=Ia_{\overline{xy}}$. On the assumption that the income is receivable annually at the end of the year, (z) will receive, at the end of the year in which the survivor of (x) and (y) dies, 1 year's income in addition to the property itself: that is, a total value of $P+I$. Therefore, it follows from the general principle stated on page 89 that the present value of (z)'s interest is $(P+I)[1-d(1+a_{\overline{xy}})]$, where $d=i\div(1+i)$, and i represents the ratio $I\div P$. As a check on the consistency of these results, the value of the combined interest of (x), (y), and (z) can be written as

$$Ia_{\overline{xy}}+(P+I)-I(1+a_{\overline{xy}})=P$$

since $P+I=P(1+i)$, $(1+i)d=i$, and $Pi=I$. This shows that the present value of the combined interest of all three heirs equals the value of the property, as would be expected.

Formulas for joint life benefits

Table R provides a reference list of formulas for net values of the more common types of joint life benefits in terms of joint life annuities and joint pure endowments. In using this table, it may be helpful to realize that the symbols used to denote net values of the different types of benefits are not merely arbitrary but follow definite rules. The symbol (x) denotes a specified individual whose age is x. The italic "a" indicates the present value of an immediate annuity; the Roman "a," of an annuity-due; and the capital "A," of an assurance. The subscripts to the right of these symbols denote the ages of the lives during whose continued existence the annuity is to be paid, or upon whose death the assurance is payable. Unless otherwise indicated, the annuity terminates, or the assurance becomes payable, upon the occurrence of the *first* death among the group of lives indicated. A subscript with an "angle" ($\overline{|}$) placed over it denotes not an age but a *term certain:* that is, a specified period of years commencing at the date of the contract. For example, the subscript "$\overline{12|}$" in the symbol $a_{35:\overline{12|}}$ denotes a 12-year period starting at the commencement of the annuity. The entire symbol represents the present value of an immediate annuity of one unit per annum to terminate as soon as (35) dies or as soon as $(\overline{12|})$ "dies," whichever occurs first. From this point of

TABLE R.—REFERENCE LIST OF FORMULAS, IN TERMS OF JOINT WHOLE LIFE ANNUITIES AND JOINT PURE ENDOWMENTS, FOR NET VALUES OF THE PRINCIPAL TYPES OF ANNUITIES AND ASSURANCE BENEFITS INVOLVING TWO OR MORE JOINT LIVES

Reference number	Symbol [1]	Description [2]	Formula for net present value or net single premium [3]
		JOINT PURE ENDOWMENT	
I	${}_nE_{xyz\ldots(m)}$	payable after n years if (x), (y), (z), . . . are all alive.	$v^n l_{x+n} l_{y+n} l_{z+n}\ldots(m)/l_x l_y l_z\ldots(m)$
		JOINT LIFE IMMEDIATE ANNUITIES	
II	${}_{n\vert}a_{xyz\ldots(m)}$	deferred n years, then payable until a death occurs among the lives (x), (y), (z), . . .	${}_nE_{xyz\ldots(m)}a_{x+n:y+n:z+n:\ldots(m)}$
III	$a_{xyz\ldots(m):\overline{n}\vert}$	payable for n years, or until a death occurs among the lives x, y, z, . . ., if earlier.	$a_{xyz\ldots(m)}-{}_{n\vert}a_{xyz\ldots(m)}$
		LAST SURVIVOR (OR JOINT AND SURVIVOR) IMMEDIATE ANNUITIES	
IV	$a_{\overline{xy}}$	payable until both (x) and (y) are dead.	$a_x+a_y-a_{xy}$
V	$a_{\overline{xyz}}$	payable until (x), (y), and (z) are all dead.	$a_x+a_y+a_z-a_{xy}-a_{xz}-a_{yz}+a_{xyz}$
VI	$a_{\overline{xy}z}$	payable until the death of either (z) or the survivor of (x) and (y).	$a_{xz}+a_{yz}-a_{xyz}$
VII	${}_{n\vert}a_{\overline{xy}}$	deferred n years, then payable until (x) and (y) are both dead.	${}_{n\vert}a_x+{}_{n\vert}a_y-{}_{n\vert}a_{xy}$
VIII	${}_{n\vert}a_{\overline{xyz}}$	deferred n years, then payable until (x), (y), and (z) are all dead.	${}_{n\vert}a_x+{}_{n\vert}a_y+{}_{n\vert}a_z-{}_{n\vert}a_{xy}-{}_{n\vert}a_{xz}-{}_{n\vert}a_{yz}+{}_{n\vert}a_{xyz}$
IX	${}_{n\vert}a_{\overline{xy}z}$	deferred n years, then payable until the death of either (z) or the survivor of (x) and (y).	${}_{n\vert}a_{xz}+{}_{n\vert}a_{yz}-{}_{n\vert}a_{xyz}$
X	$a_{\overline{xy}:\overline{n}\vert}$	payable for n years, or until both (x) and (y) are dead, if earlier.	$a_{x:\overline{n}\vert}+a_{y:\overline{n}\vert}-a_{xy:\overline{n}\vert}$
XI	$a_{\overline{xyz}:\overline{n}\vert}$	payable for n years, or until (x), (y), and (z) are all dead, if earlier.	$a_{x:\overline{n}\vert}+a_{y:\overline{n}\vert}+a_{z:\overline{n}\vert}-a_{xy:\overline{n}\vert}-a_{xz:\overline{n}\vert}-a_{yz:\overline{n}\vert}+a_{xyz:\overline{n}\vert}$
XII	$a_{\overline{xy}z:\overline{n}\vert}$	payable for n years, or until the death of either (z) or the survivor of (x) and (y), if earlier.	$a_{xz:\overline{n}\vert}+a_{yz:\overline{n}\vert}-a_{xyz:\overline{n}\vert}$
		JOINT LIFE ASSURANCES	
XIII	$A_{xyz\ldots(m)}$	payable upon [3] the first death among the lives (x), (y), (z), . . .	$1-d(1+a_{xyz\ldots(m)})$
XIV	${}_{n\vert}A_{xyz\ldots(m)}$	payable upon [3] the first death among the lives (x), (y), (z), . . ., if this occurs after n years have elapsed.	${}_nE_{xyz\ldots(m)}A_{x+n:y+n:z+n:\ldots(m)}$
XV	$A_{xyz\ldots(m):\overline{n}\vert}$	payable at [3] the end of n years, or after the first death among the lives (x), (y), (z), . . ., if earlier.	$1-d(1+a_{xyz\ldots(m):\overline{n-1}\vert})$
XVI	$A_{\overline{xyz\ldots(m)}}$	payable at [3] the death of the last survivor of (x), (y), (z), . . .	$1-d(1+a_{\overline{xyz\ldots(m)}})$
XVII	${}_{n\vert}A_{\overline{xy}}$	payable at [3] the death of the survivor of (x) and (y) if this occurs after n years have elapsed.	${}_{n\vert}A_x+{}_{n\vert}A_y-{}_{n\vert}A_{xy}$
XVIII	${}_{n\vert}A_{\overline{xyz}}$	payable at [3] the death of the last survivor of (x), (y), and (z) if this occurs after n years have elapsed.	${}_{n\vert}A_x+{}_{n\vert}A_y+{}_{n\vert}A_z-{}_{n\vert}A_{xy}-{}_{n\vert}A_{xz}-{}_{n\vert}A_{yz}+{}_{n\vert}A_{xyz}$
XIX	$A_{\overline{xyz\ldots(m)}:\overline{n}\vert}$	payable at [3] the end of n years, or at the death of the last survivor of (x), (y), (z), . . ., if earlier.	$1-d(1+a_{xyz\ldots(m):\overline{n-1}\vert})$
		REVERSIONARY (OR SURVIVORSHIP) ANNUITIES	
XX	$a_{y\vert x}$	commencing at [3] the death of (y) and continuing thereafter during the life of (x).	a_x-a_{xy}
XXI	$a_{yz\vert x}$	commencing as soon as either (y) or (z) dies,[3] and continuing thereafter during the life of (x).	a_x-a_{xyz}
XXII	$a_{z\vert xy}$	commencing at [3] the death of (z) and continuing thereafter so long as (x) and (y) are both alive.	$a_{xy}-a_{xyz}$
XXIII	$a_{\overline{yz}\vert x}$	commencing at [3] the death of the survivor of (y) and (z), and continuing thereafter during the life of (x).	$a_x-a_{xy}-a_{xz}+a_{xyz}$
XXIV	$a_{z\vert\overline{xy}}$	commencing at [3] the death of (z) and continuing thereafter until both (x) and (y) are dead.	$a_x+a_y-a_{xy}-a_{xz}-a_{yz}+a_{xyz}$

[1] The letter (m) denotes the number of lives involved.
[2] The notations (x), (y), (z) . . . denote specified individuals at ages x, y, z, etc.
[3] These formulas assume that all payments are of one unit and are made on contract anniversaries; that annuity payments are made annually at the *end* of each year which falls within the term of the annuity; and that assurance payments are made on the anniversary following death, rather than immediately after death.

view, the end of the 12-year period starting from the commencement of the annuity is regarded as the "death" of ($\overline{12|}$). Similarly, $A_{35:53:\overline{12|}}$ denotes the net single premium for an assurance of one unit payable upon the occurrence of the first "death" among (35), (53), and ($\overline{12|}$). In other words, if either (35) or (53) dies within 12 years from the date of the contract, the assurance is payable upon the first death; otherwise, the payment is made upon the "death" of ($\overline{12|}$): that is, at the end of the 12-year period. This shows why the addition of the subscript "$\overline{n|}$" to an assurance symbol indicates (unless the symbol is otherwise modified at the same time) an endowment assurance rather than a temporary assurance. These principles are illustrated by formulas III, XIII, and XV of table R.

The notation "$_{n}|$" preceding an assurance or annuity symbol indicates that the benefit in question is deferred n years. For example, $_{12}|A_{35:53}$ denotes the net single premium for an assurance of one unit payable on the occurrence of the first death among two lives now aged 35 and 53, provided such death occur after the expiration of a period of 12 years from the date of the contract. This notation is illustrated by formulas II and XIV of table R.

A horizontal bar placed over a group of subscripts representing ages denotes the last survivor of the corresponding group of lives. For example, $A_{\overline{35:53:67}}$ denotes the net single premium for an assurance of one unit payable on the death of the last survivor of three lives now aged 35, 53, and 67, and $a_{\overline{35:53}:24}$ denotes the net present value of an immediate annuity of one unit per annum which terminates either on the death of a life now aged 24 or on the death of the survivor of two lives now aged 35 and 53, whichever occurs first. This notation is illustrated by formulas IV to XII and XVI to XIX of table R.

A vertical line separating into two groups the subscripts to the right of an annuity symbol indicates that the annuity is to commence at the death indicated by the subscripts which precede the vertical line, and is to terminate at the death indicated by the subscripts which follow the vertical line. For example, $a_{\overline{35:53}|24:67}$ denotes the net present value of an annunity of one unit per annum to commence on the death of the survivor of two lives now aged 35 and 53 and to terminate on the death of either of two lives now aged 24 and 67, whichever occurs first. Of course, if either of the latter two lives should predecease the survivor of the first two, no payments would be made under the annuity. Similarly, $a_{35:53|\overline{24:67}}$ denotes the net present value of an annuity of one unit per annum to commence on the death of either of two lives now aged 35 and 53, whichever occurs first, and to terminate on the death of the survivor of two lives now aged 24 and 67. If the survivor of the latter two lives should predecease both the first two, no payments would be made. This notation is illustrated by formulas XX to XXIV of table R It will be noted that the table does not contain a formula for last survivor annuities analogous to formula II or a formula for last survivor assurances analogous to formula XIV. This is because such formulas do not hold.[5]

The symbol which represents the net single premium for a joint pure endowment (formula I of table R) follows somewhat different principles. The main part of the symbol is a capital "E." The subscript to the left of the "E" denotes a period of years starting from the date of the contract, at the end of which (if at all) the endowment is to be paid, while the subscripts following the "E" represent the ages of the various lives who must *all* survive the stated period as the necessary condition for payment of the endowment. For example, $_{12}E_{35:53:67}$ denotes the net single premium for a contract to pay one unit at the end of 12 years if three lives now aged 35, 53, and 67 are all alive at that time.

Formulas for temporary assurances are not given in table R. In any given case, the net single premium for a temporary assurance is obtained by subtracting the corresponding pure endowment premium from the corresponding endowment assurance premium.

B. ARRANGEMENT AND USE OF THE ACTUARIAL TABLES

Elementary values

In using actuarial functions derived from a life table, it is highly desirable to have the various mathematical relationships between the different functions hold as precisely as possible. Since the commutation columns, from which most other actuarial functions are derived, are based directly on the l_x and d_x columns, the desired mathematical consistency is most readily obtained by regarding l_x (rather than q_x) as the basic column of the table and deriving the others from it. This has been done in the tables of elementary values included with the actuarial tables (tables 14, 25, and 38), with the result that many of the values shown in these tables differ very slightly, in the case of white males and white females, from the corresponding figures in the life tables of part II (tables 5 and 6). A detailed statement concerning these differences is given in the appendix [6] in connection with the account of methods of construction of the actuarial tables. The values given in the makehamized mortality table for total whites (table 38) naturally differ to a much greater extent from the corresponding values in the life table previously given (table 4), since the makehamized table constitutes a different graduation of the data.

In all three cases, the tables of elementary values included with the actuarial tables give the rate of mortality on a unit basis (rather than a "per 1,000" basis), for convenience in making mathematical calculations. The average future lifetime and the functions

[5] Spurgeon, E. F., *Life Contingencies*, third edition, pp. 267-268, Cambridge University Press, London, 1938.

[6] See p. 137.

relating to the stationary population are not shown; however, two additional functions are given which did not appear in the life tables of part II. These are the probability of survival p_x and the force of mortality μ_x.

The probability of survival, or survival rate, is the complement of the rate of mortality; in algebraic terms, $p_x=1-q_x$. In other words, it is the proportion of individuals at a given exact age who survive exactly 1 year.

The force of mortality, or instantaneous rate of mortality, at age x "represents the annual rate at which the community under review is dying at the moment of attaining age x." [7] Expressed in slightly different language, it is "the proportion of persons of that age who would die in a year, if the intensity of mortality remained constant for a year, and if the number of persons under observation also remained constant, the places of those who die being constantly occupied by fresh lives." [8] In the language of mathematics, μ_x is the negative of the derivative of l_x with respect to x, expressed as a ratio to l_x itself. The values of the force of mortality are useful in evaluating annuities and other benefits involving two or more joint lives, as will be explained later.[9] In the case of the makehamized table, the radix has been taken as 1,000,000 rather than 100,000 in order to retain one more significant figure and thus take full advantage of the additional smoothness resulting from the Makeham graduation.

Use of the actuarial tables in calculating single life annuity values and net premiums for life insurance benefits

The actuarial tables based on the life tables for white males and white females (tables 14 to 35) provide the means of calculating all values ordinarily required for actuarial purposes, on the basis of the five interest rates for which tables are given. The commutation columns (tables 15 to 19 and 26 to 30) are purely mathematical devices which represent steps in the computation of annuity values, net premiums, policy values, and other actuarial figures. Their usefulness lies entirely in shortening the arithmetic: they are not susceptible of any concrete interpretation which is useful in other than exceptional cases. In using the tables of commutation functions, the reference lists of formulas given in tables P and Q (pp. 87 and 88) may be helpful.

Net values of immediate whole life annuities and net premiums for whole life assurances have been calculated, and are given in tables 20 to 24 and 31 to 35. These are the simplest forms of annuity and assurance, respectively, and correspond to the formulas appearing in line number I of tables P and Q. Formulas for dealing with varying annuities and assurances, and other benefits of a more complicated character, will be found in the standard textbooks on actuarial theory.[10]

The use of tables 14 to 35 and the application of the formulas given in tables P and Q are illustrated by the following numerical examples.

Example 1.—Find the present value at 2 percent interest of an immediate whole life annuity of $400 per annum payable to a white female now aged 63.

Solution.—As this is an immdeiate whole life annuity, it is not necessary to employ commutation columns, and the present value per dollar of annual payment can be obtained directly from table 31. There it is found that the value in question is $11.9366 per dollar of annual payment. Multiplying this figure by 400 gives $4,774.64 as the total present value of the annuity.

Example 2.—Find the present value at 3 percent interest to a white male now at age 41 of a deferred life annuity of $1,200 per annum, the first payment to be made at age 65.

Solution.—As this is a deferred whole life annuity, formula number V in table P is the correct one to use. As the first payment is made at age 65, $x+m+1=65$, while $x=41$. Therefore the total present value is:

$$\$1{,}200\frac{N_{65}}{D_{41}}$$

Table 17 shows that $N_{65}=86{,}352.4$ and $D_{41}=25{,}725$. Substituting these values in the above formula gives $4,028.10 as the total present value of the deferred annuity.

Example 3.—Find the net annual premium for a whole life assurance of $2,500 on a life aged 37 on the basis of 1939–1941 mortality of United States white males at 2½ percent interest.

Solution.—The net annual premium per dollar of insurance is taken directly from table 21, the value being $0.02118. Multiplying by 2,500 gives $52.95 as the total net annual premium.

Example 4.—Find the net single premium at age 43 for a 20-year endowment assurance of $5,000 on the basis of 1939–1941 mortality of United States white males at 2 percent interest.

Solution.—Applying the formula in line number VI of table Q gives for the net single premium:

$$\$5{,}000\frac{M_{43}-M_{63}+D_{63}}{D_{43}}$$

Reference to table 15 shows that $M_{43}=21{,}532.60$, $M_{63}=13{,}805.08$, $D_{43}=36{,}463$, and $D_{63}=17{,}907$. Substituting these values in the above formula gives $3,515 as the total net single premium.

[7] Australia Census Bureau, *Census of the Commonwealth of Australia, 3rd April 1911*, vol. 1, *Statistician's Report*, p. 319, McCarron, Bird and Co., Melbourne, 1917. The definition quoted was written by Sir George Knibbs, Commonwealth Statistician.

[8] King, George, *Institute of Actuaries' Text Book of the Principles of Interest, Life Annuities, and Assurances, and Their Practical Application, Part II, Life Contingencies*, second edition, p. 24, Charles and Edwin Layton, London, 1902.

[9] See p. 94.

[10] Menge, Walter O., and Glover, James W., *An Introduction to the Mathematics of Life Insurance*, The Macmillan Co., New York, 1935; Mackenzie, M. A., and Sheppard, N. E., *An Introduction to the Theory of Life Contingencies*, The University of Toronto Press, Toronto, 1931; Spurgeon, *op. cit.*

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Use of the actuarial tables in evaluating joint life annuities

The calculation of the values of joint life annuities is greatly facilitated when it is possible to use a mortality table which follows the mathematical formula known as Makeham's law.[11] A Makeham graduation of the life table for total whites in the United States in 1939–1941 has been prepared and appears as table 38, page 80. Tables 36 and 37, and 39 to 42 also contain values relating to or derived from this mortality table. The life table for total whites was used for this purpose, rather than the separate tables for white males and white females, because it appeared that joint life values based on the total white population would be useful for certain purposes, and because serious technical difficulties were encountered in attempting to graduate by Makeham's law the separate life tables for males and females.[12] On pages 97 to 99, a method is given by which the values of joint life annuities based on the life tables for the separate sexes can be closely approximated.

The simplification in the calculation of joint life annuity values resulting from the use of a mortality table which follows Makeham's law arises from the fact that it is necessary to tabulate only the values of joint life annuities on lives of equal age. This is feasible because it is easy to determine from any given set of m ages, x, y, z, etc., an "equivalent equal age," w such that a joint life annuity on m lives all at age w has the same value as a similar joint life annuity on m lives at the ages originally given. For example, on the basis of the makehamized mortality table included in this volume, it is found that a joint life annuity on three lives aged 27, 38, and 43 is equal in value to a joint life annuity on three lives all aged 37.75 years. Tables 39 to 42 give the values of immediate whole life annuities for single lives, and for two, three, and four joint lives of equal age, with interest at 2, 2½, 3, and 4 percent.

The most generally applicable method of arriving at the equivalent equal age involves the force of mortality. A mortality table which follows Makeham's law has the property that the value of the force of mortality at the equivalent equal age corresponding to a given set of ages is exactly the arithmetic average of the values of the force of mortality at the given ages. For example, in the illustration previously given, suppose it is required to find the present value at 2½ percent interest, on the basis of the makehamized mortality table given in this volume, of an immediate joint whole life annuity of one per annum on three lives aged 27, 38, and 43. Reference to the last column of table 38 shows that $\mu_{27}=.00212$, $\mu_{38}=.00385$, and $\mu_{43}=.00540$. Adding these three values and dividing by 3 gives $\mu_w=.00379$, where w denotes the equivalent equal age. Since $\mu_{37}=.00361$ and $\mu_{38}=.00385$, it is clear the w is an age between 37 and 38. In order to determine the exact fraction, interpolation is used. Thus,

$$\frac{.00379-.00361}{.00385-.00361}=.75$$

so that $w=37.75$. Therefore,

$$a_{27:38:43}=a_{37.75:37.75:37.75}$$

Now, table 40 shows that $a_{37:37:37}=16.2201$ and $a_{38:38.38}=15.8127$. Interpolation gives: $a_{37.75:37.75:37.75}=16.2201-.75(16.2201-15.8127)=15.9146$, which is the desired result.

When there are only two lives, it is more accurate, and usually more convenient, to use the principle of *uniform seniority*, as embodied in table 37. For example, let it be required to find $a_{35:51}$ at 3 percent interest. The difference between the two ages, 35 and 51, is 16 years. Upon entering table 37 with this difference of 16 years, 10.622 years is obtained as the addition which must be made to the *younger* age in order to obtain the equivalent equal age. Adding 10.622 to 35 gives 45.622. Reference to table 41 shows that

$$a_{35:51}=a_{45.622:45.622}=14.2012-$$
$$.622\ (14.2012-13.8180)=13.9628$$

The other method, using the values of μ_x, would give $\mu_w=\frac{1}{2}(.00320+.00986)=.00653$, whence $w=45.604$, and $a_{35:51}=a_{45.604:45.604}=13.9697$. The difference in the results is due to the fact that linear interpolation between the values of μ_x is a less accurate means of finding the equivalent equal age than the table of uniform seniority.

It is also possible to deal with four lives by repeated applications of the principle of uniform seniority. For example, if the ages of the four lives are 23, 35, 39, and 57, it is found from table 37 that the equivalent equal age corresponding to the two ages 23 and 35 is 30.523, while that corresponding to ages 39 and 57 is 51.258. Now the difference between 30.523 and 51.258 is 20.735, and interpolation gives 14.575 as the addition to be made to the younger age. Adding this quantity to 30.523 gives 45.098 as the equivalent equal age for the four lives. The result obtained by averaging the four values of μ_x is 45.099. The corresponding immediate whole life annuity values at 3 percent are 10.9172 and 10.9169, respectively. With four lives, the averaging method is slightly simpler, but of course slightly less accurate.

The application of the uniform seniority principle to three lives is inconvenient, and requires special tables which have not been included in this volume. Of course, in this case, the method based on averaging the μ_x values can be used.

The principle of uniform seniority does not hold for reversionary and last survivor annuities.[13] Values of such annuities must first be expressed in terms of simple joint life annuities, to which the uniform seniority principle can then be applied.

[11] Also called Makeham's first modification of Gompertz's law.
[12] See p. 138.

[13] Spurgeon, *op. cit.*, pp. 265-266.

Evaluation of joint life annuities involving ages under 17

The makehamized mortality table included in this volume follows Makeham's law only at ages 17 and over. Therefore, if one or more lives in the group are at ages under 17, neither of the methods described in the preceding section gives the correct annuity value. In such a case, either of two procedures may be adopted: an exact, but laborious method; and a shorter method, which is not exact but yields a close approximation. In the exact method, the annuity in question is expressed as the sum of a temporary annuity and a deferred annuity. Thus, if the ages are x, y, z, . . . (m), and if h denotes the difference between the age of the youngest life and 17, then

$$a_{xyz\ldots(m)} = a_{xyz\ldots(m):\overline{h|}} + {}_{h|}a_{xyz\ldots(m)}$$

Here the temporary annuity $a_{xyz\ldots(m):\overline{h|}}$ is limited to a maximum of h payments, and its value is given by:

$$\frac{1}{l_x l_y l_z \ldots (m)}[v l_{x+1} l_{y+1} l_{z+1} \ldots (m) + v^2 l_{x+2} l_{y+2} l_{z+2} \ldots (m) + \ldots + v^h l_{x+h} l_{y+h} l_{z+h} \ldots (m)] \qquad (7)$$

In order to evaluate this expression, it is necessary to compute each of the h individual terms within the bracket, sum them, and then divide by the product of the l_x values. The deferred annuity ${}_{h|}a_{xyz\ldots(m)}$ consists of payments commencing at the end of $h+1$ years, and then only if all m lives have survived that period. Its value is given by

$$v^h \frac{l_{x+h} l_{y+h} l_{z+h} \ldots (m)}{l_x l_y l_z \ldots (m)} a_{x+h:y+h\, z+h:\ldots(m)} \qquad (8)$$

In evaluating both these expressions, the powers of v can be obtained from compound interest tables; and the annuity value involved in the last expression can be calculated by the method of the preceding section, since it involves only ages 17 and over. For convenience, the powers of v as far as v^{17} are given in table S for the four rates of interest for which joint life annuity values appear in tables 39 to 42. Values beyond v^{17} never occur in the expressions (7) and (8) since 17 years is the maximum duration of the temporary annuity.

TABLE S.—PRESENT VALUE AT COMPOUND INTEREST OF ONE UNIT DUE AFTER t YEARS, INTEREST AT 2, 2½, 3, AND 4 PERCENT

NUMBER OF YEARS (t)	PRESENT VALUE OF ONE $v^t=(1+i)^{-t}$			
	2 percent	2½ percent	3 percent	4 percent
1	0.980392	0.975610	0.970874	0.961538
2	.961169	.951814	.942596	.924556
3	.942322	.928599	.915142	.888996
4	.923845	.905951	.888487	.854804
5	.905731	.883854	.862609	.821927
6	.887971	.862297	.837484	.790315
7	.870560	.841265	.813092	.759918
8	.853490	.820747	.789409	.730690
9	.836755	.800728	.766417	.702587
10	.820348	.781198	.744094	.675564
11	.804263	.762145	.722421	.649581
12	.788493	.743556	.701380	.624597
13	.773033	.725420	.680951	.600574
14	.757875	.707727	.661118	.577475
15	.743015	.690466	.641862	.555265
16	.728446	.673625	.623167	.533908
17	.714163	.657195	.605016	.513373

As a numerical illustration, let it be required to find the present value at 2 percent interest of an immediate joint whole life annuity of one per annum on three joint lives aged 5, 10, and 20. Now, the difference between the youngest age and 17 is 12 years; therefore, the temporary annuity will run for 12 years and the deferred annuity will have a 12-year deferment period. Table T shows the calculation of the temporary annuity. The main part of this table, which appears under the caption "numerator," represents the calculation of the expression within the square brackets in formula (7). The figures in column 6 of the table are the numerical values of the successive terms in this expression, the figures in columns 2 to 5 being the factors which must be multiplied together in order to obtain the value in column 6. For example, the sixth line (which corresponds to the sixth term inside the brackets) shows the calculation of the product $v^6 l_{11} l_{16} l_{26}$. Here, the subscripts of the "l's", 11, 16, and 26 have been obtained by adding 6 to each of the original ages 5, 10, and 20. The values are obtained from table 38. The powers of v are taken from table S. The "total" figure in column 6 of table T is the numerical value of the entire expression within the brackets in formula (7). The line under the heading "denominator" shows the calculation of the denominator of the fraction outside the brackets, and the final figure (10.3078) in column 6 is the value of the temporary annuity.

By formula (8), the deferred annuity is equal to

$$\frac{v^{12} l_{17} l_{22} l_{32}}{l_5 l_{10} l_{20}} a_{17.22.32}$$

TABLE T.—CALCULATION OF PRESENT VALUES OF A 12-YEAR IMMEDIATE TEMPORARY JOINT LIFE ANNUITY OF ONE PER ANNUM ON THREE JOINT LIVES AGED 5, 10, AND 20: MAKEHAMIZED MORTALITY TABLE FOR TOTAL WHITES IN THE UNITED STATES, 1939–1941, INTEREST AT 2 PERCENT

NUMBER OF PAYMENT (t) (1)	v^t (2)	l_{5+t} (3)	l_{10+t} (4)	l_{20+t} (5)	$10^{-12}\times$ product of columns 2 to 5 (6)
	COMPUTATION OF NUMERATOR				
1	0.980392	945,953	941,473	929,023	811,155
2	.961169	944,909	940,595	927,436	792,276
3	.942322	943,967	939,637	925,796	773,805
4	.923845	943,098	938,594	924,099	755,705
5	.905731	942,276	937,464	922,338	737,942
6	.887971	941,473	936,242	920,510	720,482
7	.870560	940,595	934,926	918,606	703,247
8	.853490	939,637	933,509	916,621	686,226
9	.836755	938,594	932,055	914,546	669,458
10	.820348	937,464	930,561	912,375	652,936
11	.804263	936,242	929,023	910,099	636,651
12	.788493	934,926	927,436	907,707	620,590
Total of numerator					8,560,473
	COMPUTATION OF DENOMINATOR				
0	1.000000	947,129	942,276	930,561	830,486
Quotient: ($a_{5\cdot10\,20:\overline{12|}}$) 8,560,473÷830,486 equals					10.3078

The arithmetic can be shortened by observing that the numerator of the fraction in this expression is identical with the final term within the brackets in the expression for the temporary annuity (and therefore with the twelfth entry in column 6 of table T), while the denominator is the same as the denominator of the temporary annuity. Therefore, the value of the fraction is 620,590÷830,486, or .747261. Since the annuity $a_{17:22:32}$ involves no ages under 17, it can be evaluated by the method of the preceding section, in which the equivalent equal age is obtained by taking the arithmetic average of μ_{17}, μ_{22}, and μ_{32}. This gives $\mu_w = .00198$, from which the equal age w is found by interpolation to be 25.44. Interpolating in table 39 then gives $a_{www} = 22.2995$. It follows that the value of the deferred annuity is .747261 × 22.2995 or 16.6635; and finally the desired value $a_{5:10:20}$ is the sum of the values of the temporary annuity and the deferred annuity: that is, 10.3078+16.6635, which gives 26.9713.

In the short method, the entire whole life annuity is first evaluated by finding an equivalent equal age, in much the same way as when no life below age 17 is involved, and the value is then corrected by means of the adjustment factors r_x given in table U. If two or four lives are involved, this approximate value may be obtained from the table of uniform seniority (table 37, p. 80) as explained on page 94. If the number of lives is other than two or four, the equal age for the approximate annuity value is obtained from the values of μ_x as follows:

First, add h to each of the ages $x, y, z, \ldots (m)$, where h is the difference between 17 and the youngest age. Next, find the equal age w' for these augmented ages by averaging the corresponding values of μ_x as explained on page 94. Then the equal age for the approximate annuity value $a_{www \ldots (m)}$ is $w = w' - h$.

This approximate annuity value is then adjusted by the formula:

$$a_{xyz \ldots (m)} = \frac{(r_w)^m}{r_x r_y r_z \ldots (m)} a_{www \ldots (m)} \qquad (9)$$

approximately. The adjustment factor r_x is defined as $l_x \div \lambda_x$, where λ_x denotes the value which would be obtained for l_x by the Makeham formula. Therefore, r_x equals unity at ages 17 and above. This method is due to George King who has given a full explanation of the rationale of the method.[14]

Taking as an illustration the same numerical example previously used, the addition of 12 years to the original ages 5, 10, and 20 gives 17, 22, and 32. The equal age corresponding to these three ages is found, just as in the evaluation of the deferred annuity in the other method, to be 25.44. Subtracting 12 years gives 13.44

[14] King, *op. cit.*, pp. 208–212. King's warning against using this approximation in connection with ages below 15 does not apply to the makehamized table published in this volume, since the present table follows Makeham's law down to a much younger age than the mortality table to which King was referring.

TABLE U.—ADJUSTMENT FACTORS FOR ESTIMATING VALUES OF JOINT LIFE ANNUITIES INVOLVING LIVES UNDER AGE 17: MAKEHAMIZED MORTALITY TABLE FOR TOTAL WHITES IN THE UNITED STATES, 1939–1941

AGE x	ADJUSTMENT FACTOR r_x
0	1.04646
1	1.00248
2	.99903
3	.99779
4	.99722
5	.99702
6	.99699
7	.99712
8	.99737
9	.99772
10	.99813
11	.99858
12	.99898
13	.99932
14	.99958
15	.99979
16	.99993
17 and over	1.00000

$$a_{xyz \ldots (m)} = \frac{(r_w)^m}{r_x r_y r_z \ldots (m)} a_{www \ldots (m)}, \text{ approximately.}$$

for the equivalent equal age w. By interpolating in table U, r_w is found to be .99943; while interpolation in table 39 gives $a_{www} = 26.9030$. Formula (9) then becomes:

$$a_{5:10:20} = \frac{(r_w)^3}{r_5 \, r_{10} \, r_{20}} a_{www}$$

which, on substituting the numerical values, gives 26.9878 as the final result. This compares favorably with the value 26.9713 obtained by the exact method, and of course involves much less computation.

Table W presents a comparison of the values of whole life annuities on two joint lives computed at 3 percent interest for various combinations of ages by both the exact and approximate method. This comparison shows that, at least in the case of two lives, the approximate method always gives sufficiently accurate results for most practical purposes. Any increase in the number of lives would decrease the value of the annuity, and therefore would, in general, reduce further the range of error.

TABLE W.—COMPARISON OF WHOLE LIFE ANNUITY VALUES ON TWO JOINT LIVES, COMPUTED BY EXACT AND APPROXIMATE METHODS:[1] MAKEHAMIZED MORTALITY TABLE FOR TOTAL WHITES IN THE UNITED STATES, 1939–1941, INTEREST AT 3 PERCENT

AGE OF OLDER LIFE	AGE OF YOUNGER LIFE: 0		5		10		15	
	Exact method	Approximate method	Exact method	Approximate method	Exact method	Approximate method	Exact method	Approximate method
5	24.7040	24.6976	--------	--------	--------	--------	--------	--------
10	24.1290	24.1555	25.0511	25.0532	--------	--------	--------	--------
20	22.6431	22.6594	23.5756	23.5899	23.2789	23.2824	22.8450	22.8450
30	20.5563	20.5733	21.4683	21.4826	21.2896	21.2931	21.0123	21.0123
40	17.8300	17.8461	18.6593	18.6732	18.5609	18.5644	18.3989	18.3990
50	14.5309	14.5462	15.2258	15.2393	15.1764	15.1796	15.0885	15.0885
60	10.8820	10.8960	11.4109	11.4241	11.3895	11.3927	11.3447	11.3446
70	7.2915	7.3024	7.6476	7.6599	7.6409	7.6443	7.6205	7.6206

[1] For description of these two methods, see text, pp. 95–96.

Calculation of net values of reversionary and last survivor annuities, and assurances involving two or more lives

Net values of various types of reversionary annuities and last survivor annuties and assurances can be calculated from joint life annuity values and joint pure endowment values by means of the formulas of table R. The symbols used in this table represent the net present value of the benefit described in the third column when the amount of each individual payment (in the case of an annuity) or of the sum insured (in the case of an assurance) is unity. When (as is usual) the payments are of some other amount, it is only necessary to multiply the value for a unit payment by the amount of the payment.[15]

It will be noted that most of the assurance formulas in table R involve the rate of discount *d*. Values of *d* corresponding to all the interest rates for which values are tabulated in this volume are given in table Y.

TABLE Y.—VALUES OF THE RATE OF DISCOUNT FOR SELECTED RATES OF INTEREST

RATE OF INTEREST	RATE OF DISCOUNT
	$d=1-v=iv$
0.02	0.019608
0.025	.024390
0.03	.029126
0.035	.033816
0.04	.038462

It should be carefully noted (as already stated on p. 92) that the formula for last survivor annuities analogous to formula II of table R, and the formula for last survivor assurances analogous to formula XIV, do not hold true. It is also important to understand (as previously mentioned on p. 92) that the principle of uniform seniority does not hold for reversionary or last survivor annuities. It is necessary first to express the values of such annuities in terms of ordinary joint life annuities, and then to evaluate the latter.

Example 5.—On the basis of the makehamized mortality table for total whites in the United States in 1939–1941 and interest at 2½ percent, find the net annual premium for a whole life last survivor assurance of $3,000 on three lives aged 35, 39, and 54, premiums being payable throughout the duration of the contract.

Solution.—Inspection of formula XVI of table R shows that the value of a last survivor annuity is first required. This, in turn, is given by fomula V. By referring to table 40 and employing the methods previously described, the values of the various annuities which enter into the latter formula are found to be as follows:

$a_{35}=22.3898$ $\quad$ $a_{35:39}=18.2354$

$a_{39}=20.8985$ $\quad$ $a_{35:54}=13.6797$

$a_{54}=14.6737$ $\quad$ $a_{39:54}=13.3686$

$a_{35:39:54}=12.5766$

Substituting in formula V gives $a_{\overline{35:39:54}}=25.2549$. Table Y shows that $d=.024390$, and substituting in formula XVI gives $A_{\overline{35:39:54}}=.35964$. Therefore,

$$P_{\overline{35:39:54}}=A_{\overline{35:39:54}}\div(1+a_{\overline{35:39:54}})=$$

$$.35964\div26.2549=.01370.$$

This is the net annual premium per unit insured. Multiplying by $3,000 gives $41.10 as the required net annual premium.

Example 6.—Find the present value, on the basis of the makehamized United States mortality table at 3 percent interest, of a reversionary annuity of $1,000 per annum to a boy now aged 17, to commence as soon as his father aged 48 and his uncle aged 42 have both died.

Solution.—This annuity is represented by the symbol $a_{\overline{42:48}|17}$. Formula XXIII of table R shows that the present value per unit of payment is $a_{17}-a_{17:42}-a_{17:48}+a_{17:42:48}$. Using table 41 and the methods previously explained gives

$a_{17}=25.2243$ $\quad$ $a_{17:48}=15.7524$

$a_{17:42}=17.7209$ $\quad$ $a_{17:42:48}=13.7237$

Substituting these values gives $a_{\overline{42:48}|17}=5.4747$. Finally, multiplying by $1,000 gives $5,474.70 as the present value of the reversionary annuity.

Estimation of joint life annuity values based on the separate life tables for white males and white females

It is often desired to take sex into consideration in the calculation of joint life annuity values: that is, to assume in the computations different rates of mortality for males and females. However, in the preparation of the joint life tables in this volume, it was found impracticable to prepare separate tables for males and females, because it was not possible, without considerable distortion of the rates of mortality, to make separate Makeham graduations of the life tables for males and females, and at the same time preserve the necessary relationship between the Makeham constants under the two tables so as to have the law of uniform seniority hold for annuities involving both male and female lives. It was desired, therefore, to devise a method of approximating the values of joint life annuities based on the separate tables which would not be laborious, and at the same time would give reasonably accurate results.

After experimenting with a number of possible methods, two were selected as meeting satisfactorily the requirements stated. Both these methods consist in entering the annuity tables based on the makehamized

[15] Strictly speaking, these symbols also imply that all payments are made on anniversaries of the original agreement or contract. In practice, this is often not the case. For example, life insurance companies usually pay the sum insured under a life assurance immediately on receipt of completed proofs of death, while payments under a reversionary annuity are frequently made on anniversaries of the death upon the occurrence of which the annuity commenced. It is usual, however, to ignore these refinements or (in the case of contracts issued by life insurance companies) to include them in the allowance for expenses and contingencies which forms part of the gross premium actually charged.

mortality table for total whites with appropriately adjusted ages. In general, the adjustment takes the form of an addition to the age in the case of males and a deduction from the age in the case of females. In the first method the adjustment is a very simple function of the age. In the second method the adjustments are more accurately determined, and the closeness of the approximation is somewhat improved.

In the first and more rough method, the addition or deduction, as the case may be, is 2 years up to and including age 50, graded down to 0 at age 90. The adjusted ages corresponding to ages 51 to 89 are given in table Z. In the second and more refined method, the *single life* annuity corresponding to each of the lives involved is first obtained from the annuity tables (not makehamized) for the separate sexes (tables 20 to 24 and 31 to 35). The next step is to enter with these single life annuity values the single life annuity column based on the makehamized mortality table for total whites, and to find the age corresponding to each annuity value. This is taken as the adjusted age for the life in question. The following illustrations will make the procedure clear.

TABLE Z.—ADJUSTED AGES TO BE USED IN ENTERING JOINT LIFE ANNUITY TABLES BASED ON THE MAKEHAMIZED MORTALITY TABLE FOR TOTAL WHITES IN ORDER TO APPROXIMATE VALUES BASED ON THE MORTALITY OF THE SEPARATE SEXES: UNITED STATES, 1939–1941

ACTUAL AGE	ADJUSTED AGE	
	Male	Female
17–50	Add 2 years	Deduct 2 years
51	52.95	49.05
52	53.90	50.10
53	54.85	51.15
54	55.80	52.20
55	56.75	53.25
56	57.70	54.30
57	58.65	55.35
58	59.60	56.40
59	60.55	57.45
60	61.50	58.50
61	62.45	59.55
62	63.40	60.60
63	64.35	61.65
64	65.30	62.70
65	66.25	63.75
66	67.20	64.80
67	68.15	65.85
68	69.10	66.90
69	70.05	67.95
70	71.00	69.00
71	71.95	70.05
72	72.90	71.10
73	73.85	72.15
74	74.80	73.20
75	75.75	74.25
76	76.70	75.30
77	77.65	76.35
78	78.60	77.40
79	79.55	78.45
80	80.50	79.50
81	81.45	80.55
82	82.40	81.60
83	83.35	82.65
84	84.30	83.70
85	85.25	84.75
86	86.20	85.80
87	87.15	86.85
88	88.10	87.90
89	89.05	88.95
90 and over	No change	No change

Example 7.—Find the approximate value of a joint life annuity of one per annum on two white male lives at ages 40 and 60 on the basis of the 1939–1941 life tables with interest at 3 percent.

Solution.—By the rough method, the adjusted ages are 42 and 61.50. The difference between these ages is 19.50 years. Entering the table of uniform seniority (table 37) with this value and interpolating gives 13.5195 years as the necessary addition to the younger age. Adding 13.5195 to 42 gives 55.5195 as the equivalent equal age. Interpolation in table 41 shows the value of a joint whole life annuity on two lives aged 55.5195 to be 10.1242 which is the required approximation by the first method.

By the second method, the values at 3 percent interest of single whole life annuities at ages 40 and 60 are found (table 22) to be 18.4247 and 10.9775. In the makehamized mortality table, the single life annuity value 18.4247 corresponds (table 41) to age 41.906, while the value 10.9775 corresponds to age 61.409. These are taken as the adjusted ages. The difference is 19.503 years, which gives 13.522 years for the addition to the younger age. Adding this to 41.906 gives 55.428 for the equivalent equal age. The value of a_{xx} at this age is 10.1594. The true value is 10.1234. In this case, it happens that the rough method gives a result closer to the true value.

Example 8.—Find the approximate value of a joint life annuity of one per annum on a white male life at age 53 and two white female lives at ages 27 and 48, on the basis of the 1939–1941 life tables for white males and white females with interest at 2 percent.

Solution.—By the rough method, the adjusted ages are 54.85, 25, and 46. By averaging the values of μ_x, the equivalent equal age is found to be 47.22, and the estimated annuity value is 12.5498.

As a first step in applying the second method, it is found (table 20) that the value of a_{53} at 2 percent interest for white males is 15.1740, while a_{27} and a_{48} at the same rate for white females are 28.4115 and 19.3285, respectively (table 31). If these are considered as single life annuity values under the makehamized mortality table with 2 percent interest (table 39), the corresponding ages would be 54.68, 24.83, and 46.04. Obtaining the values of μ_x for these ages by interpolation and averaging them gives 47.13 as the equivalent equal age. The resulting annuity value is 12.5905.

A comparison of exact values with those obtained by both methods of approximation just described for certain selected combinations of two lives is presented in table AA. As previously stated, the more refined age adjustment gives results closer to the actual values in the majority of instances, although for the case of two male lives, the rough age adjustment appears to be slightly better. The more refined method has the theoretical defect of producing values which are always in excess for two male lives and always in defect for two female lives. An improvement could no doubt be

TABLE AA.—IMMEDIATE WHOLE LIFE ANNUITIES ON TWO JOINT LIVES OF SPECIFIED SEX FOR SELECTED COMBINATIONS OF AGES—COMPARISON OF EXACT VALUES BASED ON SEPARATE LIFE TABLES FOR WHITE MALES AND WHITE FEMALES WITH APPROXIMATE VALUES OBTAINED FROM THE MAKEHAMIZED MORTALITY TABLE FOR TOTAL WHITES: UNITED STATES, 1939–1941, INTEREST AT 3 PERCENT [1]

SEX AND DIFFERENCE IN AGE	AGE OF YOUNGER LIFE								
	20			30			40		
	Exact value	Value by rough age adjustment	Value by refined age adjustment	Exact value	Value by rough age adjustment	Value by refined age adjustment	Exact value	Value by rough age adjustment	Value by refined age adjustment
Both male:									
10 years	20.0183	20.0628	20.0764	16.7529	16.7667	16.8031	13.0087	13.0177	13.0805
40 years	10.7197	10.7093	10.7419	7.1412	7.1731	7.1439	4.1470	4 2153	4.1528
Both female:									
10 years	21.3511	21.2182	21.2864	18.2946	18.1513	18.2119	14.6611	14.5549	14.5667
40 years	11.9638	11.8876	11.9531	7.9324	7.9023	7.9257	4.5751	4.5126	4.5712
Male and female (male the older):									
10 years	20.4298	20.3697	20.4145	17.1922	17.1310	17.1751	13.5055	13.4166	13.4872
40 years	10.7860	10.7481	10.7846	7.1829	7 2093	7.1804	4.1836	4.2448	4.1810

SEX AND DIFFERENCE IN AGE	AGE OF YOUNGER LIFE								
	50			60			70		
	Exact value	Value by rough age adjustment	Value by refined age adjustment	Exact value	Value by rough age adjustment	Value by refined age adjustment	Exact value	Value by rough age adjustment	Value by refined age adjustment
Both male:									
10 years	9.2616	9.2845	9.3303	5.8823	5.9324	5.9212	3.2195	3.2934	3.2466
Both female:									
10 years	10.6805	10.5446	10.5883	6.7798	6.6881	6.7185	3.6502	3.6018	3.6423
Male and female (male the older):									
10 years	9.7427	9.6887	9.7135	6.2217	6.1997	6.1909	3.3858	3.4261	3.3870

[1] The method of adjusting ages in the "rough age adjustment" and the "refined age adjustment" mentioned in the headings of this table is explained in the text, p. 98.

devised which would overcome this difficulty, but it is doubtful whether the point is of enough importance, in most practical applications, to justify sacrificing any of the simplicity and convenience of the method as given.

The estimation of joint life annuity values based on the separate life tables for white males and white females is a more complicated process when some of the lives involved are under age 17. The "exact method" described on page 95 can always be used, provided the l_x values in formulas (7) and (8) are taken from the separate life tables for males and females, and the age adjustment described in this subsection is used only in calculating the annuity value $a_{x+h:y+h:z+h:\ldots(m)}$ in formula (8). All the ages involved in this annuity are 17 or over, and the age adjustment may be made either by means of table Z (rough method) or by the more refined method just described.

If it is desired to use the shorter approximate method, described on page 96, in which an approximate value of the whole life annuity is obtained by finding an equivalent equal age and then corrected by means of the adjustment factors r_x given in table U, the equivalent equal age must be found by the more refined method last described, since the age adjustments indicated in table Z are not applicable to ages under 17. However, even the more refined method of age adjustment fails to give a definite value for the adjusted age at age 0 for males and at ages below 5 or 6 (depending on the rate of interest) for females. Here it is necessary to calculate the annuity value either by the "exact method" described on page 95, or a similar method employing in formulas (7) and (8) a small value of h sufficient to make all the augmented ages $x+h$, $y+h$, etc., at least 1 for males, and at least 5 or 6 (depending on the rate of interest) for females. The annuity value in formula (8) can then be evaluated by using the "refined" method of age adjustment to obtain an equal age and then applying the r_x factors of table U to adjust the approximate annuity value based on this equal age.

PART V

METHOD OF CONSTRUCTION AND GRADUATION OF THE LIFE TABLES

The entire process of constructing a life table consists of three major steps: (1) the preliminary adjustment of the population, birth, and death statistics which are to be used, in order to remove any errors and biases for which corrections are available or can be derived; and the approximation of certain detailed distributions of the data, needed in the computations but not available from the actual tabulations; (2) the calculation, from the adjusted data, of the rates of mortality for each year of age, which form the basis of the life tables; and (3) the computation of the remaining life table values. Of these, the first step is by far the most difficult. While the second step requires technical skill and the exercise of judgment, valuable assistance is provided by the large body of literature on the subject and the accumulated experience of actuaries in the construction of life tables. The third step involves little more than the routine application of standard formulas. However, in making the preliminary adjustment of the data, it is necessary to break new ground, as comparatively little attention has been given to this subject, and, besides, the data of each country and each epoch present their own peculiar problems, so that past experience is not a satisfactory guide.

The following description of the methods and processes used is divided into three main sections corresponding to the three major steps in the construction of a life table.

A. PRELIMINARY ADJUSTMENT OF THE DATA

In this section, the description of the various preliminary adjustments made in the data of births, deaths, and populations has been arranged in approximately the order in which the various operations were actually carried out. This order was adopted in order to avoid complicating unnecessarily the explanation of many of the steps, but does not correspond to any systematic classification of the various adjustments by either the purpose of the adjustment or the class of data involved. The adjustments made are of four types: (1) those intended to correct for incompleteness of reporting, (2) those necessitated by incomplete or inaccurate age statements, (3) those intended to eliminate roughness due to the small volume of data in certain classifications, and (4) the estimation of certain figures needed in the construction of life tables but not available from actual tabulations. Adjustments of the first type were confined to statistics of births and infant deaths. In the latter case, the adjustment of (a) the total infant deaths, and (b) the figures for subdivisions of the first year of life are separately discussed. The second type of adjustment includes the treatment of deaths for which age was not reported, and the redistribution of Negro populations and deaths at ages 55 to 69. The only adjustment of the third type was a redistribution by month of age of deaths at ages 1 month to 11 months of nonwhite infants other than Negroes. The principal adjustment of the fourth type is that made for the change in the distribution of population between April 1, 1940, the date of the census, and July 1, 1940, the date on which populations were needed for the purpose of life table construction. Also included in this category is the estimation of the distribution by single years of age of the foreign-born population under age 5, this being needed for a special purpose, as explained later.[1]

Accuracy of the data

It has been stated that the life tables in this volume are based on the results of the 1940 census of population and the tabulations of reported deaths in the continental United States for the 3 years 1939–1941. In deriving life table values for ages under 5, use was made also of the tabulations of reported births for the years 1934 to 1941, inclusive, and of deaths under 5 years of age during those years. If all these data were known to be absolutely complete and correct, the construction of life tables from them would present few problems. However, the data are affected by two main types of error: (a) incompleteness or underreporting, and (b) misstatement of age in populations and deaths, which makes the figures too large at some ages and too small at others. As will be explained later, some adjustment has been made for errors of type (b) through the graduation of the data, and, in the case of the Negro data, by a preliminary redistribution of the numbers in certain age groups for which this type of error was believed to be especially marked. Except in the case of statistics of births and infant deaths (those occurring at ages under 1 year), no attempt has been made to adjust for errors of type (a).

If it should happen that the enumeration of the population and the reporting of deaths were both deficient by exactly the same percent, the use of the unadjusted figures would produce exactly the correct mortality rates. However, if the reporting of deaths should be more complete than the enumeration of

[1] See p. 119.

population, the rates of mortality would be overstated by using the reported figures. If, on the contrary, the enumeration of population should be more complete than the registration of deaths, the mortality rates would be understated. Using the unadjusted data thus involves the assumption that the reporting of deaths and the enumeration of population have the same degree of completeness. It would be a remarkable coincidence if this were exactly true. It would be even more remarkable if it were true, not only in the aggregate but within each of the various subdivisions by sex, race, and age, for which rates of mortality have been calculated. This assumption has been made then, not because it is believed to be precisely correct, but because specific information regarding the relative completeness of death reporting and census enumeration is almost entirely lacking.

Completeness of birth registration

It has long been recognized that the census enumeration of children under 5, and particularly of those under 1 year, is markedly deficient. This is illustrated by the following figures relating to the 1940 census. The total native population enumerated as under 1 year of age on April 1, 1940, the date of the census, is closely [2] estimated as 2,019,662. The same population estimated from registered births and deaths during the year ending April 1, 1940, is 2,192,557, which exceeds the census figure by 172,895. Since it is known that birth registration is not entirely complete, the deficiency in the census enumeration of children under 1 year of age is actually greater than that num-

[2] The only estimation involved is in determining the deduction for foreign-born nonwhites which are given only by 5-year age groups and only for the principal nonwhite races. By the most liberal estimate, the number of these is less than 100.

TABLE AB.—REGISTERED AND ADJUSTED BIRTHS, 1939–1941, AND PERCENT COMPLETENESS OF BIRTH REGISTRATION, DEC. 1, 1939, TO MAR. 31, 1940, FOR WHITE AND NONWHITE, BY STATES

STATE	Registered births, 1939–1941	Percent completeness,[1] Dec. 1, 1939, to Mar. 31, 1940	Adjusted births, 1939–1941	STATE	Registered births, 1939–1941	Percent completeness,[1] Dec. 1, 1939, to Mar. 31, 1940	Adjusted births, 1939–1941
WHITE							
Alabama	116,987	86.6	135,089	Nebraska	65,183	97.0	67,199
Arizona	29,695	93.8	31,658	Nevada	5,820	97.5	5,969
Arkansas	87,231	79.6	109,587	New Hampshire	24,651	98.6	25,001
California	325,818	98.1	332,128	New Jersey	170,310	99.0	172,030
Colorado	62,242	89.8	69,312	New Mexico	42,192	91.2	46,263
Connecticut	76,401	99.4	76,862	New York	562,717	99.0	568,401
Delaware	11,655	97.2	11,991	North Carolina	165,346	88.4	187,043
District of Columbia	22,038	98.5	22,374	North Dakota	38,013	94.6	40,183
Florida	73,363	91.3	80,354	Ohio	331,037	95.3	347,363
Georgia	119,035	83.6	142,386	Oklahoma	120,695	87.0	138,730
Idaho	34,248	95.1	36,013	Oregon	52,253	97.3	53,703
Illinois	358,550	97.3	368,499	Pennsylvania	471,585	97.2	485,170
Indiana	178,893	96.6	185,189	Rhode Island	32,109	98.8	32,499
Iowa	133,517	94.7	140,989	South Carolina	68,192	82.7	82,457
Kansas	85,354	95.6	89,282	South Dakota	33,982	96.6	35,178
Kentucky	178,200	89.2	199,776	Tennessee	142,185	81.4	174,674
Louisiana	90,537	87.7	103,235	Texas	336,566	89.3	376,894
Maine	46,148	96.3	47,921	Utah	39,265	97.1	40,438
Maryland	78,610	97.8	80,378	Vermont	20,477	97.3	21,045
Massachusetts	195,356	98.9	197,529	Virginia	125,357	92.5	135,521
Michigan	288,311	97.9	294,495	Washington	83,240	98.0	84,939
Minnesota	155,394	99.3	156,489	West Virginia	121,724	86.7	140,397
Mississippi	71,006	93.8	75,699	Wisconsin	164,322	96.9	169,579
Missouri	172,456	90.7	190,139	Wyoming	15,157	95.9	15,805
Montana	32,104	98.0	32,759				
NONWHITE [2]							
Alabama	72,008	82.6	87,177	Missouri	12,521	82.7	15,140
Arizona	4,289	48.4	8,862	Montana	1,994	91.1	2,189
Arkansas	27,434	63.2	43,408	Nebraska	980	93.1	1,053
California	15,264	96.5	15,818	New Jersey	14,049	98.7	14,234
Colorado	803	90.4	888	New Mexico	1,565	40.3	3,883
Connecticut	1,938	97.9	1,980	New York	30,664	96.5	31,776
Delaware	2,297	98.6	2,330	North Carolina	78,837	81.0	97,330
District of Columbia	12,833	96.9	13,285	North Dakota	1,272	95.2	1,336
Florida	30,331	86.4	35,105	Ohio	18,648	93.7	19,902
Georgia	78,035	77.6	100,561	Oklahoma	13,572	66.9	20,287
Illinois	21,076	90.6	23,263	Oregon	779	84.1	926
Indiana	6,544	94.0	6,962	Pennsylvania	28,842	92.9	31,046
Iowa	797	90.1	885	Rhode Island	746	100.0	746
Kansas	3,236	92.9	3,483	South Carolina	66,791	71.8	93,024
Kentucky	9,819	87.6	11,209	South Dakota	2,271	79.8	2,846
Louisiana	63,784	83.7	76,205	Tennessee	25,921	75.1	34,515
Maryland	20,947	94.1	22,260	Texas	48,450	68.7	70,524
Massachusetts	2,922	98.0	2,982	Virginia	46,994	90.2	52,100
Michigan	12,470	94.0	13,266	Washington	2,079	88.7	2,344
Minnesota	2,001	97.2	2,059	West Virginia	6,297	81.3	7,745
Mississippi	88,102	86.2	102,206	Wisconsin	1,949	93.2	2,091

[1] Grove, Robert D., *Studies in Completeness of Birth Registration, Part I, Completeness of Birth Registration, United States, Dec. 1, 1939, to Mar. 31, 1940*, U. S. Bureau of the Census, Vital Statistics—Special Reports, vol. 17, No. 18, p. 228, 1943.

[2] The States of Idaho, Maine, Nevada, New Hampshire, Utah, Vermont, and Wyoming, each of which reported less than 500 nonwhite births in the period 1939–1941 are omitted.

ber. For this reason birth statistics were relied upon in obtaining a population base for the rate of mortality in the first year. This raises the question as to how completely births are reported.

Following the 1940 census, there became available for the first time reliable information as to the completeness of birth registration in the United States. This information was obtained by preparing special infant cards for all infants enumerated in the census who were under 4 months of age on April 1, 1940, and by matching these cards against copies of the birth certificates for all births reported as having occurred between December 1, 1939, and April 1, 1940. Copies of all death certificates of infants born in this 4-month period were also obtained, and matched where possible with the birth certificates. Table AB shows, for white and nonwhite separately, the number of births reported in each State in the 3-year period 1939–1941, the percent completeness of birth registration as indicated by the test just described, and the adjusted number of births obtained by dividing the number of registered births by the proportion of births registered. In the case of the nonwhite, those States in which less than 500 nonwhite births were reported in the 3-year period have been omitted from the table.

Further tabulations were made for a special sample of infant cards, which yield the completeness of birth registration by a more detailed racial classification for the United States as a whole. This sample did not include matching with death records; and, for this reason, the results obtained are probably somewhat more suitable for use in adjusting birth statistics to be employed in the construction of life tables, since those infants whose deaths are registered probably constitute a biased sample from the standpoint of birth registration. Table AC shows, for whites, Negroes, and other races separately, the number of births reported in the 3-year period in the continental United States, the percent completeness of registration as obtained from the tabulation of the sample, and the adjusted number of births obtained by dividing the registered figure by the indicated proportion of births registered.

TABLE AC.—REGISTERED AND ADJUSTED BIRTHS, 1939–1941, BY RACE, AND PERCENT COMPLETENESS OF BIRTH REGISTRATION (EXCLUDING MATCHED INFANT DEATH RECORDS), DEC. 1, 1939, TO MAR. 31, 1940: UNITED STATES

RACE	Registered births	Percent[1] completeness, Dec. 1, 1939, to Mar. 31, 1940	Adjusted births
White	6,255,527	93.98	6,656,232
Negro	843,483	81.87	1,030,271
Other races	40,404	75.05	53,836

[1] Based on tabulation of special sample.

Completeness of registration of infant deaths

It has already been mentioned that all death statistics were used without any adjustment for incompleteness of reporting, with the exception of infant deaths: that is, those occurring under 1 year of age. In the construction of all the life tables prepared by the Bureau of the Census prior to 1940, even infant deaths were not adjusted for underreporting. However, there is evidence that the proportion of infant deaths not reported is sufficiently large to have an appreciable effect on life table values, and it appears that the former practice of relating fully adjusted birth data to unadjusted infant death statistics has resulted in a substantial understatement of the rate of mortaility at age 0.

The problem of making a proper adjustment for incomplete reporting of infant deaths is a difficult one, because almost no information is available bearing directly on the point, and an indirect method of approach must be resorted to. This approach is based on an examination of infant mortality rates for subdivisions of the first year of life. Table AD shows, for each State included in table AB, the number of deaths occurring in the 3-year period 1939–1941 in each of seven subdivisions of the first year of life, per 1,000 adjusted births (table AB) in the same period. With the exception of the column pertaining to deaths under 1 day of age, these figures cannot be regarded as mortality rates in the true sense of the word, as the denominator used was, in each case, the number of births for the year, and not the number of survivors to the beginning of the age period indicated. However, this refinement would have comparatively little effect on the comparison between States, which is the chief purpose in view.

For convenience in making comparisons, the various States appear in table AD in decreasing order of the completeness of birth registration. A careful study of the table shows that there is a close relationship between the completeness of birth registration and the actual level of infant mortality in the various States. For example, if the 48 States and the District of Columbia are ranked also according to the mortality rate among white infants 9 to 11 months of age, it is found that of the 10 States having the most complete registration, 5 are also among the 10 having the lowest mortality rates. Likewise, among the 10 having least complete registration, 4 are also among the 10 having the highest mortality rates. This is not surprising, because, generally speaking, those States having the most efficient registration are States in which sanitation and public health measures have made relatively greater progress.

TABLE AD.—DEATHS UNDER 1 YEAR PER 1,000 ADJUSTED BIRTHS, BY AGE: EACH STATE, 1939–1941

STATE	AGE AT DEATH: Under 1 day	1 day to 1 week	1 week to 1 month	1 and 2 months	3 to 5 months	6 to 8 months	9 to 11 months
WHITE							
Connecticut	12.5	8.1	3.2	3.3	3.1	1.7	1.0
Minnesota	13.2	7.5	3.3	3.7	2.9	1.8	1.2
New Jersey	11.6	7.6	3.8	3.6	3.5	2.0	1.4
Massachusetts	12.3	8.1	3.9	4.2	3.5	2.2	1.6
Rhode Island	13.0	7.8	3.5	5.0	3.6	1.9	1.2
New York	12.7	7.9	3.6	4.1	3.3	1.9	1.3
New Hampshire	14.0	9.2	4.1	5.5	4.5	2.4	1.6
District of Columbia	15.2	8.0	7.5	4.2	2.8	1.7	1.1
California	13.8	7.3	3.8	4.4	4.4	2.6	1.7
Montana	14.2	7.6	4.5	4.8	4.2	2.1	1.8
Washington	12.4	8.1	3.4	3.8	3.0	1.8	1.0
Michigan	12.2	8.6	4.5	5.0	4.2	2.6	1.7
Maryland	11.8	7.7	5.0	5.4	4.8	3.1	2.2
Nevada	16.1	8.7	1.5	5.9	5.7	2.7	1.8
Illinois	12.3	7.6	3.4	3.7	3.3	2.1	1.5
Oregon	11.4	7.2	2.7	3.6	2.8	2.0	1.3
Vermont	14.4	7.9	5.9	6.1	4.5	2.5	1.8
Delaware	9.1	7.8	3.3	5.2	4.4	3.9	2.8
Pennsylvania	13.4	8.7	4.7	5.2	4.7	2.7	1.8
Utah	13.8	7.8	3.5	3.2	3.5	1.9	1.3
Nebraska	12.9	7.4	3.5	3.8	3.5	1.9	1.2
Wisconsin	13.0	7.7	3.7	4.4	3.7	2.0	1.2
Indiana	11.0	8.6	4.3	5.0	4.5	2.9	2.0
South Dakota	13.9	7.6	3.4	3.8	3.4	1.9	1.0
Maine	13.2	12.4	5.6	7.6	6.2	3.0	2.4
Wyoming	14.0	8.2	3.4	4.1	4.9	2.5	2.3
Kansas	12.9	7.3	3.3	4.2	4.0	2.3	1.6
Ohio	12.6	8.0	4.5	4.7	4.3	2.7	1.9
Idaho	14.2	8.1	3.4	5.2	4.3	2.0	1.2
Iowa	13.0	7.4	3.7	4.1	3.6	2.1	1.1
North Dakota	13.7	7.7	4.9	5.5	4.5	2.0	1.5
Arizona	14.9	8.3	6.6	11.0	14.3	9.9	5.1
Mississippi	14.5	9.2	5.4	5.6	4.5	3.5	2.5
Virginia	14.9	9.1	5.9	6.4	6.0	3.3	2.4
Florida	14.0	7.7	4.6	4.8	4.3	3.1	2.2
New Mexico	17.7	10.9	9.9	14.5	18.4	11.5	6.8
Missouri	12.2	7.3	4.8	5.4	4.8	3.0	2.2
Colorado	14.0	8.3	5.1	7.8	7.8	4.3	2.5
Texas	12.8	9.4	6.0	7.7	8.8	6.0	4.5
Kentucky	10.2	10.7	6.9	7.1	5.9	3.9	2.8
North Carolina	12.6	8.4	5.3	6.2	5.9	3.7	2.4
Louisiana	13.0	7.1	5.0	6.0	4.9	2.9	1.9
Oklahoma	12.5	7.8	5.0	5.3	4.3	2.7	2.3
West Virginia	11.4	9.1	6.1	7.5	6.8	3.9	2.7
Alabama	13.5	8.6	5.3	6.3	4.8	3.0	2.4
Georgia	11.6	9.0	4.9	5.5	4.4	2.9	2.3
South Carolina	12.5	9.3	5.0	6.5	5.8	3.4	2.4
Tennessee	10.3	7.5	5.2	6.1	5.1	3.6	2.5
Arkansas	9.2	6.6	4.2	4.7	4.4	3.2	2.5
NONWHITE [1]							
Rhode Island	21.4	6.7	5.4	8.0	8.0	5.4	6.7
New Jersey	19.0	14.4	6.3	9.6	9.3	5.7	3.9
Delaware	15.5	15.5	6.0	13.7	13.3	8.6	7.7
Massachusetts	16.4	10.4	8.7	6.0	5.7	8.0	4.0
Connecticut	17.2	13.6	7.1	5.6	5.1	5.6	2.0
Minnesota	15.5	6.8	8.3	9.7	9.7	10.7	11.2
District of Columbia	19.5	13.1	12.8	10.5	9.6	5.0	2.5
California	11.6	8.3	5.1	6.7	6.8	4.9	2.2
New York	17.8	11.3	4.8	7.9	6.6	3.7	2.2
North Dakota	13.5	12.0	15.0	15.7	18.0	9.7	9.0
Maryland	15.7	12.9	7.6	12.2	12.7	10.9	8.0
Indiana	13.5	11.8	9.0	8.2	8.6	5.5	3.4
Michigan	15.4	10.6	6.1	4.2	7.2	4.1	3.2
Ohio	16.8	10.7	7.1	7.8	7.9	5.4	3.0
Wisconsin	13.4	14.3	6.7	10.5	14.8	12.9	7.2
Nebraska	18.0	16.1	6.6	9.5	16.1	4.7	4.7
Kansas	13.5	10.3	8.3	9.2	10.6	4.9	4.6
Pennsylvania	19.1	10.7	6.2	8.1	8.8	5.7	3.3
Montana	10.1	12.3	8.2	23.8	23.3	18.3	11.0
Illinois	14.8	10.4	4.3	5.2	6.8	3.3	2.8
Colorado	16.9	10.1	3.4	11.3	4.5	9.0	2.3
Virginia	14.7	13.9	10.2	12.1	12.6	8.6	5.5
Iowa	19.2	9.0	4.5	9.0	10.2	7.9	4.5
Washington	14.5	10.2	7.3	15.8	12.4	11.9	10.2
Kentucky	11.7	17.6	10.8	11.5	10.9	7.9	5.3
Florida	14.7	14.7	10.7	9.2	7.7	5.8	3.8
Mississippi	11.1	9.2	6.4	8.2	8.5	5.4	3.9
Oregon	19.4	10.8	10.8	13.0	15.1	15.1	5.4
Louisiana	14.5	11.7	12.4	10.8	10.1	5.9	3.9
Missouri	15.4	9.4	10.0	10.2	9.8	6.5	4.8
Alabama	15.2	11.8	7.8	9.3	8.7	6.0	3.6
West Virginia	15.2	13.8	9.3	10.7	10.6	7.7	4.0
North Carolina	12.5	10.2	8.1	10.5	10.2	6.5	4.2
South Dakota	6.0	11.2	9.8	15.1	13.7	13.7	12.6
Georgia	12.4	11.3	8.3	8.8	7.8	5.0	2.9
Tennessee	11.9	10.6	6.4	8.8	8.6	6.2	4.5
South Carolina	10.8	11.8	8.1	9.3	10.9	6.9	4.0
Texas	11.1	10.7	8.7	8.1	7.9	4.6	3.1
Oklahoma	8.4	10.5	7.8	8.3	8.0	5.3	4.3
Arkansas	5.6	5.0	4.1	5.6	6.2	4.2	3.1
Arizona	7.7	9.0	12.5	16.1	20.1	15.8	13.2
New Mexico	13.1	11.1	8.0	12.4	16.0	13.4	9.5

[1] See footnote 2 to table AB, p. 102.

However, if the comparison is made with the mortality rates for infants under 1 day old, instead of those aged 9 to 11 months, just the opposite tendency is observed, the lower mortality rates being recorded, in general, in the States with less complete registration of births. For example, among the 10 States having the least complete registration of white births, 5 were also among the lowest 10 when ranked according to the mortality rate for white infants under 1 day old. It might be expected that mortality rates for infants in the first day of life would fail to show the close relationship to the completeness of birth registration which was observed in the case of the rates for infants 9 to 11 months old, because a large proportion of deaths occurring immediately after birth are due, at least in part, to mechanical causes connected with the process of childbirth. The great improvement in infant mortality in recent years has, in fact, affected the frequency of neonatal deaths to a much less degree than that of deaths occurring later in infancy.

It is not, however, to be expected that the death rate in very early infancy would be totally unaffected by varying conditions in the environment. Still less can it be thought that the normal relationship is actually reversed,[3] the lower mortality rates occurring where conditions are less favorable. It is necessary, therefore, to look for some source of error in the mortality rates for the first day of life as shown in table AD. Inasmuch as these rates were obtained from births corrected for incomplete registration, but without any corresponding adjustment in the death statistics, the most natural inference is that deaths occurring in early infancy are affected by an incompleteness of reporting having, in general, the same geographical incidence as in the case of births.

The relationships which led to this conclusion are brought out more clearly in table AE, which shows the results of arranging the States in the order of the percent completeness of birth registration and then combining them into five groups (three groups in the case of nonwhites) in such a way that the total number of reported deaths under 1 year of age is approximately the same for each group used. In the case of the data for white lives, the States of Arizona, New Mexico, and Texas have been omitted, because in these States the mortality rates for white infants in the latter part of the first year of life are so much higher than those for other States that the general relationship would be obscured by their inclusion. This condition is believed to be due to the presence, in the white population of these States, of a large number of Mexican agricultural workers in low income groups, among whom the rate of infant mortality is extremely high. Except for the omission of these 3 States from group 4, the spacing in table AD indicates the particular States included in

[3] There are certain factors tending to cause fewer deaths in the first day of life when the general infant death rate is high. For example, there are probably fewer instrumental deliveries in areas of high mortality. However, the effect of such factors is believed to be small.

each group. In making the calculations for nonwhites, the 7 States having less than 500 nonwhite births in the 3-year period, which were omitted from tables AB and AD, were again omitted here. Upon examining the part of table AE which shows data for the white population, it is observed that the percent completeness of birth registration decreases rather slowly in the first three groups and then falls at an accelerating pace as groups 4 and 5 are reached. The States in group 5 (where registration is least complete) contain only 18 percent of the adjusted white births in all five groups but contain 45 percent of the assumed unregistered (adjusted less registered) births. Above the age of 1 week, the death rates based on adjusted births rise consistently from group 1 to group 5, but in the case of deaths in the first day of life, the rates begin to decrease with group 4, and group 5 actually shows the lowest death rate of the five groups. In the age period 1 day to 1 week, a less marked but similar tendency is observed. The behavior of the rates for these two youngest age periods strongly suggests that the decline which appears in groups 4 and 5 may be spurious, and attributable, as already intimated, to incomplete reporting of deaths occurring in early infancy in these States. Among the nonwhite, the tendency toward lower apparent death rates in those States having less complete registration of births is more marked, and persists throughout the entire first year of life.

TABLE AE.—DEATHS UNDER 1 YEAR PER 1,000 ADJUSTED BIRTHS, BY AGE AND RACE, 1939–1941, FOR GROUPS OF STATES ARRANGED ACCORDING TO THE COMPLETENESS OF BIRTH REGISTRATION

	WHITE [1]					NONWHITE [2]		
	State group [3]					State group [3]		
	1	2	3	4	5	1	2	3
Percent of total deaths under 1 year for all groups	18.1	19.6	18.4	22.8	21.1	31.3	36.4	32.3
Percent completeness of birth registration	99.0	97.7	96.8	92.8	85.0	92.7	83.3	71.0
Deaths per 1,000 adjusted births:								
Under 1 day	12.6	12.7	12.9	12.8	11.8	15.8	13.3	10.6
1 day to 1 week	7.9	7.8	8.4	8.4	8.2	12.4	10.6	10.4
1 week to 1 month	3.7	3.9	4.2	5.0	5.2	7.8	8.6	7.8
1 and 2 months	4.0	4.4	4.9	5.5	6.0	9.2	9.7	8.7
3 to 5 months	3.3	3.9	4.4	4.9	5.2	9.3	9.4	8.9
6 to 8 months	2.0	2.4	2.5	3.0	3.3	6.4	6.0	5.8
9 to 11 months	1.3	1.6	1.7	2.1	2.4	4.2	3.9	3.9

[1] The States of Arizona, New Mexico, and Texas were omitted from the computations for white lives. See text, p. 104.
[2] Those States reporting less than 500 births of nonwhites in 1939–1941 were omitted from the computations. See footnote to table AB, p. 102.
[3] Higher numbers indicate less complete registration, as shown in the second line of the table.

In summary, it may be stated that the preceding analysis appears to show: (1) that there is substantial underreporting of infant deaths, (2) that this underreporting tends, in general, to be greater in those States in which underreporting of births is greater, and (3) that it is relatively greater in the case of deaths occurring in the first week of life than for those which occur later. However, it is not sufficient, for the purpose of life table construction merely to know that such a condition exists. It is necessary also to make some assumption as to the magnitude of the underreporting. As no information was available from which this could be estimated directly, an effort was made to estimate it indirectly by assuming the percent of nonreporting of infant deaths to be some fixed proportion or multiple, State by State, of the percent of nonreporting of births and adjusting the State death rates in accordance with that assumption, and then examining the death rates based on various assumed proportions or multiples to see which produced results most nearly in accordance with expectation. It was considered that, when adequately adjusted, the State death rates in each age period should show a consistent tendency to increase with decreasing completeness of birth registration, since, in general, the States having more complete registration are also those with better sanitation and public health facilities.

Such calculations were made for the first three age periods employed in table AD, which together comprise the first month of life. Since the individual State figures show minor fluctuations which make it difficult to observe the general tendency, these calculations were made for the same groups of States which were used in table AE. Three different sets of calculations were made, based on the assumption that the percent adjustment required for incomplete reporting of deaths in each age period was (a) 50 percent, (b) 100 percent, and (c) 150 percent of the corresponding percent adjustment required for births in the same State. The results of the calculations are shown in table AF.

In the case of white infant deaths under 1 day, adjustment in accordance with assumption (a) still leaves group 5 with a lower death rate than group 4. Assumption (b) produces a death rate in group 5 which is slightly higher than that in group 4, but the difference is much less than might reasonably be expected in view of the substantial difference in the completeness of birth registration in the two groups. In the case of the nonwhite, even assumption (c) fails to produce increasing death rates, although it tends in that direction, indicating that a more drastic adjustment would do so. The implication of these observations that deaths occurring in the first day of life may be less completely registered than births is less startling than it may at first appear, if one considers that there is probably a substantial number of cases of very early death, especially in rural areas and among the more underprivileged classes, in which neither the birth nor the death is registered. This group would of course constitute a much larger percent of the total infant deaths than of the total births. It is also possible that there may be a tendency, in some States, to report such cases as stillbirths.

In the case of white deaths at ages 1 day to 1 week, assumption (a) gives death rates which increase, but not by a sufficient amount, while the rates resulting from assumption (b) appear reasonable. For the nonwhite, assumption (c) at least seems to be called for. The deaths of white infants aged 1 week to 1 month yield increasing rates even without adjustment, and it is somewhat difficult to judge which assumption produces the most plausible rates. However, one would expect these deaths also to be somewhat less completely reported than those occurring after the first year of life, and to require some adjustment.

The age distribution of deaths of white infants in the period 1939–1941 was 31 percent under 1 day, 20 percent from 1 day to 1 week, and 49 percent from 1 week to 1 year. As indicated in the foregoing discussion, it may be assumed for illustrative purposes on the basis of table AF that the percent adjustment required for incomplete reporting of infant deaths was related as follows to the corresponding percent adjustment for births:

Age at death	*Number of times the percent adjustment for births*
Under 1 day	1½
1 day to 1 week	1
1 week to 1 year	½

This gives for the average percent adjustment for incomplete reporting of white infant deaths $.31\times 1.5+.20\times 1+.49\times 0.5$, or approximately 91 percent[4] of the corresponding percent adjustment for white births. Since reporting of white births was found to be about 94 percent complete (corresponding to an adjustment of 6.4 percent for incomplete reporting, see table AC), this would imply that white infant deaths were about 94.5 percent completely reported.

Similarly, the age distribution of nonwhite infant deaths was 22 percent under 1 day, 18 percent from 1 day to 1 week, 13 percent from 1 week to 1 month, and 47 percent from 1 month to 1 year; and the required percent adjustment for incomplete reporting of deaths of nonwhite infants may be assumed to be related as follows to the corresponding percent adjustment for births:

Age at death	*Number of times the percent adjustment for births*
Under 1 day	2
1 day to 1 week	1½
1 week to 1 month	1
1 month to 1 year	½

This would give for the average percent adjustment for incomplete reporting of nonwhite infant deaths $.22\times 2+.18\times 1.5+.13\times 1+.47\times 0.5$, or approximately 107.5 percent[4] of the corresponding percent adjustment for nonwhite births. Since reporting of nonwhite births was found in 1940 to be about 82 percent complete, this would mean that on the assumptions made deaths of nonwhite infants were slightly under 81 percent complete. These assumptions are, of course, rough, and such a calculation can be no more than suggestive; however, it does indicate that, in the absence of accurate information on the completeness of registration of infant deaths, it is not unreasonable to assume that for the first year of life taken as a whole the percent completeness of registration of white deaths is the same as that of white births. This assumption is probably as accurate as could be expected with the meager information available, and leads to some simplification in the numerical computation. Accordingly, it was adopted in the preparation of the life tables in this volume. As a matter of convenience, it was used for nonwhites as well as whites, although a somewhat larger correction for nonwhites might be justified.

It should be pointed out that although this assumption is considered appropriate for the data of the United States as a whole, this does not imply that it could properly be employed for separate States, areas, or regions. It is probable that the relationship between the completeness of registration of births and that of infant deaths varies widely in different localities. It is likely, for example, that in highly urban areas where registration is a well established practice, registration of infant deaths is more complete than birth registration. On the contrary, there are indications that the reverse is true in rural areas. Such an indication is found, for example, in the comparison of infant mortality rates by population groups classified according to size.[5] Although these rates tend, in general, to

TABLE AF.—DEATHS UNDER 1 MONTH PER 1,000 ADJUSTED BIRTHS, BY AGE AND RACE, 1939–1941, ON VARIOUS ASSUMPTIONS AS TO THE COMPLETENESS OF DEATH REGISTRATION, FOR GROUPS OF STATES ARRANGED ACCORDING TO THE COMPLETENESS OF BIRTH REGISTRATION

	WHITE [1]					NONWHITE [2]		
	State group [3]					State group [3]		
	1	2	3	4	5	1	2	3
Percent completeness of birth registration	99.0	97.7	96.8	92.8	85.0	92.7	83.3	71.0
Deaths under 1 day:								
Unadjusted	12.6	12.7	12.9	12.8	11.8	15.8	13.3	10.6
Adjusted [4] according to:								
Assumption (a)	12.7	12.8	13.1	13.3	12.8	16.4	14.7	12.7
Assumption (b)	12.7	12.9	13.3	13.8	13.9	17.1	16.0	14.0
Assumption (c)	12.8	13.1	13.5	14.3	14.9	17.7	17.3	16.6
Deaths after 1 day to 1 week:								
Unadjusted	7.9	7.8	8.4	8.4	8.2	12.4	10.6	10.4
Adjusted [4] according to:								
Assumption (a)	7.9	7.9	8.6	8.8	8.9	12.9	11.7	12.6
Assumption (b)	7.9	8.0	8.7	9.1	9.6	13.4	12.8	14.7
Assumption (c)	8.0	8.1	8.8	9.4	10.3	13.9	13.9	16.4
Deaths after 1 week to 1 month:								
Unadjusted	3.7	3.9	4.2	5.0	5.2	7.8	8.6	7.8
Adjusted [4] according to:								
Assumption (a)	3.7	3.9	4.3	5.2	5.6	8.2	9.4	9.4
Assumption (b)	3.8	4.0	4.4	5.4	6.1	8.5	10.3	11.1
Assumption (c)	3.8	4.0	4.4	5.7	6.5	8.8	11.2	12.3

[1] The States of Arizona, New Mexico, and Texas were omitted from the computations for white lives. See text, p. 104.
[2] Those States reporting less than 500 births of nonwhites in 1939–1941 were omitted from the computations. See footnotes to table AB, p. 102.
[3] Higher numbers indicate less complete registration of births.
[4] Assumptions (a), (b), and (c) suppose that the percent adjustment needed to correct for incompleteness of reporting of deaths in each State in the indicated age period is, respectively, 50, 100, and 150 percent of that required for births in the same State.

[4] Strictly speaking, the proportions of infant deaths occurring in the three age periods used in this calculation should be based on total infant deaths (after adjustment for underreporting). Allowance for this factor would slightly increase the resulting average.

[5] See, for example, Forrest E. Linder and Robert D. Grove, *Vital Statistics Rates in the United States, 1900–1940*, table 28, p. 578, Government Printing Office, Washington, D. C., 1943.

increase steadily with diminishing population size, the rates for rural areas are usually somewhat lower than those for the smallest urban places.[6] It is doubtful if this can be wholly explained on the basis of faulty allocation by residence, since the rates are based not on census populations but on births, which should be affected by errors in allocation in the same direction as infant deaths.

Method of adjustment of infant data

Inasmuch as the statistics of births and infant deaths were assumed to be equally complete, mortality rates at age 0 were obtained directly from the reported figures. However, as previously stated, the populations at ages 1 to 4 used in determining the number exposed to risk at those ages were not obtained from the census, but were calculated from birth and death statistics. To the extent that they entered into the calculation of populations at these subsequent ages, the statistics of births and infant deaths required some adjustment. The method followed was to compute, from reported figures only, the number of survivors to the exact age of 1 year from each year's births, and then to increase this number of survivors by the desired percentage before extending the calculations to higher ages. The method of determining the adjustment factors to be applied to the number of survivors at age 1 will now be described.

On first consideration, it might appear that the percents of completeness of birth registration obtained from the birth registration study could be used as divisors to obtain the corrected number of survivors. However, such a procedure would not be consistent with the assumptions being made in connection with ages 5 and above. At these ages it is not assumed that the census figures and the registered deaths are 100 percent complete, but rather that both have the same percent of incompleteness. Since it is not considered that deaths at ages 1 to 4 are reported any more completely than those at ages 5 and above, the populations to be used in rate computations at ages 1 to 4 should not be corrected to a higher degree of completeness than the census populations at ages 5 and over, if a consistent series of mortality rates is to be produced.

In order to determine the proper adjustment factors, a calculation was made, by two independent methods, of the survivors to exact age 1 out of the births corresponding to the 1940 census population at each single year of age from 1 to 9, inclusive. For example, the native population at age 5 (that is, between the fifth and sixth birthdays) on the census date, April 1, 1940, are survivors of babies born in the year April 1, 1934, to April 1, 1935. The survivors to exact age 1 of this group of births were estimated (a) by subtracting from the reported births of that period the reported infant deaths occurring among this group of lives and (b) by adding to the native population aged 5 on April 1, 1940, as enumerated in the census, the reported deaths among this group of lives after age 1, but before April 1, 1940. Similar calculations were made for the groups at each of the other ages under 10 in the 1940 census. Table AG shows the results, which are given separately for the three racial groups: whites, Negroes, and other races. It will be observed that the ratio of estimate (a) to estimate (b) falls sharply from birth to age 3, but from age 3 to age 9 merely fluctuates without showing any consistent trend. It shows, however, a marked tendency to be low at even ages and high at odd ages. This suggests that the fluctuation may be principally due to preference for certain ages in the census and that the ratio might be very nearly constant except for this disturbance. At the very young ages, where the ratio is particularly high, the census enumeration is known to be markedly deficient.

[6] The suggestion has sometimes been made that this may be a genuine phenomenon. See, for example, Herbert J. Sommers, *Infant Mortality in Rural and Urban Areas*, Public Health Reports, vol. 57, No. 40, p. 1498, October 1942.

TABLE AG.—COMPARISON OF SURVIVORS TO AGE 1 AS ESTIMATED BY TWO METHODS: UNITED STATES, 1930–1939

Time period in which births occurred (Apr. 1 to Mar. 31)	Age on April 1, 1940, in completed years	Survivors to first birthday: Method (a) (based on registered births and deaths)	Survivors to first birthday: Method (b) (based on 1940 census enumeration and registered deaths)	Ratio (a) ÷ (b)
WHITE				
1939–1940	0	[1] 1,924,622	[2] 1,777,738	1.083
1938–1939	1	1,907,032	1,820,840	1.047
1937–1938	2	1,859,609	1,932,336	.962
1936–1937	3	1,776,542	1,862,403	.954
1935–1936	4	1,797,748	1,892,182	.950
1934–1935	5	1,807,799	1,894,413	.954
1933–1934	6	1,722,081	1,808,031	.952
1932–1933	7	1,784,366	1,862,620	.958
1931–1932	8	1,862,495	1,964,620	.948
1930–1931	9	1,911,381	1,974,105	.968
1930–1937	3–9	12,662,412	13,258,347	.9551
NEGRO				
1939–1940	0	[1] 255,798	[2] 229,795	1.113
1938–1939	1	247,842	230,601	1.075
1937–1938	2	243,215	267,545	.909
1936–1937	3	230,240	263,205	.875
1935–1936	4	233,585	272,955	.856
1934–1935	5	236,128	266,885	.885
1933–1934	6	224,318	270,927	.828
1932–1933	7	229,476	265,838	.863
1931–1932	8	227,439	272,604	.834
1930–1931	9	225,908	256,179	.882
1930–1937	3–9	1,607,094	1,868,593	.8601
OTHER RACES				
1939–1940	0	[1] 12,137	[2] 12,129	1.001
1938–1939	1	11,882	11,576	1.026
1937–1938	2	11,315	13,511	.837
1936–1937	3	10,474	13,345	.785
1935–1936	4	10,609	13,866	.765
1934–1935	5	10,799	13,335	.810
1933–1934	6	10,214	12,983	.787
1932–1933	7	10,571	12,805	.826
1931–1932	8	10,743	13,103	.820
1930–1931	9	10,980	12,332	.890
1930–1937	3–9	74,390	91,769	.8106

[1] Survivors to Apr. 1, 1940.
[2] 1940 census population under 1 year of age.

An average ratio was therefore obtained for each racial group based on the totals of estimates (a) and (b) for the entire age group 3 to 9 in 1940. These average ratios (also shown in table AG) were then used as divisors, in the construction of the life tables, to inflate the number of each group of survivors to age 1, as calculated from births and deaths, to the general level of completeness of the census. The populations at age 1, 2, 3, and 4 used in the actual life table calculations were derived from age 1 survivors adjusted in this manner.

In this method of adjustment it is implicitly assumed that the completeness of birth registration, relative to that of enumeration in the census, did not improve during the decade 1930 to 1940. Similar calculations were also made on the assumption of a progressive improvement in birth registration during the decade, adjusting the reported births of earlier years up to the level of completeness of 1940. This produced a series of ratios (of survivors calculated by the two methods) decreasing with increasing age, which would imply that the enumeration in the 1940 census at ages under 10 became less complete with advancing age. This seems absurd; but, on the other hand, it appears unlikely that there was no improvement during the decade in the completeness of birth registration. As the number of deaths entering into the calculation is small in relation to the total survivors, the completeness of death registration is not an important factor. In view of these inconsistencies in the data, it seemed expedient to adopt the simplest course and assume, for this purpose, no change during the decade in the completeness of birth registration.

Adjustment for incomplete reporting of infant deaths by subdivisions of the first year of life

Statistics of infant deaths for subdivisions of the first year of life were used in computing life table values for such subdivisions, as will be explained later.[7] It has already been mentioned that neither births nor infant deaths were corrected for underreporting in obtaining mortality rates for the first year of life as a whole, the assumption being made that reported statistics of births and of deaths under 1 year of age are equally complete. Since births were assumed to be deficient in the proportions indicated in table AC, this is equivalent to the assumption that total infant deaths are deficient in the same proportions. However, in dealing with subdivisions of the first year, consideration must be given to any age variation *within* the year in the assumed completeness of death reporting. It has already been stated that the evidence indicates a progressive improvement with increasing age from birth up to the first birthday. In order to give effect to this condition, the admittedly rough assumption was made that the *percent* addition which must be made to the reported deaths at any specific age during the first year of life in order to correct for underregistration is directly proportional to the time interval remaining up to the first birthday. It can only be said for this assumption that it gives plausible results, and, in the absence of any real information as to the specific age incidence of nonreporting of infant deaths, it seems as reasonable as any other assumption which might be made. Naturally, the resulting life table values for subdivisions of the first year cannot be considered as reliable as those for integral ages, but it is believed that they serve a useful purpose in indicating the general trend of mortality and survival in this important period of life; and, in any case, these values are not an essential part of the life table. The values for integral ages were computed quite independently of the assumption just stated, the supplementary values for the first year being then inserted at a later stage.

In carrying out the numerical work under this assumption as to nonreporting of infant deaths, the remaining portion of the first year of life was taken, for each of the subdivisions in which infant deaths are tabulated, as the interval of time between the *middle* of such subdivision and the end of the year of age. The length of the entire year was taken as 365 1/3 days, this being the average length of the three calendar years (1939–1941) covered by the experience. For this purpose, 1 month was regarded as being exactly one-twelfth of a year or 30 4/9 days. Table AH shows, on these assumptions, the number of days remaining in the year after the middle of each subdivision of the first year of life. The assumption that the *percent* additions required in the various age periods are proportional to these numbers implies that the *actual numbers* of deaths assumed to be unreported will be proportional to the products obtained by multiplying the time intervals indicated in table AH by the numbers of deaths actually reported in the corresponding age periods. These products were obtained separately by sex and for whites, Negroes, and other races; and in proportion to them the total number of deaths assumed unreported for the entire first year of life was distributed by age, in each of the six classifications. These total numbers, in turn, were obtained by dividing the total deaths reported for the year by the proportion assumed to be

TABLE AH.—ASSUMED NUMBER OF DAYS REMAINING IN THE FIRST YEAR OF LIFE FOLLOWING THE MIDDLE OF EACH OF THE AGE PERIODS INDICATED

Age period	Number of days remaining in year after middle of period
Under 1 day	364 5/6
1 day	363 5/6
2 days	362 5/6
3 to 6 days	360 1/3
1 week	354 5/6
2 weeks	347 5/6
3 weeks to 1 month	339 11/18
1 month	319 2/3
2 months	289 2/9
3 months	258 7/9
4 months	228 1/3
5 months	197 8/9
6 months	167 4/9
7 months	137
8 months	106 5/9
9 months	76 1/9
10 months	45 2/3
11 months	15 2/9

[7] See p. 133.

registered, as indicated in table AC, and subtracting the reported number from the result. Within each classification by race, the same percents of completeness were assumed to hold for both males and females. The figures resulting from this adjustment are shown in part III of table AM, except those for "other races" aged 1 to 11 months, in which case a further adjustment was made as described later.

Redistribution of "other races" deaths under 1 year of age

The reported deaths for subdivisions of the first year of life for the group of nonwhites other than Negroes show serious irregularities, due apparently to the small size of the experience, which, if not adjusted for, would cause a marked lack of smoothness in the life table values. Accordingly, the deaths occurring at ages between 1 month and 1 year, after being adjusted for assumed underreporting, were redistributed by fitting a second degree curve to the monthly values by the method of least squares, subject to the condition that the total for the 11-month period must be reproduced. If y_x denotes the original, and y_x' the adjusted number of deaths at the age of x months, and if x' stands for $x-6$, then it is found by applying the usual least squares criterion that y_x' is given by the equation:

$$y_x' = a + bx' + cx'^2$$

where

$$a = \frac{1}{429}(89\Sigma y_x - 5\Sigma x'^2 y_x)$$

$$b = \frac{1}{110}\Sigma x' y_x$$

$$c = \frac{1}{858}(\Sigma x'^2 y_x - 10\Sigma y_x)$$

all the summations being from $x=1$ to 11: that is, from $x' = -5$ to $+5$. Writing the equation in terms of x' rather than x makes the 11-month total a symmetrical expression and leads to results of a simpler form than would otherwise be obtained. Table AJ shows the calculated number of deaths in each of the 11 months, both before and after the least squares adjustment.

TABLE AJ.—LEAST SQUARES ADJUSTMENT OF DEATHS OF OTHER RACES[1] AT AGES 1 TO 11 MONTHS: UNITED STATES, 1939–1941

AGE	MALE DEATHS—After adjustment for nonreporting but before smoothing	MALE DEATHS—After smoothing	FEMALE DEATHS—After adjustment for nonreporting but before smoothing	FEMALE DEATHS—After smoothing
Total 1 to 11 months	1,510	1,510	1,391	1,391
1 month	251	244	251	228
2 months	216	217	164	199
3 months	180	192	184	174
4 months	174	169	149	151
5 months	138	148	128	130
6 months	133	128	116	113
7 months	116	110	97	98
8 months	89	95	80	86
9 months	100	81	96	76
10 months	56	68	71	70
11 months	57	58	55	66

[1] All except white and Negro.

Unreported ages at death

For a small proportion of deaths the age is not specified. In order not to understate the total mortality, these deaths must be distributed in some manner among the various age groups. The method used was to divide them in proportion to the numbers actually reported in each age group. While this is probably not strictly correct, the entire number of deaths involved is so small a fraction of the total that little error could result. This problem does not arise in connection with the population figures, because in the 1940 census probable ages were assigned by a special process to all persons whose age was not reported, so that no unknown ages appear in the final tabulations.[8]

Estimation of July 1, 1940, populations

For ages 5 and above, the populations required in the construction of life tables for the 3-year period 1939–1941 are those at the middle of the period: that is, on July 1, 1940. Since the census was taken as of April 1, 1940, an adjustment is necessary to arrive at the July 1, 1940, figures. For this purpose the following formula was applied to each subdivision of the population by race and sex for each 5-year age group from age 5 to age 100, and for the final group consisting of ages 100 and over. Estimates for the age group 3–4 years were also obtained, to be used in the interpolation process as described later.

$$P^{7/1}_{x/x+n-1} = P^{4/1}_{x/x+n-1} - kD^{1940}_{x/x+n-1} + \tfrac{1}{4}(P^{4/1}_{x-1} - P^{4/1}_{x+n-1}) + M_{x/x+n-1}$$

Here, $P^{4/1}_{x/x+n-1}$ denotes the population on April 1, 1940, at ages x to $x+n-1$, inclusive (that is, between exact age x and exact age $x+n$); and $P^{7/1}_{x/x+n-1}$ denotes the corresponding population on July 1, 1940. Similarly, $P^{4/1}_{x-1}$ denotes the April 1 population at age $x-1$, and $P^{4/1}_{x+n-1}$ denotes the April 1 population at age $x+n-1$. $D^{1940}_{x/x+n-1}$ denotes the number of reported deaths occurring in 1940 at ages x to $x+n-1$; and $M_{x/x+n-1}$ denotes the estimated net immigration (positive or negative) during the period April 1 to July 1, 1940, at ages x to $x+n-1$. The symbol k denotes the ratio, for both sexes and all races combined, of the reported deaths occurring in April, May, and June, 1940, to the total for the year.

The term $kD^{1940}_{x/x+n-1}$ represents the estimated deaths occurring in the particular age group between April 1 and July 1, 1940. This approximation had to be used, as deaths were not tabulated simultaneously by month of occurrence and by race or sex. The term $\tfrac{1}{4}(P^{4/1}_{x-1} - P^{4/1}_{x+n-1})$ is an adjustment for the fact that in the 3 months between April 1 and July 1 some individuals passed out of the group by reaching age $x+n$, while others entered from the next lower age group by reaching age x. In dealing with the final age group "100 and over," this term reduced to merely $\tfrac{1}{4}P^{4/1}_{99}$, and the subscript "$x/x+n-1$" in the other terms

[8] U. S. Bureau of the Census, *Sixteenth Census of the United States: 1940, Population, Vol. II, Characteristics of the Population, Part I*, p. 9, Government Printing Office, Washington, D. C., 1943.

was interpreted as "100 and over." The net immigration was estimated on the basis of information furnished by the Immigration and Naturalization Service, Department of Justice. For the white population, the migration adjustment never exceeded 0.06 of 1 percent of the corresponding enumerated population in any classification.

While the total nonwhite population was available by single years of age, Negroes were tabulated separately only by 5-year age groups up to age 75 and also for a few selected single years of age under 21. The single age figures for Negroes were obtained by assuming that, for each sex separately, the ratio of Negroes to total nonwhites was the same in each single year of age as in the smallest age group containing that year of age for which separate figures for Negroes and other nonwhites were available. In each classification, estimated figures for "other races" were obtained by subtracting Negroes from total nonwhites. A further difficulty was encountered in that the migration estimates used were furnished only for total nonwhites, and not for Negroes separately. As the movement of Negroes into and out of the United States is believed to be exceedingly small, and as the migration estimates for total nonwhites were small in any case, never reaching 100 for either sex in any 5-year age group, they were assumed to relate wholly to races other than Negroes, no migration adjustment being made in the Negro populations.

The estimates of July 1, 1940, population resulting from the application of the above formula are shown in part II of table AM, except those for Negroes between ages 55 and 70, in which case a further adjustment was made as explained in the next subsection. These estimated populations differ only slightly from those previously published by the Bureau of the Census.[9] It was decided not to use the previously published estimates in the construction of the life tables because they were based on a graduated, or smoothed distribution by single years of age of the April 1 population. While such a procedure was entirely appropriate in preparing population estimates for general use, it was felt that, in the construction of the life tables, the smoothness of the rates of mortality was adequately provided for by the graduation of the rates themselves,[10] and that there were some objections to graduating the enumerated populations. The single year populations, since they arise, in the beginning, from fluctuating numbers of annual births, cannot be expected to form a perfectly smooth series, and any genuine irregularities will be reflected also in the death statistics, so that the smooth progression of the rates of mortality will not be disturbed. Moreover, this appears to be true also, in large measure, of the irregularities which are not genuine, since the analysis of digit preference later in this report[11] indicates that, in the usual system of 5-year age grouping (5–9, 10–14, etc.), errors of this type in the populations and deaths tend to cancel out in the computation of mortality rates. Therefore, if the populations were partially smoothed, without subjecting the death statistics to some similar treatment, the result might only be to diminish the smoothness of the mortality rates.

Special adjustment of Negro data

Both population and death statistics in the neighborhood of age 65 show evidence of substantial misstatement of age. In the case of the data for Negroes, this error appeared sufficiently marked to seriously affect life table values. This condition is brought out in table AK in which the 1940 Negro populations actually enumerated in the various age groups are compared with those expected on the basis of the 1930 populations of the same groups of individuals (then 10 years younger) and the deaths of the intervening period. It will be noted that while these population figures show, on the whole, a steady decrease with advancing age, the *enumerated* 1940 populations level off sharply at age 65. The 1930 figures do not show any such tendency. Moreover, the *expected* 1940 populations, from the 1930 enumeration and the deaths during the decade, are free from the leveling off effect. This strongly suggests an overstatement in the 1940 census of the age groups just beyond 65 at the expense of those just under that age. This phenomenon is probably attributable to the enactment of social security legislation providing benefits to persons over 65.

TABLE AK.—COMPARISON OF NEGRO POPULATIONS IN CERTAIN AGE GROUPS: UNITED STATES, 1930 AND 1940

[Numbers given in thousands]

Age in 1930 (1)	Age in 1940 (2)	Population enumerated in 1930 (3)	1940 population estimated from 1930 population and deaths (4)	Population enumerated in 1940 (5)	Discrepancy in 1940 estimates (4)−(5)
MALE					
40–44	50–54	339	264	283	−19
45–49	55–59	323	245	207	+38
50–54	60–64	278	207	154	+53
55–59	65–69	174	109	152	−43
60–64	70–74	133	76	84	−8
65–69	75–79	83	40	40	0
70–74	80–84	51	20	19	+1
75–79	85–89	29	7	9	−2
FEMALE					
40–44	50–54	348	285	267	+18
45–49	55–59	307	242	190	+52
50–54	60–64	227	169	142	+27
55–59	65–69	135	83	145	−62
60–64	70–74	109	63	79	−16
65–69	75–79	72	36	42	−6
70–74	80–84	48	22	22	0
75–79	85–89	29	10	11	−1

The conclusion that such misstatement of age has occurred is reinforced by the observation that mortality rates calculated from the reported data without adjustment also level off sharply at 65, in the case of the females actually showing a temporary decrease

[9] U. S. Bureau of the Census, *Estimated Population in Continental United States, by Age, Color, and Sex: 1940–1942*, Population—Special Reports, Series P-44, No. 9, 1944.

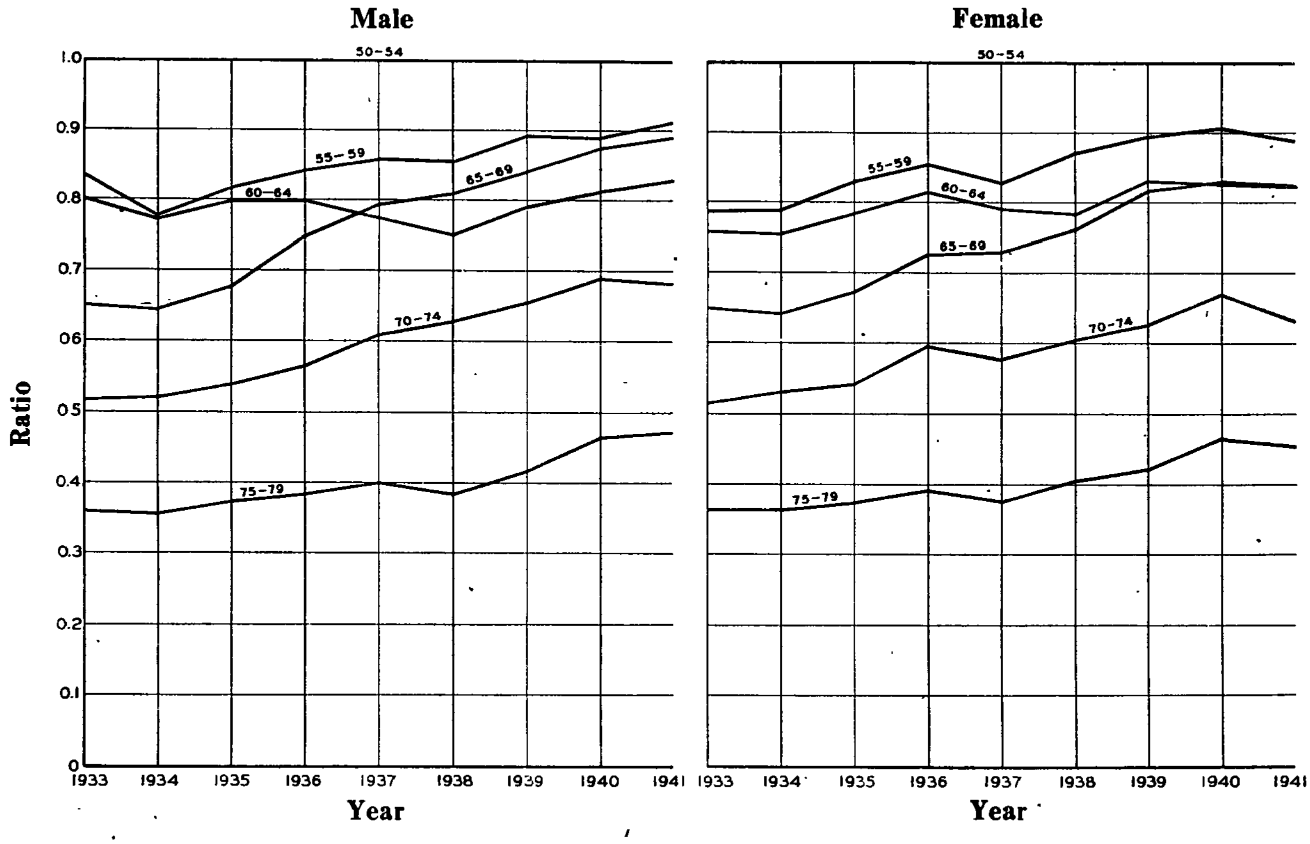
[10] See pp. 122–126.

[11] See pp. 120–122.

with increasing age. There is evidence also that the death statistics have been affected in the same way. This is indicated by figure 12, which shows the trend for the period 1933–1941 of the ratio of Negro deaths in certain selected age groups to those occurring in the same year in the age group 50–54. The age groups selected extend from age 55 to age 80. The general tendency of each of these ratios over any fairly long period is to increase gradually, because of the steadily increasing proportion of the population in the older age groups. In every year of the period covered by the graph, the Negro deaths by 5-year age groups have reached a maximum in the group 50–54, and prior to 1937 have decreased steadily thereafter to the end of life. However, in 1937 and each subsequent year, the reported deaths for Negro males in the age period 65–69 have exceeded those in the group 60–64. In this connection it will be remembered that the Social Security Act was enacted in 1935 and that State old-age assistance programs as provided by the act did not go into operation until 1936 or, in a few States, even later. In the case of Negro females the effect is less noticeable, although the number of deaths in these two age groups began in 1937 to move closer together, and in 1940 there was actually a larger number in the group 65–69. In view of the magnitude of this disturbance, it seemed advisable to make a preliminary redistribution of the Negro populations and deaths between 55 and 70. After experimenting with various empirical methods of redistribution, the one described in the next paragraph was adopted as giving the most plausible results.

From the estimated July 1, 1940, populations, obtained as previously described, the ratio of the Negro population 50 and over to the corresponding white population was obtained, for males and females separately. Similar ratios were obtained for the population 55 and over, 60 and over, and so on up to and including 75 and over: a total of six ratios for each sex. The calculated ratios for ages 60 and over and 65 and over were rejected, and corrected values of these ratios were obtained by interpolation from the remaining four ratios, using Waring's formula.[12] By applying these corrected ratios to the white populations 60 and over and 65 and over, corrected Negro populations were obtained. By inserting these corrected values in the original series of Negro populations 50 and over, 55 and over, etc., and differencing, corrected populations by 5-year age groups were obtained. By this method, only

FIGURE 12.—RATIO OF NEGRO DEATHS IN SELECTED AGE GROUPS TO NEGRO DEATHS AT AGES 50–54 IN THE SAME YEAR: UNITED STATES, 1933–1941



[12] Also known as Lagrange's formula. See E. T. Whittaker and G. Robinson, *The Calculus of Observations*, second edition, pp. 28–32, Blackie and Son, Ltd., London and Glasgow, 1937; also T. N. E. Greville, *A Generalization of Waring's Formula*, Annals of Mathematical Statistics, vol. 15, No. 2, pp. 218–219, June 1944.

the figures for the age groups 55–59, 60–64, and 65–69 are changed, and these automatically add to the original total for the entire 15-year age period. The method does not assume that the white and Negro populations have similar age distributions, but merely that the ratio between them progresses fairly smoothly by age. The Negro deaths reported in these three age groups were redistributed by relating them to the corresponding white deaths in the same manner. Table AL shows the original figures for Negro populations and deaths and also the adjusted figures obtained in the redistribution. For comparison, the figures for the two adjacent age groups on each side are also shown.

TABLE AL.—ORIGINAL AND REDISTRIBUTED NEGRO STATISTICS FOR AGES 55 TO 69:[1] UNITED STATES, 1939–1941

AGE	DEATHS, 1939–1941				ESTIMATED POPULATIONS, JULY 1, 1940			
	Male		Female		Male		Female	
	Reported	Adjusted	Reported	Adjusted	Original[2]	Adjusted	Original[2]	Adjusted
50–54	25,041	25,041	20,677	20,677	285,012	285,012	270,679	270,679
55–59	22,485	23,335	18,531	19,162	208,656	218,324	191,534	203,048
60–64	20,306	21,452	17,038	17,540	154,632	168,242	142,381	155,619
65–69	21,760	19,764	16,956	15,823	151,407	128,129	144,314	119,562
70–74	16,938	16,938	13,286	13,286	84,436	84,436	79,945	79,945
55–69	64,551	64,551	52,525	52,525	514,695	514,695	478,229	478,229

[1] Adjacent 5-year age groups also shown for comparison.
[2] Calculated from reported data by the formula given on p. 109.

Estimation of foreign-born population under 5

As will be explained later when the method of calculating mortality rates under 5 is described, the distribution by nativity of the 1940 population in this age group, separately by sex, race, and single years of age, was required. For whites, the census tabulations give, for males and females separately, the number of foreign-born under 1 year, the number at ages 1–4, and the number at age 5. Single year figures for both sexes were obtained by assuming that the figures for single years starting with age 1 and ending with age 5 formed an arithmetic progression. This assumption was suggested by a study of the data of previous censuses, in which the complete detail was available. The resulting values were then distributed by sex in the same ratio as the entire age group 1–4, and rounded to add to the correct total.

Nativity was tabulated for Negroes by 5-year age groups only, and the foreign-born Negroes under age 5 were distributed by single years of age on the assumption that, for each sex separately, the numbers for the first 5 years of age formed an arithmetic progression in which the common difference was equal to the number under 1 year of age. In the case of the remaining races, foreign-born were given by age only for Chinese and Japanese. Hence, it was assumed that there were no foreign-born under age 5 of races other than white, Negro, Chinese, and Japanese. In actual fact, the number of such children is believed to have been very small. The estimated native population in each classification was, of course, obtained by subtracting the estimated foreign-born from the total.

B. CALCULATION OF THE RATES OF MORTALITY

The description of the process of obtaining rates of mortality divides itself naturally into two main parts, corresponding to ages 0 to 4 and ages 5 and over, since the methods used in the two cases were very different. In connection with the calculation of mortality rates for ages 0 to 4, two subordinate topics are discussed under separate subheadings. These are (1) the derivation of separation factors for estimating the distribution of deaths by calendar year of birth, and (2) the adjustment of the mortality rates to allow for the effect of migration.

Basically, the method employed in obtaining rates at ages 5 and over consisted of three steps. First, populations and deaths were estimated by interpolation for the middle age of each of the 5-year age groups in which the data were tabulated. Secondly, rates of mortality for these middle ages (at 5-year intervals) were computed from the interpolated populations and deaths. Finally, osculatory interpolation was applied to the mortality rates derived in the second step in order to obtain rates for all ages. In the discussion which follows, each of these three steps is treated under a separate subheading. Additional subsections are devoted to (1) a justification of the basic procedure just described, as against other procedures which have sometimes been employed, and (2) a description of the tests which were applied to the final rates of mortality in order to be sure that the graduation was satisfactory. Further subsections deal with two digressions from the main theme: (a) an analysis of preferences shown in the reporting of age for figures ending with certain digits and the effect of this bias on mortality rates, with reference to the selection of a particular way of combining single ages into 5-year age groups; and (b) the method used in obtaining mortality rates at the very old ages, where the ordinary methods fail to give satisfactory results, because of unreliable age reporting and the small volume of data.

All the basic data actually used in the construction of the various life tables are given in table AM. Part I of that table contains the data required in the computation of mortality rates for ages 0 to 4, inclusive; while part II contains the data used in deriving mortality rates for ages 5 and over. Part III contains certain additional data required in obtaining life table values for subdivisions of the first year of life.

Table AM.—Data Employed in the Computation of Mortality Rates for the United States, 1939–1941

Part I—Ages under 5

A—Registered births, and registered deaths at certain ages under 5, by race and sex, 1934–1941

Race, sex, and item tabulated	1941	1940	1939	1938	1937	1936	1935	1934
WHITE MALES								
Registered births	1,133,394	1,064,067	1,019,021	1,030,398	991,356	966,332	969,916	975,804
Registered deaths:								
Age:								
Under 1	52,191	51,477	50,201	54,121	55,540	56,970	56,424	60,319
1	4,717	4,929	5,292	6,366	6,781	7,491	7,183	
2	2,517	2,592	2,750	3,255	3,671	3,834		
3	1,756	1,731	2,012	2,334	2,461			
4	1,363	1,432	1,572	1,729				
WHITE FEMALES								
Registered births	1,071,509	1,003,886	963,650	975,557	937,081	915,551	918,096	922,697
Registered deaths:								
Age:								
Under 1	38,742	38,013	37,683	40,411	41,575	42,601	41,548	45,302
1	3,995	4,124	4,542	5,574	5,906	6,165	6,137	
2	1,954	2,130	2,181	2,780	3,098	3,158		
3	1,444	1,442	1,598	1,850	2,042			
4	1,105	1,131	1,276	1,416				
NEGRO MALES								
Registered births	149,147	140,675	137,072	135,328	132,990	127,017	129,578	130,795
Registered deaths:								
Age:								
Under 1	12,180	11,482	11,201	11,636	11,951	12,067	11,700	13,053
1	1,339	1,361	1,388	1,660	1,771	1,720	1,618	
2	615	605	595	724	728	710		
3	341	341	415	445	457			
4	261	275	329	356				
NEGRO FEMALES								
Registered births	145,407	138,194	132,988	132,372	129,472	124,081	125,546	126,311
Registered deaths:								
Age:								
Under 1	9,708	8,920	8,598	9,269	9,613	9,605	9,263	10,410
1	1,211	1,071	1,170	1,407	1,441	1,399	1,406	
2	534	490	530	601	621	660		
3	344	304	359	413	428			
4	263	270	298	338				
OTHER RACES, MALES								
Registered births	7,193	6,942	6,507	6,815	6,295	6,116	5,995	6,104
Registered deaths:								
Age:								
Under 1	689	691	642	771	750	795	761	684
1	159	116	175	149	192	179	212	
2	66	69	65	68	68	55		
3	33	29	40	43	41			
4	22	22	30	31				
OTHER RACES, FEMALES								
Registered births	6,777	6,635	6,350	6,492	6,143	5,693	5,974	5,925
Registered deaths:								
Age:								
Under 1	554	549	611	594	607	625	567	589
1	160	124	157	161	191	166	194	
2	71	62	68	65	68	68		
3	38	36	32	53	41			
4	22	26	26	29				

B—Estimated distribution by nativity, race, and sex of the enumerated population under 5 on Apr. 1, 1940

Nativity and age	White		Negro		Other races	
	Male	Female	Male	Female	Male	Female
NATIVE						
Age:						
Under 1	906,653	871,085	113,809	115,986	6,085	6,044
1	925,801	889,247	114,602	114,509	5,752	5,642
2	977,608	940,835	131,392	132,779	6,589	6,538
3	933,924	908,668	127,357	131,223	6,379	6,456
4	953,265	914,098	134,509	132,855	6,730	6,532
FOREIGN-BORN						
Age:						
Under 1	244	251	1	3	8	5
1	597	579	3	5	15	10
2	862	834	4	8	23	16
3	1,126	1,091	5	10	30	21
4	1,390	1,347	7	13	38	26

TABLE AM.—DATA EMPLOYED IN THE COMPUTATION OF MORTALITY RATES FOR THE UNITED STATES, 1939–1941—Continued

PART II—AGES 5 AND OVER

Registered deaths, 1939–1941, and estimated population on July 1, 1940, by race and sex, for ages 3 and over

SEX AND AGE	WHITE		NEGRO		OTHER RACES	
	Registered deaths, 1939–1941	Estimated population, July 1, 1940	Registered deaths, 1939–1941	Estimated population, July 1, 1940	Registered deaths, 1939–1941	Estimated population, July 1, 1940
MALE						
3–4	9,866	1,894,925	1,962	260,949	176	13,142
5–9	16,716	4,736,987	3,003	646,283	242	30,765
10–14	17,062	5,234,717	3,438	658,972	222	31,773
15–19	28,507	5,511,945	7,043	633,259	420	34,184
20–24	35,522	5,131,965	10,661	551,484	475	28,808
25–29	37,146	4,905,853	12,472	530,348	471	28,938
30–34	42,405	4,588,155	13,602	470,605	561	29,031
35–39	53,285	4,253,778	15,927	457,586	670	28,303
40–44	72,956	4,021,881	18,961	408,541	676	24,272
45–49	105,256	3,841,840	21,830	346,047	758	18,404
50–54	142,217	3,461,903	25,041	285,012	894	17,802
55–59	173,192	2,808,550	23,335	218,324	1,030	14,140
60–64	201,341	2,238,579	21,452	168,242	1,120	11,104
65–69	229,887	1,749,889	19,764	128,129	1,031	7,260
70–74	235,612	1,190,567	16,938	84,436	804	3,848
75–79	208,875	683,763	11,302	41,108	667	2,266
80–84	157,479	342,554	7,048	18,709	443	1,042
85–89	76,515	114,282	4,296	8,902	248	494
90–94	23,084	25,165	2,060	3,279	131	181
95–99	4,396	4,292	961	1,274	41	71
100 and over	626	573	628	747	42	41
FEMALE						
3–4	7,996	1,831,178	1,838	263,942	180	13,015
5–9	12,109	4,576,540	2,579	652,833	216	30,786
10–14	11,334	5,060,216	3,012	665,957	213	30,405
15–19	10,140	5,436,705	8,525	675,628	414	30,838
20–24	25,475	5,241,255	11,246	644,690	479	24,087
25–29	29,490	5,030,298	12,253	617,641	309	18,265
30–34	33,709	4,651,966	12,930	528,854	283	14,085
35–39	39,774	4,267,585	15,520	517,645	312	13,968
40–44	50,335	3,969,185	17,503	426,087	292	13,201
45–49	68,003	3,699,217	18,194	342,504	328	11,155
50–54	87,083	3,242,931	20,677	270,679	342	8,330
55–59	107,050	2,658,635	19,162	203,048	345	5,757
60–64	135,810	2,191,641	17,540	155,619	371	4,611
65–69	171,664	1,776,057	15,823	119,562	387	3,745
70–74	193,091	1,227,732	13,286	79,945	376	2,218
75–79	189,795	740,120	9,237	42,764	321	1,501
80–84	159,109	395,970	6,061	21,721	252	774
85–89	88,451	145,982	4,217	11,370	158	407
90–94	31,981	37,184	2,380	4,911	104	174
95–99	7,365	6,825	1,144	2,011	39	73
100 and over	1,064	929	1,133	1,383	44	50

PART III—SUBDIVISIONS OF THE FIRST YEAR OF LIFE

Estimated total deaths under 1 year by age, race, and sex

AGE	WHITE		NEGRO		OTHER RACES	
	Male	Female	Male	Female	Male	Female
Total	163,592	121,704	42,467	33,178	2,688	2,278
Under 1 day	52,275	38,122	9,913	7,487	420	332
1 day	13,507	9,437	2,802	2,222	93	84
2 days	8,555	5,558	2,102	1,372	86	77
3 to 6 days	12,773	8,997	3,639	2,516	218	135
1 week	8,263	6,347	2,632	2,208	159	113
2 weeks	5,418	4,231	1,662	1,378	99	74
3 weeks to 1 month	4,749	3,475	1,423	1,195	103	72
1 month	11,823	8,624	3,607	2,894	244	228
2 months	9,281	7,130	2,735	2,255	217	199
3 months	7,460	5,895	2,335	1,978	192	174
4 months	5,906	4,750	2,017	1,631	169	151
5 months	5,045	3,998	1,607	1,317	148	130
6 months	4,119	3,483	1,508	1,200	128	113
7 months	3,711	2,914	1,197	921	110	98
8 months	3,224	2,600	1,066	778	95	86
9 months	2,768	2,292	857	733	81	76
10 months	2,446	1,935	699	535	68	70
11 months	2,269	1,916	666	558	58	66

Basic process for obtaining mortality rates at ages 0 to 4

The basic equation employed in obtaining mortality rates at ages 0 to 4 is based on the interpretation of the rate of mortality as a probability of death. For example, the rate of mortality[13] at age x, denoted by q_x, can be regarded as the probability that a person exactly x years old will die before reaching exact age $x+1$. Similarly, the complement $p_x=1-q_x$ represents the probability that an individual exactly x years of age will survive to exact age $x+1$. In order to facilitate its calculation from the data available, p_x may be expressed as the product of two separate probabilities. Thus:[14]

$$p_x={}_ap_x\,{}_bp_x$$

where ${}_ap_x$ denotes the probability that an individual alive at exact age x will survive to the end of the calendar year in which this exact age was attained, and ${}_bp_x$ denotes the probability that an individual who is alive at the end of the calendar year in which he attained age x will survive to exact age $x+1$. It follows that:

$$q_x=1-{}_ap_x\,{}_bp_x \tag{10}$$

this being the basic formula employed in computing mortality rates at ages 0 to 4. In order to derive expressions for the partial probabilities ${}_ap_x$ and ${}_bp_x$ in terms of the data as given, the following special symbols will be employed:

E_x^z denotes the number reaching exact age x during the calendar year z.

P_x^z denotes the number living on January 1 of the year z whose age in completed years is x.

D_x^z denotes the number dying in the year z whose age in completed years at the time of death is x.

${}_aD_x^z$ denotes that portion of D_x^z consisting of cases in which exact age x was reached during the year z.

${}_bD_x^z$ denotes that portion of D_x^z consisting of cases in which exact age x was reached during the year $z-1$.

E_x denotes the total number reaching exact age x during the entire period of observation, which is assumed to be an integral number of years.

P_x' denotes the total number who, after attaining exact age x during the period of observation, are still alive at the end of the year in which exact age x was attained.

P_x'' denotes the total number who are alive at the end of the year in which age x was attained, and whose $(x+1)$th birthday falls within the period of observation.

u and v denote, respectively, the first and last years included in the period of observation.

Certain relationships between these symbols are immediately apparent. For example,

$$E_x^z-{}_aD_x^z=P_x^{z+1} \tag{11}$$

and

$$P_x^z-{}_bD_x^z=E_{x+1}^z \tag{12}$$

If birth and death statistics were available in the necessary detail, it would be possible, by successive applications of formulas (11) and (12), to obtain values of E_x^z and P_x^z for any desired ages. It is to be noted that E_0^z denotes the number reaching age 0: that is, the number of births, in the year z.

For example, suppose it is desired to find the number alive on January 1, 1940, at age 4 in completed years, and also the number reaching exact age 5 in 1940. Anyone whose age in completed years on January 1, 1940, is 4, or who reaches exact age 5 in 1940, must have been born in 1935. Therefore, one would start with E_0^{1935}, the number of births occurring in that year. Formula (11) gives:

$$E_0^{1935}-{}_aD_0^{1935}=P_0^{1936}$$

and formula (12) gives:

$$P_0^{1936}-{}_bD_0^{1936}=E_1^{1936}$$

By continuing in this fashion and applying formulas (11) and (12) alternately, the desired values would eventually be reached, provided, of course, the necessary birth and death statistics are available.

It is obvious from the definition of E_x, P_x', and P_x'' that

$$E_x=\sum_{z=u}^{v}E_x^z \tag{13}$$

$$P_x'=\sum_{z=u+1}^{v+1}P_x^z \tag{14}$$

and

$$P_x''=\sum_{z=u}^{v}P_x^z \tag{15}$$

Finally, the values of the partial probabilities ${}_ap_x$ and ${}_bp_x$, on the basis of the experience which is being employed, are given by:

$${}_ap_x=\frac{P_x'}{E_x} \tag{16}$$

and

$${}_bp_x=\frac{E_{x+1}}{P_x''} \tag{17}$$

Formulas (11) to (17) and formula (10) would seem to provide the means of computing mortality rates up to any age desired, if adequate birth and death statistics are available. There remain, however, two difficulties. In the first place, deaths are not ordinarily tabulated so as to give the separate parts denoted by ${}_aD_x$ and ${}_bD_x$; and, secondly, the effect of migration has been ignored. The methods employed in order to overcome these two

[13] The rates of mortality shown in the life tables which appear in this volume (except in the case of tables 14, 25, and 38) are values of $1{,}000q_x$, the rate of mortality per 1,000 survivors at age x. However, in developing the mathematical theory of the life table, it is more convenient to use the rate of mortality per single survivor.

[14] The notation employed in this development follows, with slight modifications, that of Hugh H. Wolfenden in *Population Statistics and Their Compilation (Actuarial Studies, No. 3)*, pp. 70–84, Actuarial Society of America, New York, 1925. The basic formula (10) given here is Wolfenden's formula (12), p. 76.

difficulties form the subject of the next two subsections. However, it will be useful, before taking up these rather technical points, to give a numerical illustration of the application of the formulas just derived. In this illustration, the required values of ${}_aD_x$ and ${}_bD_x$ will be given without explanation as to how they were obtained; and, inasmuch as the correction for migration was made as a final adjustment in the mortality rates, after the calculations had been otherwise completed, the consideration of this point can easily be postponed.

Another point which needs to be mentioned at this time concerns the method of applying the correction for underreporting of births and infant deaths. Since these were assumed to be equally complete,[15] the rates of mortality at age 0 were obtained from registered figures without applying any correction. To this end, the calculations were begun by taking as the values of E_0^z the number of births registered in the various years. By the subtraction of registered deaths, values of P_0^z and E_1^z were obtained. The values of q_0 were computed from these three sets of quantities as indicated by formulas (13) to (17) and formula (10). Next, the values of E_1^z were corrected for underreporting by dividing by the ratios derived for that purpose,[16] which were based on comparison with census populations in the age period 3 to 9. These adjusted values of E_1^z were taken as the starting point in obtaining corrected values of P_x^z and E_x^z for subsequent ages, it being assumed that deaths occurring at ages 1 and over required no correction. Mortality rates at ages 1 to 4 were then computed entirely on the basis of corrected figures.

The calculation of mortality rates at ages 0 to 4 for white males will be taken as a numerical illustration of the process. The registered births for each of the 8 years 1934 to 1941 are given in part I of table AM, page 113. Those values of ${}_aD_x$ and ${}_bD_x$ which will be needed in the computations are shown in table AN. The calculation of the values of P_0 and E_1 and the adjustment of E_1 for underreporting are shown in table AO. For the births of the years 1934 to 1937, the number of survivors to the end of the year of birth is not required, since the children concerned will have reached age 1 before January 1, 1939, the commencement of the period of observation. Therefore, for the births of these years, the total number of infant deaths to be subtracted, although the sum of two figures in table AN, is shown as a single figure in table AO. It will be noted that each of these totals contains deaths occurring in two different calendar years. In each case, the number of survivors to exact age 1 of the *registered* births is corrected for underregistration by dividing by .9551, the ratio previously derived for that purpose.[17]

[15] See p. 106.
[16] These ratios are given in the final column of table AG, p. 107.

[17] See table AG, p. 107.

TABLE AN.—DEATHS OF WHITE MALES AT AGES 0 TO 4, BY AGE AND YEAR OF DEATH, SEPARATED ACCORDING TO WHETHER DEATH OCCURRED IN THE SAME YEAR AS THE LAST BIRTHDAY ATTAINED, OR IN THE FOLLOWING YEAR: UNITED STATES, 1934–1941

CLASS OF DEATHS[1]	YEAR OF DEATH							
	1934	1935	1936	1937	1938	1939	1940	1941
${}_aD_0$	49, 039	45, 196	46, 658	44, 654	44, 542	41, 165	43, 138	43, 423
${}_bD_0$	(2)	11, 228	10, 312	10, 886	9, 579	9, 036	8, 339	8, 768
${}_aD_1$	(2)	4, 238	4, 420	4, 001	3, 756	3, 122	2, 908	2, 783
${}_bD_1$	(2)	(2)	3, 071	2, 780	2, 610	2, 170	2, 021	1, 934
${}_aD_2$	(2)	(2)	2, 032	1, 946	1, 725	1, 462	1, 374	1, 334
${}_bD_2$	(2)	(2)	(2)	1, 725	1, 530	1, 297	1, 218	1, 183
${}_aD_3$	(2)	(2)	(2)	1, 280	1, 214	1, 046	900	913
${}_bD_3$	(2)	(2)	(2)	(2)	1, 120	966	831	843
${}_aD_4$	(2)	(2)	(2)	(2)	899	817	745	709
${}_bD_4$	(2)	(2)	(2)	(2)	(4)	755	687	654

[1] For explanation of the symbols in this column, see text, p. 115.
[2] Value not needed in life table calculations.

TABLE AO.—NUMBER OF REGISTERED BIRTHS OF WHITE MALES, NUMBER SURVIVING SPECIFIED PERIODS, AND ADJUSTMENT FOR UNDERREPORTING, BY YEAR OF BIRTH (z): UNITED STATES, 1934–1941

	1934	1935	1936	1937	1938	1939	1940	1941
Registered births (E_0^z)	975, 804	969, 916	966, 332	991, 356	1, 030, 398	1, 019, 021	1, 064, 067	1, 133, 394
Deaths to be subtracted (${}_aD_0^z$)	49, 039	45, 196	46, 658	44, 654	44, 542	41, 165	43, 138	43, 423
Survivors to end of year of birth (P_0^z)	(1)	(1)	(1)	(1)	985, 856	977, 856	1, 020, 929	1, 089, 971
Deaths to be subtracted (${}_bD_0^{z+1}$)	11, 228	10, 312	10, 886	9, 579	9, 036	8, 339	8, 768	(1)
Survivors to exact age 1 (E_1^{z+1})	915, 537	914, 408	908, 788	937, 123	976, 820	969, 517	1, 012, 161	(1)
Survivors to exact age 1 (corrected for underreporting)	958, 577	957, 395	951, 511	981, 178	1, 022, 741	1, 015, 095	1, 059, 743	(1)

[1] Not needed in life table calculations.

Continuation of the process of subtracting the appropriate groups of deaths, in accordance with formulas (11) and (12), gives the various numbers shown in table AP. In the case of the births of the years 1934 to 1936, the deaths occurring between the attainment of age 1 and January 1, 1939, can be lumped together, as it is not necessary to know the number of survivors on any prior date. It will be noted that the successive death figures to be subtracted from a given year's births form a sort of broken diagonal extending downward and to the right in table AN, consisting of ${}_aD_0$ from the column for the given year itself, ${}_bD_0$ and ${}_aD_1$ from the column for the following year, ${}_bD_1$ and ${}_aD_2$ from the column for the next following year, and so on. After January 1, 1939, has been reached, the successive death figures must be subtracted one by one, noting the remainder after each subtraction, until the cohort has been carried to January 1, 1942, after which no further values are needed. The various numbers of survivors shown in table AP are arranged not according to the year of birth, but according to the calendar year in which the indicated exact age is attained, or at the beginning of which the indicated population exists. In those lines of the table which give values of P_x^z, the total for 1939–1941 is, of course, P_x'', while the total for 1940–1942 is P_x'.

Values of ${}_ap_x$ and ${}_\delta p_x$ for ages 1 to 4 obtained from the figures in the last two columns of table AP are given in table AQ which also shows the calculation of the mortality rates except for the final adjustment for migration. The calculations for age 0 are not shown, since in that case the adjustment for migration was introduced at an earlier stage in the computation. This point is explained in detail on pages 119 and 120.

In the case of the life tables for combinations of classes such as total whites or total males, the values of E_x, P_x', and P_x'' for the component parts were combined before computing the partial probabilities of survival, the remainder of the calculation being exactly the same as for the separate classes.

TABLE AP.—NUMBER OF WHITE MALES SURVIVING SPECIFIED PERIODS OF LIFE BETWEEN BIRTH AND AGE 5: UNITED STATES, 1939–1941

CLASS OF SURVIVORS [1]	CALENDAR YEAR IN WHICH INDICATED BIRTHDAY IS REACHED, OR AT THE BEGINNING OF WHICH INDICATED POPULATION EXISTS (z)					
	1939	1940	1941	1942	Total 1939–1941	Total 1940–1942
E_1^z [2]	1,022,741	1,015,095	1,059,743	----------	3,097,579	----------
P_1^z	-977,422	1,019,619	1,012,187	1,056,960	3,009,228	3,088,766
E_2^z	975,252	1,017,598	1,010,253	----------	3,003,103	----------
P_2^z	943,175	973,790	1,016,224	1,008,919	2,933,189	2,998,933
E_3^z	941,878	972,572	1,015,041	----------	2,929,491	----------
P_3^z	945,505	940,832	971,672	1,014,128	2,858,009	2,926,632
E_4^z	944,539	940,001	970,829	----------	2,855,369	----------
P_4^z	944,212	943,722	939,256	970,120	2,827,190	2,853,098
E_5^z	943,457	943,035	938,602	----------	2,825,094	----------

[1] For explanation of symbols in this column, see text, p. 115.
[2] Corrected for underreporting.

TABLE AQ.—CALCULATION OF RATES OF MORTALITY [1] FOR WHITE MALES AT AGES [2] 1 TO 4: UNITED STATES, 1939–1941

	1	2	3	4
${}_ap_x=P_x'/E_x$	0.99715487	0.99861144	0.99902406	0.99920466
${}_\delta p_x=E_{x+1}/P_x''$	.99796459	.99873926	.99907628	.99925863
$p_x={}_ap_x{}_\delta p_x$	.99512525	99735245	.99810124	.99846388
$q_x=1-p_x$	.00487475	.00264755	.00189876	.00153612

[1] Unadjusted for effect of migration.
[2] Age denoted by x.

Derivation of separation factors for deaths

In the preceding section, mention was made of the necessity of separating the deaths of each calendar year into two groups according to whether death occurred in the same calendar year as the last birthday attained, or in the following year. This could evidently be accomplished by sorting on the year of birth. To illustrate this, consider the case of children dying in 1940 at age 3. In this group, all those who reached exact age 3 in 1939 were obviously born in 1936, while those who reached exact age 3 in 1940 were born in 1937. However, deaths in the United States are not tabulated by year of birth; and it was therefore necessary to estimate, in each case, the subdivision of D_x^z into ${}_aD_x^z$ and ${}_\delta D_x^z$.

This is accomplished by employing what may be called "separation factors." The separation factor, denoted by f_x^z, is defined as

$$f_x^z=\frac{{}_\delta D_x^z}{D_x^z} \qquad (18)$$

In dealing with death statistics not tabulated by year of birth, it is customary to employ values of this ratio obtained from other data, so that the working formulas are:

$${}_aD_x^z=(1-f_x^z)D_x^z \qquad (19)$$

and

$${}_\delta D_x^z=f_x^zD_x^z \qquad (20)$$

Tabulations of deaths from which values of f_x^z can be obtained directly have never been made in the United States, and are found in only a few countries, notably Germany.[18] Such a tabulation is now being undertaken in the Bureau of the Census based on a 10-percent sample of all 1944 deaths under age 5; and the values derived from it will be available for use in the preparation of future life tables.

It is not always satisfactory to use values of f_x^z based on the statistics of other countries, particularly if such statistics are, in addition, not very recent, as the values of this ratio have been observed to vary as between different countries and to change markedly over periods of time. Another alternative is to approximate the values of f_x^z by making use of tabulations of deaths by month of age, if these are available. In the United States, such tabulations have been made in recent years only for the first year of life. However, it is in the first year of life that the values of f_x^z are most subject to change, so that reliance on values obtained from outside sources is most unsatisfactory. Accordingly, the values of f_0^z used in connection with the life tables in this volume were all estimated from the tabulations of deaths by subdivisions of the first year of life.

The method of arriving at such estimates is best illustrated by a numerical example. This example will be based on the tabulation of infant deaths for males of all races in 1935. The data to be used are given in table AR. In this table, attention is called to the figures in bold-face type which extend across the table more or less diagonally. It is evident that all the figures below and to the left of the bold-face figures represent deaths of infants born in 1934. Similarly, all the figures above and to the right of the bold-face figures refer to deaths of infants born in 1935. However, the bold-face figures themselves include some deaths of infants born in 1934 and some deaths of infants born in 1935. In the case of all these figures except those which represent deaths in the month of January, it was assumed that an equal number were born in each of the 2 years. When one of these numbers was an odd number, the extra infant was assumed to have been born in the year of death (in this case, 1935).

[18] See U. S. Bureau of the Census, *United States Life Tables, 1890, 1901, 1910, and 1901-1910*, p. 339, Government Printing Office, Washington, D. C., 1921.

TABLE AR.—DEATHS OF MALES UNDER 1 YEAR OF AGE, BY MONTH OF DEATH AND BY AGE: UNITED STATES, 1935

AGE	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Total under 1 year	7,145	6,376	6,691	5,740	5,747	5,489	5,413	5,219	4,920	5,107	5,143	5,815
Under 1 day	**1,548**	1,451	1,610	1,561	1,680	1,689	1,746	1,631	1,405	1,502	1,486	1,470
1 day	**413**	394	413	379	380	373	391	421	301	375	310	414
2 days	**314**	327	304	264	238	235	260	231	238	218	251	292
3 to 6 days	**624**	508	553	502	409	442	406	420	409	417	422	485
1 week	**382**	352	371	280	309	287	297	290	274	265	291	350
2 weeks	**303**	245	270	189	174	178	149	177	168	156	195	224
3 weeks to 1 month	**282**	233	193	184	161	142	162	142	162	154	158	190
1 month	717	**558**	528	435	450	354	321	366	381	447	412	476
2 months	531	431	**429**	347	333	310	268	259	281	324	356	390
3 months	416	352	346	**296**	293	216	228	202	259	257	261	347
4 months	322	272	279	204	**206**	239	231	197	190	209	198	237
5 months	267	237	249	189	184	**177**	171	171	154	159	184	212
6 months	237	253	242	182	186	170	**187**	146	123	152	149	148
7 months	186	188	220	184	161	155	141	**147**	128	113	112	142
8 months	158	172	200	151	166	135	133	124	**120**	110	121	115
9 months	159	162	186	162	170	165	133	107	98	**87**	93	108
10 months	124	113	141	120	115	118	102	83	73	77	**82**	112
11 months	160	128	157	111	123	104	87	105	66	85	62	**94**

In the month of January, the assumption was made that, within each age period shown, $\frac{1}{31}$ of the total deaths occurred on each day of the month. In the case of deaths under 1 day, an infant included in this group who was born in 1934 must have died on January 1. However, even among those dying on January 1 at an age under 1 day, some were born in 1935. Therefore it was assumed that $\frac{1}{2}$ of $\frac{1}{31}$, or $\frac{1}{62}$ of the deaths under 1 day occurring in January were of infants born in 1934. Multipliers for the other age periods under 1 month were obtained by similar reasoning, and are shown in table AS. It will be sufficient to give one further illustration. Those infants dying in January 1935 at the age of 1 week (exact age 1–2 weeks), who were born in 1934 include all those dying in this age interval on January 1 to 7, inclusive, and a portion of those dying on January 8 to 14, inclusive. The number of deaths on January 1 to 7 is assumed to be $\frac{7}{31}$ of the total for the month. The number occurring on January 8 to 14 is likewise assumed to be $\frac{7}{31}$, and it is further assumed that one-half of these are of infants born in 1934. Therefore, the proportion of the total January 1935 deaths at the age of 1 week which are assumed to represent 1934 births is $\frac{7}{31}$ plus $\frac{1}{2}$ of $\frac{7}{31}$, or $\frac{21}{62}$.

By the application of these rules, the estimated total number of deaths under 1 year in 1935 of infants born in 1934 is found to be 14,236 to the nearest integer, while the total number of deaths under 1 year in 1935, irrespective of the year of birth, is 68,805. Therefore, the value of f_0^{1935} is the quotient of 14,236 by 68,805, or .207.

TABLE AS.—PROPORTION OF JANUARY DEATHS UNDER 1 MONTH ASSUMED TO REPRESENT BIRTHS OF THE PREVIOUS YEAR

Age at death	Under 1 day	1 day	2 days	3 to 6 days	1 week	2 weeks	3 weeks to 1 month
Assumed proportion born in previous year	$\frac{1}{62}$	$\frac{3}{62}$	$\frac{5}{62}$	$\frac{19}{62}$	$\frac{21}{62}$	$\frac{35}{62}$	$\frac{52}{62}$

However, this value applies to all males of all races combined; and it is desired to obtain values for the different races separately, as f_0^z is known to vary significantly by race. A difficulty is encountered in that the tabulation of infant deaths in the United States by age and month of death was further subdivided only by sex prior to 1939; and commencing with that year, even the sex classification was eliminated.[19] However, for all the years involved in the life table calculations, another tabulation was available giving infant deaths for the United States by age, race, and sex (but not by month of death). Separation factors at age 0 by race and sex for the years 1939 to 1941 were obtained by making the assumption that, within each age period, the distribution of deaths by race and sex was the same in each calendar month of death as for the entire calendar year. The values for the years 1934 to 1938 had previously been calculated by a somewhat less refined method, and were not recomputed. The values of f_0^z actually employed for each of the years 1934 to 1941 are given in table AT.

TABLE AT.—SEPARATION FACTORS AT AGE 0 (VALUES OF f_0^z) BY RACE AND SEX: UNITED STATES, 1934–1941

YEAR	WHITE		NEGRO		OTHER RACES	
	Male	Female	Male	Female	Male	Female
1934	0.187	0.198	0.216	0.226	0.291	0.319
1935	.199	.210	.210	.215	.302	.304
1936	.181	.191	.216	.221	.275	.315
1937	.196	.204	.214	.219	.277	.310
1938	.177	.138	.222	.223	.296	.332
1939	.180	.191	.226	.231	.304	.348
1940	.162	.174	.202	.209	.270	.320
1941	.168	.180	.223	.230	.328	.310

As no data were available for the United States from which separation factors for ages 1 to 4 could be estimated, the values employed by Glover [20] were again used. These are given in table AU.

TABLE AU.—SEPARATION FACTORS USED AT AGES[1] 1 to 4

	1	2	3	4
Separation factor f_z	0.410	0.470	0.480	0.480

[1] Age denoted by z.

It will be noted that the values are given by age only, and are assumed independent of sex or race. As the values used by Glover were based on German

[19] This was resumed in the tabulation of infant deaths for 1943.

[20] U. S. Bureau of the Census, *op. cit.*, p. 340.

statistics of 1911 and prior years, their appropriateness for use in connection with recent data for the United States was tested before they were used for this purpose. A technical explanation of the test which was applied is given in section A of the appendix.[21]

Adjustment of mortality rates at ages 0 to 4 for the effect of migration

In the method previously described for obtaining rates of mortality at ages 0 to 4, it was assumed that the population under observation was not affected by migration during the period and at the ages considered, and that the deaths allocated to each annual cohort of births included all the deaths occurring in the cohort, and no deaths outside the cohort. Actually, it must be supposed that the deaths reported included some deaths of children born outside the continental limits of the United States, and failed to include some deaths of infants born in the United States who died outside. Some indication of the effect of immigration can be gained from the census tabulations of foreign-born population. The effect of emigration is more difficult to appraise, but is believed to have been negligible at the ages and during the period under consideration, and was therefore ignored. In other words, it was assumed that the native population under age 5 on the date of the census included all the survivors of births of the 5-year period ending on that date.

The method employed to allow for the effect of immigration involves certain concepts which make it necessary to refer briefly to the calculation of death rates at ages 5 and over. The central death rate is defined in terms of the life table as[22]

$$m_x=\frac{d_x}{L_x} \qquad (21)$$

In other words, it is the number of deaths occurring during a year in the stationary life table population at age x last birthday, divided by the total number of persons at age x last birthday in the stationary population. When the life table covers a short period, such as 1 or 3 years, it is usually assumed that this is equal to the central death rate computed from the actual data: that is,

$$m_x=\frac{D_x}{nP_x} \qquad (22)$$

where D_x denotes the number of deaths in the period of observation at age x last birthday, P_x denotes the population at age x last birthday at the middle of the period, and n denotes the number of years in the period. This assumption serves to bridge the gap between the actual population and the ideal life table population. Under this method, migration presents no difficulty if it can be assumed that the net migration has been uniformly spread over the period. For, in that event, the adjustment required in the number of person-years of exposure to the risk of dying is $n/2$ times the net migration, and since the population at the *middle* of the period has already been subjected to about half the net migration for the entire period (and is multiplied by n in the formula), the necessary adjustment is automatically taken care of.

This method of obtaining mortality rates was not used at the very young ages because of the known deficiency in the census enumeration. However, the procedure actually followed, while designed to produce estimated populations corrected for underenumeration, yields an estimate of the native population only (ignoring emigration). Now, formula (22) can be written in the form:

$$m_x=\frac{D_x}{nP_x^N}\,\frac{P_x^N}{P_x}=m_x^N\,\frac{P_x^N}{P_x}$$

when P_x^N denotes the native population at age x last birthday at the middle of the period, and m_x^N denotes an approximate value of m_x, in which the native population, rather than the total population, has been used as the denominator. Since the value of m_x obtained from births and deaths by the process described is really m_x^N, it needs to be corrected by multiplying by the factor P_x^N/P_x.

If it is assumed (as it usually is) that, in the life table, $L_x=l_x-\frac{1}{2}d_x$, it follows that[23]

$$m_x=\frac{2q_x}{2-q_x} \qquad (23)$$

or, solving for q_x,

$$q_x=\frac{2m_x}{2+m_x} \qquad (24)$$

Therefore, it would be possible to convert the values of q_x obtained without considering migration into values of m_x by formula (23), multiply them by P_x^N/P_x, and then convert them back to q_x values by formula (24). However, this lengthy procedure is unnecessary, for the ratio P_x^N/P_x is always very close to unity, and thus represents only a slight adjustment; and putting equation (24) in the form:

$$q_x=m_x\left(1+\frac{1}{2}m_x\right)^{-1}$$
$$=m_x-\frac{1}{2}m_x^2+\ \ldots$$

shows that a slight adjustment in the value of m_x results in a very nearly proportional adjustment in q_x. Therefore, the adjustment factor P_x^N/P_x may, without appreciable error, be applied to the values of q_x directly.

In the case of the life tables in this volume, P_x^N and P_x should properly represent populations on July 1, 1940, the midpoint of the 3-year period 1939–1941. However, since the adjustment involved is small in any case, it

[21] See p. 135.

[22] See pp. 21–22 for definition and explanation of the life table functions.

[23] Spurgeon, E. F., *Life Contingencies*, third edition, pp. 4–5, Cambridge University Press, London, 1938.

was felt that little error would result in calculating this ratio from populations on the census date (April 1, 1940). Hence, the actual procedure at ages 1 to 4, was merely to multiply the unadjusted rate of mortality by the ratio of the native population to the total population, as enumerated in the census, at the corresponding age and in the same classification by race and sex. This, of course, involves the assumption that the enumeration was equally complete for the native and foreign-born elements of the population. The method used in estimating the distribution of the foreign-born under age 5 by single years of age has already been described,[24] and the resulting distribution by nativity, race, and sex of the population on April 1, 1940, is given in part I of table AM.[25]

The above method is not appropriate for adjusting the mortality rate at age 0, because in that case, the small amount of immigration which occurs is believed to be heavily concentrated in the latter part of the year of life, while the mortality is very much heavier in the early part. Therefore, the application of the ratio P_0^N/P_0 to the mortality rate q_0 would greatly overstate the amount of the necessary correction. Hence, the expedient was adopted of applying the adjustment ratio to the mortality rate for the second portion only of the first year of life: that is, to the probability ${}_\delta q_0=1-{}_\delta p_0$.

The numerical illustration showing the calculation of mortality rates for white males in the United States in 1939–1941 is completed, for ages 1 to 4, in table AW which exhibits the adjustment for the effect of migration.

TABLE AW.—ADJUSTMENT OF RATES OF MORTALITY FOR WHITE MALES AT AGES[1] 1 TO 4, TO ALLOW FOR IMMIGRATION: UNITED STATES, 1939–1941

	1	2	3	4
Unadjusted q_x	0.00487475	0.00264755	0.00189876	0.00153612
Adjustment factor[2]	.99935557	.99911903	.99879579	.99854398
Adjusted q_x	.0048716	.0026452	.0018965	.0015339

[1] Age denoted by x.
[2] Estimated native white male population at age x divided by total white male population at age x, April 1, 1940. See table AM, part I, p. 113.

In the case of age 0, formulas (16) and (17) give ${}_\alpha p_0=.96029016$ and ${}_\delta p_0=.99124082$. It follows that ${}_\delta q_0$, the complement of ${}_\delta p_0$, is .00875918. Multiplying this value by the adjustment factor .99973095, which is the quotient of the number of native white males enumerated at age 0 by the total white males so enumerated, gives .00875682 as the corrected value of ${}_\delta q_0$. The complement ${}_\delta p_0$, which is .99124318, multiplied by ${}_\alpha p_0$ gives .9518811 as the adjusted value of p_0. The complement .0481189 is the final value of q_0.

There is a criticism of the theory underlying the method adopted in correcting for the effect of migration the mortality rates at ages under 5, in that the deaths which were deducted from the recorded births in order to obtain the number of survivors at the various ages include some deaths of children born outside the United States, so that the number of survivors of the native births is understated. As the deaths improperly deducted are very few, the resulting error is slight, and in any case serves as a partial offset to the failure to take account of emigration.

Grouping of ages for the computation of rates of mortality at ages 5 and over

Deaths at ages 5 and over were not tabulated by single years of age during the period 1939–1941, but only in the 5-year age groups 5–9, 10–14, etc., with a final group at ages 100 and over. As a matter of fact, it has frequently been considered preferable, in the construction of national life tables, to work with grouped data for the reason that statements of age, both in death reports and in the census, usually show what is known as "heaping": that is, marked preference for ages ending in certain digits, at the expense of other digits. This preference is especially noticeable in the case of ages which are multiples of five; while, to a lesser degree, even numbers tend to be given more frequently than odd numbers. A notable exception to the latter rule is observed at age 21, where a marked concentration is commonly found. The use of grouped data tends to smooth out the irregularities resulting from digit preference by averaging together ages at which the reported figures are excessive and other ages where a deficiency appears.

However, the particular grouping in which the 1939–1941 deaths were tabulated has not often been found the most satisfactory from the point of view of life table construction.[26] Glover had both deaths and populations tabulated by single years of age, and made an exhaustive study [27] of the results of all the possible methods of grouping in 5-year periods, finally deciding on the grouping 4–8, 9–13, etc. Wolfenden [28] has also given a very full discussion of the general problem of heaping and the conclusions reached by a number of actuaries as to the best method of age grouping for the data of various countries. In dealing with the 1939–1941 data, there was, however, no choice as to the mode of grouping, insofar as deaths are concerned. While the census populations were available by single years of age, the estimated populations on July 1, 1940, were much more easily obtained for the age groups in which deaths were available, and the computation of rates of mortality is appreciably simplified by having deaths and populations similarly grouped.

Nevertheless, it was thought advisable to study the nature of the heaping present in the population data of the 1940 census and to test the effect of various

[24] See p. 112.
[25] See p. 113.

[26] See, however, Nathan Keyfitz, *Census Monograph No. 13, Canadian Life Tables, 1931*, p. 8, Dominion Bureau of Statistics, Ottawa, 1937. Here, the "5–9" grouping was decided upon, even though both populations and deaths were available by single years of age.
[27] U. S. Bureau of Census, *op. cit.*, pp. 356–364.
[28] Wolfenden, *op. cit.*, pp. 32–44, 54–57. See also Wolfenden's discussion in the Transactions, Actuarial Society of America, vol. 42, Part 1, No. 105, pp. 78–86, May 1941.

possible groupings. This was done by summing the reported figures for ages ending with the same digit and comparing the totals by means of Myers' "blended" method.[29] For comparison, the deaths of the year 1935, the most recent year for which deaths have been tabulated by single years of age, were analyzed in the same way. In this method of analysis, the ages below 20 are omitted, because they exhibit a pattern of digit preference which differs markedly from that observed at adult ages. The ages in the immediate neighborhood of age 21 may also be omitted because of the peculiar form of heaping usually present there.[30] Myers' blended method is designed to eliminate any bias due to a particular choice of the starting age.

In this case, ages 23 to 32 were employed as starting ages and the summations were not carried beyond age 99.[31] The results are shown in table AY. In this table, Negroes and other races are not shown separately, because these separate races were not tabulated by single years of age in the 1940 census. In interpreting the table, it should be noted that the extent of heaping or deficiency at any particular digit is indicated by the amount by which the percent shown for that digit differs from 10 percent. The "index of preference," which is the sum of the absolute deviations from 10 percent, is a useful general measure of the amount of bias present. The smaller the index, the less error is present, since if there were no bias, all the percentages would be exactly 10 percent, and the index would be 0.

TABLE AY.—PREFERENCE FOR DIGITS OF AGE BY RACE AND SEX, IN THE UNITED STATES, FOR 1935 DEATHS AND 1940 CENSUS POPULATIONS: NUMBERS REPORTED AT EACH DIGIT OF AGE [1] AS PERCENT OF TOTAL NUMBER

DIGIT OF AGE	1935 DEATHS					1940 POPULATIONS				
	Total deaths	White		Nonwhite		Total population	White		Nonwhite	
		Male	Female	Male	Female		Male	Female	Male	Female
0	11.1	10.5	10.6	15.8	15.9	11.6	11.0	11.5	14.6	15.0
1	8.7	9.0	8.9	7.4	7.3	8.5	8.8	8.6	6.8	6.3
2	10.0	10.0	10.0	9.8	9.6	10.4	10.5	10.4	10.1	9.9
3	9.7	9.9	9.8	8.4	8.4	9.6	9.8	9.6	8.3	8.2
4	10.1	10.2	10.3	8.9	9.1	9.7	9.9	9.8	9.0	8.8
5	11.4	11.0	10.9	14.5	14.1	10.7	10.5	10.6	12.5	12.4
6	9.6	9.7	9.8	8.4	8.6	9.6	9.7	9.7	9.0	9.0
7	9.6	9.8	9.7	8.2	8.4	9.6	9.7	9.6	8.8	8.7
8	10.1	10.1	10.1	9.7	9.7	10.3	10.1	10.3	10.6	11.2
9	9.7	9.8	9.9	8.9	8.9	10.0	10.0	9.9	10.3	10.5
Index of preference [2]	5.4	3.6	3.8	20.6	20.0	6.0	4.2	5.6	16.2	18.2

[1] Computed by Myers' blended method, using starting ages 23 to 32 and ending at age 99 in all cases.
[2] Sum of deviations from 10 percent, taken without regard to sign.

Inspection of the values of the index of preference shows, as might be expected, that the error is much more serious for the nonwhite than for the white races. Among white persons, there is slightly greater bias in the populations than in the death statistics; but among the nonwhite the reverse is true. In fact, in the nonwhite deaths, the heaping on digits 0 and 5 is so pronounced that all the other digits show a deficiency. Table AZ shows the value of the index of preference for the total population in each census from 1880 to 1940. With the exception of the 1940 figure, these values are taken from Myers' article.[32] This table indicates a steady improvement over the entire period in the accuracy of age statements. The relatively low figure for 1900 is due to the fact that in that census both age and date of birth were asked for, while in other censuses only age was obtained.

TABLE AZ.—INDEX OF PREFERENCE IN STATEMENTS OF AGE IN THE CENSUS OF POPULATION: UNITED STATES, 1880–1940

CENSUS	Index of preference	CENSUS	Index of preference
1880	20.8	1920	9.0
1890	15.6	1930	8.6
1900	9.4	1940	6.0
1910	11.2		

The percents in table AY may be used to test the effectiveness of different grouping methods by adding the percentages for the five digits which are combined in the particular grouping method. The closer the resulting total is to 50 percent, the better is the given method. Table BA shows the results obtained with the data of table AY. If it can be assumed that the pattern of digit preference among the 1939–1941 deaths was similar to that found in 1935, evaluation of table BA purely on the basis of the proximity of the totals to 50 percent would indicate the best groupings for deaths to be "1–5" for whites and "2–6" for nonwhites; while for the populations the preferred groupings would be either "4–8" or "5–9" for whites and "4–8" for nonwhites. However, in computing rates of mortality, if the same grouping is to be used for both populations and deaths, it is of little avail to select the most effective grouping for populations if this grouping produces marked bias in the death figures, and vice versa. On the other hand, the correct mortality rates will be obtained, even with considerable error in both population and death statistics, if both are deficient or both excessive in the same proportion. This suggests choosing as the best age grouping for mortality rate calculations the one in which the smallest difference is found between the percents in table BA for deaths and populations. This criterion indicates as the best groupings "5–9" for whites and "4–8" for nonwhites. Since the "5–9" grouping appears to be an advantageous one for the data of white lives, and no other grouping is actually available in the census for Negroes and other races separately, and in view of the simplification which results from employing the same grouping for both populations and deaths, it was decided to use the "5–9" grouping throughout.

[29] Myers, Robert J., *Errors and Bias in the Reporting of Ages in Census Data*, Transactions, Actuarial Society of America, vol. 41, Part 2, No. 104, pp. 395–415, October–November 1940. See especially pp. 402–407, 411–415.

[30] See p. 120.

[31] For the details of Myers' method, see his article, previously cited.

[32] Myers, *op. cit.*, p. 403.

TABLE BA.—PERCENTAGE OF TOTAL REPORTED IN VARIOUS QUINQUENNIAL AGE GROUPINGS IN THE UNITED STATES, FOR 1935 DEATHS AND 1940 CENSUS POPULATIONS[1]

DIGIT GROUPING	1935 DEATHS					1940 POPULATIONS				
	Total deaths	White		Nonwhite		Total population	White		Nonwhite	
		Male	Female	Male	Female		Male	Female	Male	Female
1-5	49.0	50.1	49.9	49.0	48.5	48.9	49.5	49.0	46.7	45.6
2-6	50.8	50.8	50.8	50.0	49.8	50.0	50.4	50.1	48.9	48.3
3-7	50.4	50.6	50.5	48.4	48.6	49.2	49.6	49.3	47.6	47.1
4-8	50.8	50.8	50.8	49.7	49.9	49.9	49.9	50.0	49.9	50.1
5-9	50.4	50.4	50.4	49.7	49.7	50.2	50.0	50.1	51.2	51.8

[1] The figures in this table were obtained by summing the appropriate ones in table AY.

General procedure used in obtaining rates of mortality at ages 5 and over

The method used in obtaining mortality rates for individual years at age from the grouped data at ages 5 and over was that of osculatory interpolation. This method has been used for many years in the construction of the national life tables of England and Wales, and the United States, and was adopted in the most recent official life tables of Canada and Australia. It produces a satisfactory degree of smoothness while at the same time yielding mortality rates which fit the original data closely. Osculatory interpolation may be defined as that method of interpolation which insures smooth junction between the curves representing the interpolated values in adjacent tabular intervals by requiring that such adjacent curves have the same first derivative (or, sometimes, the same first and second derivatives) at the point of junction.[33]

In applying the principle of osculatory interpolation to the construction of life tables, there are two possible methods of approach. In the first method, osculatory interpolation is applied to the populations and deaths separately in order to obtain smooth interpolated values for single years of age. The rates of mortality are then computed by relating the interpolated values for deaths and population at each age. In the second method, "pivotal" rates of mortality are obtained at specified intervals, and osculatory interpolation is then applied directly to the mortality rates, in order to fill in the intermediate values. The pivotal rates are obtained by first deriving pivotal values of populations and deaths separately from quinquennial (or other) sums of data, usually by ordinary interpolation, the interpolation process being sometimes combined with a certain amount of graduation, or smoothing.

There has been much discussion of the relative merits of these two methods of approach. The first method was introduced by Dr. John Tatham and used by him in constructing the English Life table number 6, covering the period 1891–1900. It was improved by George King, and in this improved form was adopted in this country by Glover and Foudray and has been used in all previous United States life tables. The second method was introduced by George King in connection with the English Life tables numbers 7 and 8, and has been followed by Sir Alfred Watson in preparing the subsequent tables numbers 9 and 10. It has also been used in the most recent official life tables for Canada and Australia. For the former method it is argued that by its use the investigator is enabled to keep closer to the original data, and can test the reasonableness of the interpolated results in the light of his knowledge of the basic characteristics of the populations he is dealing with. The method also has the practical advantages that it requires no decision as to the ages at which pivotal values are to be calculated or the formula to be used in obtaining them, and that mortality rates for any combination of the original population classes can be readily obtained without performing a new interpolation. Such a case, for example, would be the preparation of a life table for total whites, after separate tables for white males and white females had been completed.

For the second method it may be argued that all mathematical formulas of interpolation, particularly those of the osculatory variety, are based on the assumption that the values being estimated can properly be expected to form a smooth series. Now, it can reasonably be expected that, with a large enough body of data, the rates of mortality should exhibit a smooth progression from age to age. However, the populations and deaths at single ages, arising as they do from fluctuating annual cohorts of births, and affected to a considerable extent by the incidence of past migration, can hardly be expected to be perfectly smooth. Hence, the assumption underlying the use of an interpolation formula is not entirely valid when it is applied to such data. There is also a practical advantage in that only one complete interpolation is required, as against the two separate interpolations needed in the other method. Also, the second method is found, in general, to produce a smoother series, because the graduating effect of the osculatory formula is applied directly to the mortality rates. A further point is made by Sir George Hardy, who states[34] that in "graduating separately the numbers in the two series of 'exposed to risk' and 'died' rather than their ratio, . . . we thereby discard our previous knowledge of the nature of the curve expressing that ratio—our general knowledge, that is, of the nature of the curve q_x or μ_x."

In the preparation of the present life tables, careful consideration was given to the choice as between the two general methods of procedure, and experimental calculations were made by both methods. In the end, the method of operating directly on the rates of mortality was adopted, as it was found to produce smoother

[33] For a synopsis of the theory of osculatory interpolation and of the historical development of the subject, see Hugh H. Wolfenden, *The Fundamental Principles of Mathematical Statistics*, pp. 124-132, Actuarial Society of America, New York, 1942.

[34] Hardy, G. F., *The Theory of the Construction of Tables of Mortality and of Similar Statistical Tables in Use by the Actuary*, p. 21, Charles and Edwin Layton, London, 1909.

values, and the theoretical arguments in its favor seemed more cogent. Pivotal values of both populations and deaths were obtained by interpolation for the middle age of each of the age groups used: that is, at ages 7, 12, 17, etc., and the corresponding pivotal rates of mortality were obtained by the usual formula:

$$q_x = \frac{D_x}{nP_x + \frac{1}{2}D_x} \qquad (25)$$

where D_x and P_x denote the pivotal values of deaths and populations, respectively, and n is the number of years in the period of observation: in this instance, 3. This formula is obtained at once by substituting in formula (24) the value of m_x given by formula (22). On the basis of these pivotal rates, values of q_x were obtained by osculatory interpolation for all integral ages from age 5 to the limiting age of each life table. The formulas used in obtaining pivotal values and in performing the osculatory interpolation, the method of securing smooth junction with the mortality rates at ages under 5, and the special devices adopted to extend the tables into the very high ages where the use of actual data leads to unreasonable results, are described in the sections which follow.

Pivotal value formulas employed

The pivotal value formula employed in the majority of cases was the usual King formula, which, written in central difference notation, is:[35]

$$v_x = .2w_x - .008\delta^2 w_x \qquad (26)$$

where v_x denotes an interpolated value for the single year of age x; w_x denotes a quinquennial sum of data centered on age x: in other words, $w_x = \sum_{t=-2}^{2} u_{x+t}$, where the "$u$'s" denote unadjusted single year values; and the symbol δ denotes a central difference[36] taken at quinquennial intervals. In other words, if data (e. g., deaths or populations) are available for three consecutive 5-year age groups, this is a formula for estimating the number at the single age in the middle of the middle group. If the single year values for all 15 ages are exactly fitted by a third degree polynomial, this formula gives exactly the correct value. The assumption is, therefore, that the single year values would be approximately fitted by a third degree polynomial if they were unaffected by age heaping or sampling error. To facilitate the numerical computation, the formula was put in the alternative form:

$$v_x = -.008w_{x-5} + .216w_x - .008w_{x+5} \qquad (27)$$

which was used (with certain exceptions to be noted later) to compute pivotal values of populations and deaths at each fifth age from age 12 to 97. The pivotal values for populations were taken to the nearest integer; those for deaths, to two places of decimals. In applying formula (27) to obtain pivotal values at age 97, figures for the age group 100 and over were used as though they represented the age group 100–104.

Applying King's formula to obtain a pivotal value at age 7 would involve substituting in the formula a value of w_2, which would be a sum of data for the age group 0–4. It was not considered proper to regard such a figure as belonging to the same series with the other "w" values: in the case of the deaths, because of the special mortality conditions prevailing in the first year of life; and in the case of the populations, because of the substantial underenumeration of infants and small children in the census. Hence, the pivotal values at age 7 were obtained by the following special formula based on ordinary interpolation from sums of data for the three age groups 3–4, 5–9, and 10–14, assuming that the 12 single year values can be fitted by a second degree curve:

$$v_7 = \frac{1}{700}\left[-25(u_3+u_4) + 157w_7 - 7w_{12}\right] \qquad (28)$$

To derive this formula, suppose that $u_{7+x} = a + bx + cx^2$. Then,

$$\begin{aligned} u_7 &= a \\ w_7 &= 5a + 10c \\ w_{12} &= 5a + 25b + 135c \\ u_3 + u_4 &= 2a - 7b + 25c \end{aligned}$$

Now if it be assumed that $u_7 = m(u_3+u_4) + nw_7 + rw_{12}$, substituting the above expressions and equating coefficients of a, b, and c gives:

$$\begin{aligned} 2m + 5n + 5r &= 1 \\ -7m + 25r &= 0 \\ 25m + 10n + 135r &= 0 \end{aligned}$$

Solving these equations yields $m = -1/28$, $n = 157/700$, and $r = -1/100$, which are precisely the coefficients in formula (28).

The other exceptions made to the use of King's pivotal value formula were confined to the life tables for Negroes and other races. In working with Negro data it has often been found that the substantial amount of heaping present tends to produce cyclical fluctuations or waves which give to certain portions of the graph of the q_x function somewhat the appearance of a sine curve superimposed on the basic mortality curve. This condition is quite apparent in the published graphs of the q_x function in certain previous United States life tables.[37] However, this peculiarity can scarcely be considered a genuine characteristic of the data and there would seem to be little justification for reproducing it in the life table.

It will be remembered that in the discussion of digit preference in age statements[38] the "5–9" grouping was found to be not the most desirable for the nonwhite

[35] For a derivation of King's formula, see pp. 109–110 of Wolfenden's Actuarial Study, previously cited.

[36] Freeman, Harry, *Mathematics for Actuarial Students*, vol. 2, p. 76, Cambridge University Press, London, 1939.

[37] U. S. Bureau of the Census, *op. cit.*, p. 245; and *United States Life Tables, 1930–1939 (Preliminary), for White and Nonwhite, by Sex*, pp. 12–14, July 1941.

[38] See p. 121.

data. In fact, table BA shows that in the digit grouping 5–9, the nonwhite populations are overstated, while the nonwhite deaths are understated. In the digit grouping 0–4, the reverse would of course be true. This would mean that the rate of mortality would be consistently understated in the groups consisting of ages ending with the digits 5–9, and consistently overstated in the "0–4" groups, producing just the sine curve effect so frequently observed. When pivotal values were obtained by King's formula, this tendency was clearly observed from age 30 to about age 60, where it became obscured by more serious errors in age statement.[39] Although the osculatory interpolation formula used has a moderate graduating effect, this was found not to eliminate the waviness entirely. Therefore, it was decided to use also a pivotal value formula which incorporates an element of graduation.

The formula selected for this purpose was [40]

$$v_x=\frac{1}{7}\left[.696w_x+.488(w_{x+5}+w_{x-5})-.136(w_{x+10}+w_{x-10})\right] \quad (29)$$

This formula gives the middle term of a 25-term series summed in five groups of five, on the assumption that the individual terms can be represented by a third degree curve. However, it is not unique in this respect, as an infinite number of other formulas exist which have the same property. Its uniqueness lies in the fact that, of the entire class of such formulas, this is the one for which the mean square error of the interpolated value, v_x, is least, on the assumption that the mean square errors of the five sums of "u" values are all equal.[41]

This formula involves the assumption that the "true values," after adjusting for errors in the data, of any five consecutive age groups will be exactly fitted by a third degree curve. There are certain portions of the mortality curve in which this assumption is unsuitable. For both Negroes and "other races," this is true of the ages under 30, where the death statistics form a curve with very rapidly changing curvature, and where, in any case, the tendency to "waviness" is not apparent. Here the use of formula (29) was found to produce unwarranted distortion in the mortality rate; accordingly, King's formula was used. For the Negroes, a similar situation exists beyond age 75, where both populations and deaths are decreasing so rapidly that the assumption of fitting a third degree curve to the data of five consecutive age groups was clearly inappropriate. In the case of the data for "other races," populations and deaths also decrease rapidly above age 75, but the figures are so irregular, because of the small size of the data, that the smoothing effect of the special formula (29) was needed, and the values are so rough, in any case, that any distortion resulting from the use of this formula is not of much importance. To sum up, formula (29) was used instead of King's formula in obtaining pivotal values of populations and deaths at ages 32 to 72, inclusive, for Negroes; and at ages 32 to 87, inclusive, for "other races."

Derivation of pivotal rates of mortality

Pivotal rates of mortality were computed at every fifth age from age 7 to age 97 by applying formula (25) to the pivotal values of populations and deaths. They were carried out to seven decimal places on a unit basis: that is, to four decimal places on a per 1,000 basis. The progression of these rates at the very high ages was carefully studied, and unsuitable values were rejected by inspection. In the end, the originally calculated rates were retained through age 92 for white males and females and Negro males, and through age 87 for Negro females and "other races" males and females. In the case of the life tables for combinations of classes, pivotal rates of mortality were obtained by summing separately the values used as numerators and denominators in obtaining pivotal rates for the individual classes, at all ages at which the originally calculated rates were retained for all the individual classes included.

Treatment of the very old ages

At the very old ages (those above age 90, approximately) mortality rates obtained in the conventional manner from the data as reported frequently appear unreasonable or even absurd. This condition is probably due in part to inaccuracies in age statements, and in part to random irregularities made possible by the very small size of the experience at these ages. It is customary, therefore, to reject those values which are considered unsuitable, and to end the life table in some more or less artificial manner. From a practical standpoint, it probably makes little difference what method is used for this purpose, as little reliance is placed on the values obtained at the very old ages, and they affect only slightly other life table values which are extensively used. The question may properly be raised as to why it is necessary to show life table values at all beyond those ages at which they can be considered reliable. It may be answered that, in order to obtain values of the average future lifetime and of life annuity and assurance premiums, it is necessary to assume *some* values of the rate of mortality at the oldest ages, and the user of the tables may properly wish to be informed as to what values were assumed.

In connection with the life tables included in this volume, the use of a fifth difference interpolation formula (as described in the next subsection) made it desirable to extend the series of pivotal rates of mortality in some manner, prior to performing the interpolation. This was done, in each case, by fitting a third degree curve to the last four pivotal rates retained. In carrying out the actual arithmetic, each pivotal rate

[39] See p. 110.

[40] This formula was first published in an unsigned book review in the Journal of the Institute of Actuaries, vol. 51, No. 272, p. 368, October 1919. It is also given by Wolfenden in his Actuarial Study (previously cited), p. 113.

[41] See Wolfenden's derivation of this formula, already referred to.

beyond those retained from the original series was computed from the four preceding ones by the formula:

$$u_x = 4u_{x-5} - 6u_{x-10} + 4u_{x-15} - u_{x-20}$$

In the case of the life tables for combinations of classes, pivotal rates of mortality were not calculated beyond age 92. A special problem arose at age 92 when individual classes for which the originally calculated rate had been rejected were included in the combination. In such cases the pivotal value of the number of deaths, as originally calculated, was regarded as the correct numerator, and an adjusted denominator was obtained by dividing this numerator by the extrapolated pivotal rate of mortality. These adjusted denominators were carried out to two decimal places in order to avoid inconsistency between the life tables for combinations of classes and those for the individual classes included.

Osculatory interpolation formulas used

The osculatory interpolation formula used for the main body of the life tables in this volume was Jenkins' modified fifth difference formula.[42] The word "modified" in the name of this formula indicates that, although satisfying the conditions of smooth junction, it does not exactly reproduce the pivotal rates of mortality, but has a moderate graduating effect. The advantages of using a formula of this type have been aptly expressed by the Scottish actuary, James Buchanan, who says:[43]

> The weak point of the osculatory method, regarded as a smoothing agent, rests on the fact that the graduated curve is required to pass through certain predetermined points. The curve will in fact be constrained to take a form similar to that assumed by a flexible steel wire which is clamped at fixed points, so that, while the curve is free from discontinuities, any departure of these points from the smooth curve will be reproduced with resulting undulations. To remove this tendency to waviness, Jenkins has devised his modified osculatory method, which, while requiring the successive interpolation curves to have the same slope and curvature at their common points at the end of each interval, does not require the curves to pass through the points corresponding to the calculated values.

The practice of employing such a formula in the construction of national life tables has been slow to gain general acceptance, perhaps because it has been considered that fidelity to the original data is here more fundamental than smoothness. However, experience has shown that a well chosen modified osculatory formula can usually be depended on to preserve the basic underlying trend of the mortality curve, only local irregularities being smoothed out. National life tables are being increasingly used for population projections, valuation of old-age pensions and survivors' benefits, and other calculations in which a lack of smoothness in the life table is likely to produce irregularities and inconsistencies which, although minor, can be awkward and inconvenient. Also, it may justly be argued that it is better to produce a smooth table which, in all likelihood, represents the true underlying conditions as precisely as they can be inferred from a careful analysis of the data, rather than a table which merely reproduces the data along with all the errors they are known to contain. It is a virtue of the better modified osculatory formulas that when applied to a series containing many undulations, such as rates of mortality for Negroes in the United States, they exert a considerable smoothing effect, and yet when applied to a series which is already fairly smooth, such as the corresponding rates for white persons, they produce only an insignificant change.

In the case of 5-year age intervals, Jenkins' modified fifth difference formula can be written in the form:[44]

$$v_{a+t} = \frac{s}{5}\left(u_a - \frac{1}{36}\delta^4 u_a\right) + \frac{s(s^2-25)}{750}\left(\delta^2 u_a - \frac{1}{6}\delta^4 u_a\right) + \frac{t}{5}\left(u_{a+5} - \frac{1}{36}\delta^4 u_{a+5}\right) + \frac{t(t^2-25)}{750}\left(\delta^2 u_{a+5} - \frac{1}{6}\delta^4 u_{a+5}\right) \quad (30)$$

where u_a and u_{a+5} denote consecutive pivotal values, δ denotes a central difference as before, t is a number between 0 and 5, $s=5-t$, and v_{a+t} denotes the interpolated value obtained by the formula. This formula produces contact of the second order: that is, the interpolation curves in any two adjacent age intervals have equal ordinates and equal first and second derivatives at their point of junction. It may be noted that this formula gives, on substituting $t=0$ and 5, respectively:

$$v_a = u_a - \frac{1}{36}\delta^4 u_a \quad (31)$$

$$v_{a+5} = u_{a+5} - \frac{1}{36}\delta^4 u_{a+5} \quad (32)$$

These results show that the pivotal values are adjusted by the formula to the extent of 1/36 of the negative of the corresponding fourth central difference. Substituting the expressions (31) and (32) and writing $\delta^2 y_a$ for $\delta^2 u_a - \frac{1}{6}\delta^4 u_a$ the equation (30) becomes:

$$v_{a+t} = \frac{s}{5}v_a + \frac{s(s^2-25)}{750}\delta^2 y_a + \frac{t}{5}v_{a+5} + \frac{t(t^2-25)}{750}\delta^2 y_{a+5} \quad (33)$$

In using a formula which appears in this symmetrical form, the arithmetic can be considerably shortened by

[42] Jenkins, W. A., *Graduation Based on a Modification of Osculatory Interpolation*. Transactions, Actuarial Society of America, vol. 28, Part 2, No. 78, p. 202, October 1927. The formula is also given (in a form more closely resembling that employed in this volume) by Robert Henderson, *Mathematical Theory of Graduation (Actuarial Studies No. 4)*, second edition, p. 22, Actuarial Society of America, New York, 1938.

[43] Buchanan, James, *Recent Developments of Osculatory Interpolation, With Applications to the Construction of National and Other Life Tables*, Transactions of the Faculty of Actuaries (Scotland), vol. 12, Part 5, No. III, pp. 117–160, 1929.

[44] The form given here differs from that given by Jenkins and Henderson for the reason that here the single year of age is taken as the unit of reckoning, while in the other formulations the unit is the entire interval of interpolation (in this instance, 5 years). The formula given here is readily obtained from Henderson's expression upon replacing x by $t/5$ and y by $s/5$. Jenkins' original statement of the formula was in terms of advancing differences rather than central differences.

employing a special computation process in which the results of certain calculations are used twice.[45]

In the construction of all the life tables in this volume, this formula was used for interpolation from age 32 to the end of the table. As stated in the preceding subsection, the series of pivotal rates of mortality was extended to the very old ages by fitting a third degree curve to the last four of the original pivotal rates actually used, which is, of course, equivalent to assuming fourth differences to be 0. Under these conditions, formula (30) reduces to:

$$v_{a+t}=\frac{s}{5}u_a+\frac{s(s^2-25)}{750}\delta^2u_a+\frac{t}{5}u_{a+5}+\frac{t(t^2-25)}{750}\delta^2u_{a+5}$$

which is merely the ordinary Everett interpolation formula[46] for quinquennial intervals. This shows the special convenience, in connection with Jenkins' modified fifth difference formula, of the particular method chosen for terminating the life tables. It may be noted that, in carrying out the extrapolation for the very old ages, the second differences δ^2u_a were values of a first degree curve (or straight line), and could therefore be obtained by the formula:

$$\delta^2u_a=2\delta^2u_{a-5}-\delta^2u_{a-10} \qquad (34)$$

This formula holds at the last age for which the calculated pivotal rate was retained, and at subsequent ages.

In the case of the life tables for combinations of classes, it was found that interpolation of the rates of mortality beyond age 92 would, in some instances, give results inconsistent with the rates for the component classes. Therefore, in all these tables, the interpolation was terminated at that point, and mortality rates for subsequent ages were obtained from the l_x column of the life table, which was itself derived by a special process to be explained later. The value of δ^2q_{92} to substitute in the interpolation formula was obtained by equation (34). This, of course, implicitly assumes the existence of an extrapolated pivotal rate at age 97.

Because of the rapid change of curvature of the q_x curve at ages under 30, and the small size of the rate of mortality at these ages, the fourth differences of q_x are quite large in relation to the values of q_x itself, and an excessive adjustment is introduced by Jenkins' formula, which has the effect of replacing the pivotal values originally calculated by adjusted values obtained by formula (31), involving a fourth difference correction. Moreover, the mortality curve commonly displays genuine irregularities at these ages, which it is not desirable to remove by a smoothing process. Therefore, it seemed the wisest course to use a formula which would reproduce the pivotal values. The formula selected was the familiar Karup-King formula,[47]

$$v_{a+t}=\frac{s}{5}u_a+\frac{s^2(s-5)}{250}\delta^2u_a+\frac{t}{5}u_{a+5}+\frac{t^2(t-5)}{250}\delta^2u_{a+5} \qquad (35)$$

This formula was used for interpolation in all the life tables between ages 12 and 27.

Between ages 4 and 12 and between 27 and 32, special extensions were devised in order to secure smooth junction, in the one case with the mortality rates under age 5 specially computed from birth and death statistics, and in the other case with the rates above age 32 interpolated by Jenkins' formula. Inasmuch as both the two interpolation formulas are of the third degree, third degree curves were employed for the special extensions as well. The curve used for ages 5 to 11 was required to reproduce the calculated rates of mortality at ages 4, 7, and 12, and to have the same derivative at age 12 as the Karup-King curve used between ages 12 and 17. The curve used for ages 28 to 31 was required to have its ordinate and first derivative equal to those of the adjoining Karup-King curve at age 27 and to those of the adjoining Jenkins curve at age 32. In both cases, there are four conditions imposed, and this is enough to determine a third degree curve. In each case also, it was possible to regard the interpolation by the special curve as merely a further application of the Karup-King formula, by utilizing a suitable artificial extension of the series of pivotal values.[48]

Seven decimal places were retained throughout the interpolation process, and the resulting interpolated rates of mortality were rounded to six places. They are further rounded to five places (or two places on a per 1,000 basis) in the published tables.

Test of the graduation of the rates of mortality

Tests were applied to the final rates of mortality in each of the six life tables for individual classes of the population to determine whether the graduation could be deemed satisfactory. It was not considered necessary to test separately the mortality rates for combinations of classes. In making such tests, there are two chief points to be considered: (1) conformity to the original data, and (2) smoothness. Conformity to the original data is usually tested by calculating, for each age group, the number of deaths expected on the basis

[45] Freeman, *op. cit.*, pp. 73–75. See also T. N. E. Greville's discussion in the Record, American Institute of Actuaries, vol. 32, Part 1, No. 65, pp. 86–87, June 1943. See also Louis I. Dublin and Alfred J. Lotka, *Length of Life*, pp. 338–339, The Ronald Press Co., New York, 1936.

[46] Freeman, *op. cit.*, p. 66. The form given here may be obtained from Freeman's expression by substituting central differences for advancing differences, changing the origin so that a corresponds to Freeman's "0," and replacing x by $t/5$ and ξ by $s/5$.

[47] This formula was first published by Johannes Karup in his article, *On a New Mechanical Method of Graduation*, Transactions of the Second International Actuarial Congress, p. 83, Charles and Edwin Layton, London, 1899. It was discovered independently by George King who published it in the Journal of the Institute of Actuaries, vol. 41, p. 545, October 1907. Since its publication by King, it has been used extensively in the construction of national life tables, both in England and elsewhere. The formula is also given, in three different forms, by Wolfenden in his Actuarial Study (previously cited), p. 105. The expression given here is obtained at once from Wolfenden's form (c) upon replacing x by $t/5$ and y by $s/5$, and changing the origin so that a corresponds to Wolfenden's "0."

For a discussion of computation methods, see John Boyer, *Osculatory Interpolation in Practice*, Record, American Institute of Actuaries, vol. 31, Part 2, No. 64, pp. 337–338, October 1942. A method similar to that mentioned in connection with the Jenkins formula can also be employed.

[48] The formulas which were used for this purpose are derived in the appendix, p. 136.

of the calculated rates of mortality, and comparing this with the number of deaths actually reported. This would seem to be a simple enough procedure, but, in dealing with grouped data, questions immediately arise as to the proper method of calculating the expected deaths. The traditional method consists in multiplying the population at each single age by the number of years in the period of exposure and by the value of m_x at that age, based on the life table. In the present case, however, the populations used were estimated populations on July 1, 1940, and were not obtained by single years of age. Nor could such values be made available without considerable additional work, and without making some assumption as to the distribution of deaths by single ages. As an approximation to this procedure, experiments were made with the expedient of distributing the population in each 5-year age group into single years of age in the same proportion as the corresponding population on April 1, 1940, the date of the census. In the case of white males and white females, this method gave numbers of expected deaths consistently smaller than the corresponding number of reported deaths, although the differences were extremely small in most cases. This condition resulted from the fact that the greatest "heaping" occurs at the ages ending with the digits 0 and 5, and in the "5–9" mode of grouping these ages are, in every case, the youngest ages of the 5-year age groups in which they fall, and therefore, in general, the ages having the lowest mortality rate in the group. This padding at ages where mortality rates are lower results in understatement of the expected deaths.

Another possible method of computing the expected deaths would be to compute, from the life table, an average central death rate for each 5-year period by the formula:

$$ {}_5m_x = \frac{l_x - l_{x+5}}{T_x - T_{x+5}} \tag{36} $$

and to apply this rate to the total population in the age group, multiplying also, of course, by the number of years in the period of exposure. In the case of white males and white females, this method has a tendency to produce expected deaths which are consistently very slightly in excess of the actual deaths. This results from the assumption underlying the method: namely, that the proportionate distribution by single years within the 5-year age group is the same in the actual population as in the hypothetical life table population. This assumption is not exactly fulfilled, as the numbers decrease more rapidly with age in the actual population, because of the effect of past migration and of a steadily declining birth rate in past years.

The fact that the general tendency of the relation between reported and expected deaths is completely reversed by making only a slight change in the method of computation of the expected deaths is in itself evidence that an excellent fit has been secured; and, by either method, the differences are in most cases small fractions of 1 percent of the numbers of deaths involved. However, it was felt that a more meaningful comparison would be obtained by estimating the populations at single years of age by an osculatory interpolation formula which preserves the 5-year totals. For this purpose, the Karup-King formula was used. In this connection the interpolation in the age group 5–9 was performed by a special extension by means of a curve having the property of reproducing the enumerated population in the age group 3–4. The resulting comparison is shown in table BB. No comparison is made for the ages under 5, where the methods used in deriving mortality rates should, at least in theory, produce exact agreement between actual and expected deaths.

TABLE BB.—COMPARISON OF REPORTED DEATHS AND EXPECTED DEATHS ON THE BASIS OF LIFE TABLES, BY RACE AND SEX: UNITED STATES, 1939–1941

RACE AND AGE	MALE				FEMALE			
	Reported deaths	Expected deaths	Excess of expected over reported deaths		Reported deaths	Expected deaths	Excess of expected over reported deaths	
			+	−			+	−
WHITE								
5–9	16,716	16,590		126	12,109	12,049		60
10–14	17,062	17,273	211		11,334	11,438	104	
15–19	28,507	28,485		22	19,140	19,045		95
20–24	35,522	35,277		245	25,475	25,382		93
25–29	37,146	37,288	142		29,490	29,536	46	
30–34	42,405	42,390		15	33,709	33,664		45
35–39	53,285	53,196		89	39,774	39,703		71
40–44	72,956	73,042	86		50,335	50,435	100	
45–49	105,256	105,074		182	68,003	67,809		194
50–54	142,217	142,493	276		87,083	86,991		92
55–59	173,192	172,725		467	107,050	106,940		110
60–64	201,341	200,945		396	135,810	135,278		532
65–69	229,887	229,905	18		171,664	171,962	298	
70–74	235,612	236,137	525		193,091	193,359	268	
75–79	208,875	208,614		261	189,795	189,577		218
80–84	157,479	157,683	204		159,109	159,346	237	
85–89	76,515	76,336		179	88,451	88,250		201
90–94	23,084	23,091	7		31,981	31,943		38
95 and over	5,022	5,990	968		8,429	9,264	835	
Total 5 and over	1,862,079	1,862,534	2,437	1,982	1,461,832	1,461,971	1,888	1,749
Total of absolute values			4,419				3,617	
Net total			+455				+139	
NEGRO								
5–9	3,003	2,976		27	2,579	2,561		18
10–14	3,438	3,473	35		3,012	3,138	126	
15–19	7,043	7,074	31		8,525	8,399		126
20–24	10,661	10,566		95	11,246	11,215		31
25–29	12,472	12,502	30		12,253	12,286	33	
30–34	13,602	13,400		202	12,930	12,758		172
35–39	15,927	16,243	316		15,520	15,915	395	
40–44	18,961	18,954		7	17,503	17,032		471
45–49	21,830	22,173	343		18,194	18,850	656	
50–54	25,041	24,359		682	20,677	20,025		652
55–59	22,485	22,302		183	18,531	18,079		452
60–64	20,306	19,846		460	17,038	16,105		933
65–69	21,760	23,641	1,881		16,956	19,387	2,431	
70–74	16,938	16,772		166	13,286	13,163		123
75–79	11,302	11,282		20	9,237	9,182		55
80–84	7,048	7,009		39	6,061	6,058		3
85–89	4,296	4,297	1		4,217	4,222	5	
90–94	2,060	2,063	3		2,380	2,517	137	
95 and over	1,589	2,109	520		2,277	2,991	714	
Total 5 and over	239,762	241,041	3,160	1,881	212,422	213,883	4,497	3,036
Total of absolute values			5,041				7,533	
Net total			+1,279				+1,461	

TABLE BB.—COMPARISON OF REPORTED DEATHS AND EXPECTED DEATHS ON THE BASIS OF LIFE TABLES, BY RACE AND SEX: UNITED STATES, 1939–1941—Continued

Race and age	Male				Female			
	Reported deaths	Expected deaths	Excess of expected over reported deaths +	Excess of expected over reported deaths −	Reported deaths	Expected deaths	Excess of expected over reported deaths +	Excess of expected over reported deaths −
OTHER RACES								
5–9	242	240		2	216	214		2
10–14	222	226	4		213	215	2	
15–19	420	418		2	414	412		2
20–24	475	471		4	479	477		2
25–29	471	475	4		399	401	2	
30–34	561	556		5	283	302	19	
35–39	670	654		16	312	295		17
40–44	676	705	29		292	308	16	
45–49	758	711		47	328	324		4
50–54	894	944	50		342	340		2
55–59	1,030	1,015		15	345	338		7
60–64	1,129	1,124		5	371	371		
65–69	1,031	1,022		9	387	410	23	
70–74	804	789		15	376	344		32
75–79	667	681	14		321	341	20	
80–84	443	423		20	252	231		21
85–89	248	278	30		158	176	18	
90–94	131	152	21		104	128	24	
95 and over	83	192	109		83	244	161	
Total 5 and over	10,955	11,076	261	140	5,675	5,871	285	89
Total of absolute values			401				374	
Net total			+121				+196	

In the case of Negroes and "other races," the differences between reported and expected deaths are larger, and the comparison shows about the same relationships, regardless of how the expected deaths are computed. The method used in the case of white lives seemed, however, entirely suitable, and was therefore adopted. Table BB shows, for both Negro males and Negro females, a very large excess of expected over reported deaths in the age group 65–69, which is offset only to a small extent by deficiencies in the neighboring age groups. This is because the expected deaths were computed on the basis of populations as actually reported, while the rates of mortality are based on a redistribution by age of the population and deaths between ages 55 and 70. This redistribution was made in the belief that a substantial number of persons actually between ages 55 and 65 had been reported at ages between 65 and 70. If this is true, the expected deaths for the entire 15-year age period would be greatly overstated, because the rates of mortality are much higher at the ages incorrectly given than at the true ages of the groups affected by this error. Table BC shows how the comparison would be altered if based on the redistributed populations and deaths, and indicates that the calculated rates of mortality conform satisfactorily to the redistributed data.

The traditional procedure for testing the smoothness of the graduation of a series of rates of mortality calls for examination of the third differences of the graduated rates. If these are reasonably small and change sign fairly often, the smoothness of the graduation is considered satisfactory. The sum of the absolute values of the third differences over some specified range of ages is often taken as a criterion of smoothness. It is not, however, entirely clear why third differences, rather than differences of some other order, should always be used for this purpose; and in fact, there are strong arguments, at least from a theoretical standpoint, to support the view that the most appropriate order of differences to be so used depends on the characteristics of the particular data, and on the graduation formula employed. For example, in connection with the life tables in this volume, it can reasonably be argued that fourth differences are more suitable at ages 32 and above.

TABLE BC.—COMPARISON OF ASSUMED AND EXPECTED DEATHS FOR NEGROES AT AGES 50 TO 74, BASED ON REDISTRIBUTED POPULATIONS AND DEATHS: UNITED STATES, 1939–1941

Age	Male				Female			
	Assumed[1] deaths	Expected deaths	Excess of expected over assumed deaths +	Excess of expected over assumed deaths −	Assumed[1] deaths	Expected deaths	Excess of expected over assumed deaths +	Excess of expected over assumed deaths −
50–54	25,041	24,367		674	20,677	20,035		642
55–59	23,335	23,352	17		19,162	19,182	20	
60–64	21,452	21,566	114		17,540	17,577	37	
65–69	19,764	19,976	212		15,823	16,039	216	
70–74	16,938	16,826		112	13,286	13,202		84
Total 5 and over[2]	239,762	240,208	1,622	1,176	212,422	213,159	2,339	1,602
Total of absolute values			2,798				3,941	
Net total			+446				+737	

[1] Redistributed by age as described on p. 111.
[2] Using the values in this table for ages 50 to 74.

The argument is based on the fact that the interpolation formula employed above age 32 (Jenkins' fifth difference modified formula) has the property of reproducing a third degree curve. In other words, if it should happen that the guiding values at quinquennial ages were exactly the values of some third degree polynomial for the corresponding ages, then all the interpolated values would also be the corresponding values of the same polynomial. This implies that when a third degree curve can be fitted to the guiding values, such a curve constitutes an entirely satisfactory graduation, and does not require adjustment. Now, the third differences of a third degree polynomial are constant; therefore, they need not be small, and obviously do not change sign. Thus, the conventional test for smoothness employing third differences is inconsistent with the philosophy underlying the interpolation formula used. On the other hand, the fourth differences of a third degree polynomial are 0, so that there is no inconsistency in testing for smoothness by an examination of fourth differences.

The interpolation formulas used at ages under 32 have the property of reproducing second degree polynomials only, so that the same line of reasoning would justify the application of a third-difference test for smoothness. Table BD gives both the third and fourth differences of the rates of mortality for each of the six single classes of the population for ages 4 to 87, inclusive. The rates for ages under 5 were not graduated, but age 4 is included in the table because the value of q_4 was used to secure smooth junction with the rates for subsequent ages. As the method used in extrapolating mortality rates at the old ages resulted in employing a single third degree curve for all ages above 87, the mortality rates at these ages do not need to be tested for smoothness.

The range of ages covered by the table has been divided into three intervals of 28 ages each, for which separate totals are shown in table BD. The first of these intervals, including ages 4 to 31, is precisely the area in which it was argued on theoretical grounds that a criterion of smoothness based on third differences is appropriate. In general, it appears that in the two younger age intervals the differences of both orders change sign frequently, and the sum of the absolute values is satisfactorily small in both cases, being somewhat smaller for third differences than for fourth differences. However, in the oldest age interval, 60 to 87, the third differences show a marked tendency to form clusters of positive and negative values, and the sums of their absolute values are large, so that the graduation would probably be rejected as not sufficiently smooth if strict reliance were placed on third differences as the criterion of smoothness. On the other hand, the fourth differences in this interval change sign frequently and have small numerical values. Hence, on the basis of fourth differences, the smoothness would be judged satisfactory throughout.

TABLE BD.—THIRD AND FOURTH DIFFERENCES OF GRADUATED RATES OF MORTALITY,[1] AGES 4 TO 87: UNITED STATES, 1939–1941

PART I—WHITE

Age (x)	Male $10^5\Delta^3q_x$ +	Male $10^5\Delta^3q_x$ −	Male $10^5\Delta^4q_x$ +	Male $10^5\Delta^4q_x$ −	Female $10^5\Delta^3q_x$ +	Female $10^5\Delta^3q_x$ −	Female $10^5\Delta^4q_x$ +	Female $10^5\Delta^4q_x$ −
4	3			5		1	–	–
5		2	4		–	–	–	–
6	2			4	–	–	–	–
7		2	3		–	–		1
8	1		–	–		1	1	
9	1			2	–	–	1	
10		1	3		1		–	–
11	2			4	1			4
12		2		2		3	3	
13		4	4		–	–		2
14	–	–	1			2	2	
15	1			2	–	–	2	
16		1	2		2			4
17	1			3		2	3	
18		2	2		1			2
19	–	–		1		1	2	
20		1	1		1			3
21	–	–	2			2	4	
22	2			1	2			2
23	1		3		–	–	1	
24	4			6	1			1
25		2	1		–	–		1
26		1	2			1	2	
27	1			2	1			1
28		1	3		–	–	1	
29	2			3		1	3	
30		1	1		2			4
31	–	–	–	–		2	4	
32	–	–	1		2			4
33	1			1		2	5	
34	–	–		1	3			6
35		1	3			3	6	
36	2			2	3			4
37	–	–	–	–		1	2	
38	–	–	1		1			2
39	1			2		1	3	
40		1	3		2			3
41	2			4		1	2	
42		2	5		1			2
43	3			6		1	1	
44		3	5		–	–	–	–
45	2			2	–	–	1	
46	–	–	2		1		–	–
47	2			3	1			2
48		1	3			1	3	
49	2			2	2			1
50	–	–	1		1			2
51	1			2		1	4	
52		1	1		3			4
53	–	–	–	–		1	3	
54	–	–		1	2			2
55		1	3		–	–	2	
56	2			1	2			2
57	1		1		–	–	1	
58	2			1	1		1	
59	1		1		2			3
60	2		–	–		1	7	
61	2		2		6			4
62	4			1	2		2	
63	3		–	–	4		1	
64	3		1		5			2
65	4			1	3		–	–
66	3		3		3		–	–
67	6			3	3		1	
68	3		2		4			3
69	5			2	1		3	
70	3			3	4			4
71	–	–	1		–	–	2	
72	1			3	2			3
73		2	2			1	2	
74	–	–	–	–	1		–	–
75	–	–	3		1		2	
76	3		1		3		2	
77	4			2	5			2
78	2		2		3		2	
79	4			2	5			5
80	2			8	–	–		1
81		6	–	–		1		8
82		6		2		7	2	
83		8	3			5	–	–
84		5		4		5	–	–
85		9	4			5	–	–
86		5		1		5	2	
87		6	1			3		2
Total 4–31	21	20	32	35	12	16	30	25
Total of absolute values	41		67		28		55	
Net total	+1		−3		−4		+5	
Total 32–59	22	10	30	28	27	12	34	37
Total of absolute values	32		58		39		71	
Net total	+12		+2		+15		−3	
Total 60–87	54	47	25	32	55	33	28	34
Total of absolute values	101		57		88		62	
Net total	+7		−7		+22		−6	
Total 4–87	97	77	87	95	94	61	92	96
Total of absolute values	174		182		155		188	
Net total	+20		−8		+33		−4	

[1] Rates were taken to the nearest fifth decimal place and multiplied by 10^5.

TABLE BD.—THIRD AND FOURTH DIFFERENCES OF GRADUATED RATES OF MORTALITY,[1] AGES 4 TO 87: UNITED STATES, 1939–1941—Continued

PART II—NEGRO

AGE (x)	MALE				FEMALE			
	$10^5\Delta^3q_x$		$10^5\Delta^4q_x$		$10^5\Delta^3q_x$		$10^5\Delta^4q_x$	
	+	−	+	−	+	−	+	−
4	–	–		1	1		1	
5		1	2		2			2
6	1			1	–	–	2	
7	–	–	–	–	2			1
8	–	–	–	–	1			2
9	–	–	2			1	9	
10	2			1	8			9
11	1			5		1		9
12		4	1			10	1	
13		3	3			9	–	–
14	–	–	1			9	8	
15	1		3			1	8	
16	4			12	7			7
17		8	5		–	–	1	
18		3		4	1		–	–
19		7	4		1			2
20		3	7			1		2
21	4		1			3	8	
22	5			2	5			1
23	3		3		4			2
24	6			7	2		–	–
25		1		3	2			6
26		4	6			4	5	
27	2			2	1			1
28	–	–	1		–	–	1	
29	1			1	1			2
30	–	–		1		1	3	
31		1	3		2			6
32	2			2		4	6	
33	–	–		1	2			3
34		1	1			1	–	–
35	–	–	4			1	3	
36	4			5	2		–	–
37		1	4		2		–	–
38	3			2	2		–	–
39	1		2		2		–	–
40	3			6	2			4
41		3	1			2	1	
42		2	2			1	–	–
43	–	–		3		1		2
44		3	3			3	4	
45	–	–		3	1			4
46		3	4			3	6	
47	1			3	3			5
48		2	2			2	3	
49	–	–		2	1			1
50		2	2		–	–		2
51	–	–		1		2		1
52		1	1			3	1	
53	–	–		2		2	–	–
54		2	2			2		1
55	–	–	3			3	6	
56	3		–	–	3		–	–
57	3			1	3			1
58	2		3		2		1	
59	5			4	3			1
60	1		–	–	2		–	–
61	1		–	–	2			2
62	1		–	–	–	–	–	–
63	1			2	–	–	1	
64		1	4		1		–	–
65	3		3		1		4	
66	6		2		5		1	
67	8		1		6			3
68	9			1	3		4	
69	8			3	7			5
70	5			6	2			5
71		1		7		3		3
72		8	5			6		1
73		3		3		7	1	
74		6	–	–		6	3	
75		6		2		3	7	
76		8	–	–	4		–	–
77		8		3	4		4	
78		11	4		8			3
79		7	2		5		2	
80		5	13		7			4
81	8		6		3		3	
82	14			2	6			2
83	12		2		4		–	–
84	14			2	4		2	
85	12			1	6			2
86	11			2	4		1	
87	9		3		5			1
Total 4–31	30	35	42	40	40	40	47	52
Total of absolute values	65		82		80		99	
Net total	−5		+2		0		−5	
Total 32–59	27	20	34	35	28	30	31	25
Total of absolute values	47		69		58		56	
Net total	+7		−1		−2		+6	
Total 60–87	123	64	45	34	89	25	33	31
Total of absolute values	187		79		114		64	
Net total	+59		+11		+64		+2	
Total 4–87	180	119	121	109	157	95	111	108
Total of absolute values	299		230		252		219	
Net total	+61		+12		+62		+3	

[1] Rates were taken to the nearest fifth decimal place and multiplied by 10^5.

TABLE BD.—THIRD AND FOURTH DIFFERENCES OF GRADUATED RATES OF MORTALITY,[1] AGES 4 TO 87: UNITED STATES, 1939–1941—Continued

PART III—OTHER RACES

AGE (x)	MALE $10^5\Delta^3q_x$ +	MALE $10^5\Delta^3q_x$ −	MALE $10^5\Delta^4q_x$ +	MALE $10^5\Delta^4q_x$ −	FEMALE $10^5\Delta^3q_x$ +	FEMALE $10^5\Delta^3q_x$ −	FEMALE $10^5\Delta^4q_x$ +	FEMALE $10^5\Delta^4q_x$ −
4		2	2			4	1	
5	–	–	–	–		3	1	
6	–	–		3		2	–	–
7		3	5			2		3
8	2			5		5	5	
9		3	8		–	–	1	
10	5			4	1		2	
11	1			7	3			8
12		6		2		5		1
13		8	5			6	2	
14		3	2			4	6	
15		1	6		2		2	
16	5			7	4			9
17		2		2		5	2	
18		4	2			3		1
19		2		3		4	2	
20		5	7			2	6	
21	2		4		4			2
22	6		2		2			2
23	8			1	–	–	4	
24	7			8	4			6
25		1		1		2	–	–
26		2		1		2	5	
27		3	2		3			2
28		1		3	1			1
29		4	4		–	–	2	
30	–	–	1		2			2
31	1		1		–	–	–	–
32	2			3	–	–	2	
33		1	3		2			1
34	2			1	1			1
35	1			2	–	–	–	–
36		1	2		–	–	–	–
37	1			1	–	–	–	–
38	–	–		1	–	–		1
39		1	3			1	2	
40	2			2	1			2
41	–	–	1			1	5	
42	1			2	4			4
43		1	3		–	–	2	
44	2			1	2			2
45	1			3	–	–	1	
46		2	1		1			2
47		1	1			1	1	
48	–	–		2	–	–	–	–
49		2	1		–	–		1
50		1	6			1	–	–
51	5			4		1	–	–
52	1		5			1		1
53	6			4		2	3	
54	2		1		1			3
55	3			4		2	3	
56		1		4	1		–	–
57		5	2		1		–	–
58		3	1		1		–	–
59		2		1	1		1	
60		3	8		2		–	–
61	5		2		2			2
62	7			2	–	–	4	
63	5		2		4			3
64	7			1	1		3	
65	6			3	4		1	
66	3		–	–	5		3	
67	3		2		8		1	
68	5			3	9			2
69	2			2	7			5
70	–	–		10	2			17
71		10		6		15		5
72		16	1			20		2
73		15	1			22	2	
74		14	5			20	7	
75		9	20			13	31	
76	11		6		18		9	
77	17		–	–	27			1
78	17			1	26			1
79	16		2		25		3	
80	18			5	28			9
81	13		–	–	19		3	
82	13		–	–	22			1
83	13		–	–	21			1
84	13		2		20		1	
85	15			4	21			1
86	11		3		20		2	
87	14			1	22			1
Total 4–31	37	50	51	47	26	49	41	37
Total of absolute values	87		98		75		74	
Net total	−13		+4		−23		8+	
Total 32–59	29	21	30	35	16	10	20	18
Total of absolute values	50		65		26		38	
Net total	+8		−5		+6		+2	
Total 60–87	214	67	54	38	313	90	70	51
Total of absolute values	281		92		403		121	
Net total	+147		+16		+223		+19	
Total 4–87	280	138	135	120	355	149	131	106
Total of absolute values	418		255		504		237	
Net total	+142		+15		+206		+25	

[1] Rates were taken to the nearest fifth decimal place and multiplied by 10^5.

As regards the relative magnitude of third and fourth differences in the two younger age intervals, it should be pointed out that rounding errors contribute the greater part of the numerical values of differences at these ages, and their effect becomes more marked as the order of differences increases. This point is illustrated in table BE, which shows both the third and fourth differences obtained from the mortality rates for white males by retaining all the seven decimal places to which these rates were originally computed. Comparison of table BD and table BE shows that the absolute values of the differences are greatly reduced by using the two additional decimal places. (Note that figures in table BD must be multiplied by 100 to make them comparable with those of table BE.) The reduction in the sum of the absolute values over the entire age range shown amounts to 23 percent in the case of third differences and 74 percent in the case of fourth differences. Table BE shows that, in each of the three age subdivisions, the sum of the absolute values is less for fourth than for third differences when the effect of rounding is eliminated. It also shows that the third differences are in reality predominantly positive and do not change sign at all frequently above age 30. Therefore, the mortality rates would not have been considered satisfactory above age 30 if third differences had been taken as the criterion. This seems to reinforce the suggestion made earlier that the order of differences to be employed for this purpose should be varied according to the characteristics of the basic data and the graduation procedure used.

TABLE BE.—THIRD AND FOURTH DIFFERENCES OF GRADUATED RATES OF MORTALITY [1] FOR WHITE MALES, AGES 4 TO 87: UNITED STATES, 1939–1941

AGE (x)	$10^7\Delta^3 q_x$ +	$10^7\Delta^3 q_x$ −	$10^7\Delta^4 q_x$ +	$10^7\Delta^4 q_x$ −
4	8			4
5	4		2	
6	6		1	
7	7			2
8	5		2	
9	7		124	
10	131			134
11		3		195
12		198		2
13		200	3	
14		197	251	
15	54		77	
16	131			211
17		80	-	-
18		80		1
19		81		43
20		124	178	
21	54		131	
22	185		1	
23	186			2
24	184			204
25		20		100
26		120	146	
27	26			1
28	25		1	
29	26			12
30	14			7
31	7		8	
32	15		3	
33	18			4
34	14		7	
35	21		17	
36	38		5	
37	43		-	-
38	43			1
39	42			6
40	36			21
41	15			6
42	9			1
43	8			1
44	7		15	
45	22		49	
46	71		10	
47	81		4	
48	85			4
49	81			14
50	67			75
51		8		11
52		19		8
53		27	9	
54		18	19	
55	1		112	
56	113		23	
57	136		6	
58	142			6
59	136		33	
60	169		122	
61	291		32	
62	323			4
63	319		2	
64	321		20	
65	341		72	
66	413		21	
67	434			3
68	431		1	
69	432			79
70	353			325
71	28			80
72		52	4	
73		48		6
74		54	68	
75	14		261	
76	275		66	
77	341			2
78	339		-	-
79	339			171
80	168			691
81		523		173
82		696	-	-
83		696	1	
84		695	18	
85		677	87	
86		590	22	
87		568		4
Total 4–31	1,060	1,103	925	918
Total of absolute values	2,163		1,843	
Net total	−43		+7	
Total 32–59	1,244	72	312	158
Total of absolute values	1316		470	
Net total	+1,172		+154	
Total 60–87	5,331	4,599	797	1,538
Total of absolute values	9,930		2,335	
Net total	+732		−741	
Total 4–87	7,635	5,774	2,034	2,614
Total of absolute values	13,409		4,648	
Net total	+1,861		−580	

[1] Rates were taken to the nearest seventh decimal place and multiplied by 10^7.

C. CALCULATION OF OTHER LIFE TABLE FUNCTIONS

Calculation of l_x and d_x

The values of l_x and d_x were obtained by successive multiplication and subtraction commencing with a radix of 100,000 at birth, by the usual elementary formulas:

$$\left.\begin{aligned} d_x&=l_x q_x \\ l_{x+1}&=l_x-d_x \end{aligned}\right\} \qquad (37)$$

Values of q_x were used to seven decimal places, and three decimals were retained in the l_x and d_x values. At the very old ages, the number was increased, so as to give seven significant figures in every case. Although the published tables are terminated, in each case, at that age where l_x, taken to the nearest integer, first becomes 0, nevertheless, for reasons which will be explained later,[49] it was desired to have l_x values computed further for use in calculating the values of $\mathring{e}_x$, the average future lifetime. Accordingly, the process was continued so long as l_x values in excess of .0025 were obtained.

In the life tables for combinations of classes, this process could be carried only to age 93, as interpolated rates of mortality were not obtained beyond age 92. These tables were completed by a special process designed to avoid inconsistencies between the q_x values for the combination and those for the component classes. The value of l_{92} in the combined table was divided into as many parts as there were separate classes included, the division being made in proportion to the denominators used in computing the pivotal values of q_{92} for the corresponding life tables. Each separate part of the l_{92} figures was then carried forward by applying the mortality rates for the corresponding separate class, the results being summed to obtain the subsequent l_x values for the combined table. This is equivalent to expressing the value of l_x for the combined table as a weighted sum of the l_x values from the separate life tables for the component parts. Using the latter process shortens the arithmetic. For example, let $l_x^{(1)}$, $l_x^{(2)}$, etc., denote the l_x values in the separate life tables for the component classes, let l_{92}', l_{92}'', etc., denote the corresponding parts into which l_{92} for the combined table is divided, and let $w_1=l_{92}^{(1)}/l_{92}'$, $w_2=l_{92}^{(2)}/l_{92}''$, etc. Then, at ages above 92, l_x for the combined table is given by

$$l_x=w_1 l_x^{(1)}+w_2 l_x^{(2)}+\ldots$$

The q_x values were then obtained by the formula:

$$q_x=1-\frac{l_{x+1}}{l_x}$$

In the published tables, all the l_x values have been rounded to the nearest integer, while the published d_x values have been obtained by differencing the published l_x column, and, for that reason, differ slightly in some cases from the figures which would result from rounding the d_x values as originally calculated.

In view of the necessarily rough nature of the adjustments made in the data for subdivisions of the first year of life,[50] it was not felt that much refinement was justified in calculating the life table functions for these subdivisions. Accordingly, the value of d_0 obtained for the main life table was merely divided among the various age periods within the first year in proportion to the numbers of deaths in each age period during the 3 years (after adjustment for underreporting, and smoothing of the "other races" data). The intermediate l_x values were then obtained by subtraction, and the mortality rates by division.

Calculation of L_x

At ages 5 and over, it was considered sufficiently accurate to assume that

$$L_x=\frac{1}{2}(l_x+l_{x+1})$$

At ages 1 to 4, L_x was obtained by the formula:

$$L_x=l_x-(1-f_x)d_x=l_{x+1}+f_x d_x$$

where f_x denotes the separation factor previously referred to.[51] In justification of this formula, it may be pointed out that, in the hypothetical stationary population, L_x represents the number of persons at age x last birthday who would be enumerated by a census taken at any time. Naturally this is equal to the number who have reached exact age x during the preceding year less those who have died in the meantime: that is, l_x less a part of d_x. If the incidence of deaths during a calendar year is the same in the stationary population as in the actual experience, the fraction of d_x to be taken is[52] ${}_aD_x/D_x=1-f_x$.

The value of L_0 was obtained by making separate calculations for the various subdivisions of the first year of life and adding. The process used is most readily explained by adopting the point of view which considers L_0 as the total number of person-years of life lived, between birth and exact age 1, by l_0 infants born alive. The function T_x-T_{x+t} for a particular age interval, x to $x+t$, within the first year of life represents the number of person-years lived between these exact ages by the survivors of l_0 live births. This would be given by

$$T_x-T_{x+t}=tl_x-\frac{1}{2}t\,d_x=\frac{1}{2}t(l_x+l_{x+t}) \qquad (38)$$

on the assumption that those who die between ages x and $x+t$ live, on the average, half the period. It was necessary to express the values of t as fractions of a year,

[49] See p. 134.

[50] See p. 108.

[51] See p. 117.

[52] See p. 115.

on the same assumption previously made[53] that the total length of the year was 365¼ days. The value of L_0 was taken as the sum of the values of $T_x - T_{x+t}$ for all subdivisions of the first year of life.

With the exception of L_0, all the values of L_x were retained to not less than one place of decimals and to not less than six significant figures, for use in subsequent calculations. Values of $T_x - T_{x+t}$ for subdivisions of the first year of life were rounded to the nearest integer before addition. Values of L_x were obtained in each case up to, but not including the last age for which l_x was computed, but are shown in the published tables only when the value is at least 1 to the nearest integer. The published L_x values (except in the first year of life) were obtained by differencing the published T_x column, and therefore differ slightly in some cases from the figures which would result from rounding the originally calculated L_x values directly.

Calculation of T_x and $\mathring{e}_x$

Values of T_x were obtained by accumulating the computed values of L_x from the oldest age available down to age 0. The values of $T_x - T_{x+t}$ for subdivisions of the first year of life were added one by one, proceeding backward from age 1, in order to obtain T_x values within the first year. In the calculation of $\mathring{e}_x$ each value of T_x was used to the smallest number of decimal places retained in any of the L_x values included in it. However, the published values of T_x have all been rounded to the nearest integer.

The values of $\mathring{e}_x$, carried out to two decimal places in all cases, were computed by the formula:

$$\mathring{e}_x = \frac{T_x}{l_x}$$

the l_x values being used to the full number of decimal places originally retained. In order to obtain plausible values of $\mathring{e}_x$ at the oldest ages shown in the published tables, the actual computation of l_x values was continued so long as the values obtained exceeded .0025. In arriving at this limit, it was reasoned that the ages for which figures appear in the tables are those for which l_x is at least 1 to the nearest integer: that is, the exact value is at least .5. Therefore, accuracy to two decimal places will be secured, in most cases, by using, in the computation of T_x, all values of l_x which, when divided by .5, give a quotient of at least ½ of .01, or .005: that is, all values of l_x in excess of .0025.

[53] See p. 108.

APPENDIX

A. METHOD USED IN TESTING THE APPROPRIATENESS OF GLOVER'S SEPARATION FACTORS

It was stated in part V[1] that in dividing the deaths D_x occurring in each calendar year at ages 1 to 4 into the two parts ${}_aD_x$ and ${}_bD_x$ according to the year of birth, it was necessary to employ the same separation factors f_x used by Glover in connection with the 1910 life tables, as no statistics were available on which a new determination could be based. However, the appropriateness of Glover's factors for use with the 1939–1941 data was first tested in the manner described below.

Let $\theta_{x+t}dt$ denote the number of deaths which occur during a specified period of observation (assumed to be an integral number of years) between age $x+t$ (x being an integer and t a fraction) and age $x+t+dt$, let D_x denote the total deaths during the period at age x last birthday, and let K_{x+t} denote the total deaths at all ages under $x+t$, so that $K_x=\sum_{s=0}^{x-1}D_s$. It follows immediately that

$$K_{x+t}=\int_0^{x+t}\theta_z\,dz$$

therefore, $\frac{d}{dt}K_{x+t}=\theta_{x+t}$. On the assumption that the $\theta_{x+t}dt$ deaths occurring at exact age $x+t$ are uniformly distributed over each of the calendar years covered, these would include $t\theta_{x+t}dt$ deaths where exact age x was attained in the calendar year preceding the year of death, and $(1-t)\theta_{x+t}dt$ deaths where exact age x was attained in the year of death. The total number of deaths in the calendar year following the attainment of exact age x, but before attaining age $x+1$, which may be denoted by ${}_bD_x$, would therefore be:

$$\int_0^1 t\theta_{x+t}dt$$

Considering this expression to be of the form $\int UdV$, where $U=t$ and $dV=\theta_{x+t}dt$, and integrating by parts gives:

$${}_bD_x=K_{x+1}-\int_0^1 K_{x+t}dt$$

Dividing by D_x gives an average separation factor for the entire period of observation, which may be represented by f_x. Thus,

$$f_x=\frac{1}{D_x}\left(K_{x+1}-\int_0^1 K_{x+t}dt\right) \qquad (39)$$

[1] See p. 118.

Values of the expression (39) were obtained for ages 1, 2, and 3 by using the deaths of the 3-year period 1939–1941 and employing an approximate integration formula to evaluate the integral. In the case of ages 2 and 3, the formula used for this purpose was the symmetrical formula:

$$\int_0^1 K_{x+t}dt=\frac{1}{24}(-K_{x-1}+13K_x+13K_{x+1}-K_{x+2})$$

which is obtained by fitting a third degree polynomial to four consecutive integral values of K_x. When this expression is substituted in formula (39), the latter reduces to:

$$f_x=\frac{1}{2}-\frac{D_{x-1}-D_{x+1}}{24D_x}$$

This formula was not considered suitable for age 1 because of the very large difference between K_0 and K_1, and accordingly the following unsymmetrical formula was derived by fitting a third degree polynomial to the values of $K_{\frac{9}{12}}$, K_1, K_2, and K_3:

$$\int_0^1 K_{1+t}dt=\frac{1}{180}(-64K_{\frac{9}{12}}+165K_1+84K_2-5K_3)$$

The values so obtained are shown in table BF.

TABLE BF.—ESTIMATED SEPARATION FACTORS FOR AGES 1, 2, AND 3, OBTAINED BY APPROXIMATE INTEGRATION: UNITED STATES, 1939–1941

AGE	Separation factors used by Glover	SEPARATION FACTORS OBTAINED BY APPROXIMATE INTEGRATION					
		White		Negro		Other races	
		Male	Female	Male	Female	Male	Female
1	0.410	0.399	0.404	0.387	0.392	0.415	0.406
2	.470	.450	.446	.431	.434	.428	.431
3	.480	.474	.474	.464	.470	.448	.450

In interpreting these results, it must be remembered that the values which are being compared with Glover's values have been obtained by a method which is not only rough, but is also based on assumptions which are likely not to be exactly fulfilled. It may be mentioned also that a moderate change in the values of the separation factors affects the value of the mortality rate only minutely. Therefore, the results obtained are considered satisfactorily close to Glover's values, except perhaps in the numerically unimportant group of "other races," where the data are too scanty, in any case, to yield reliable results.

B. DERIVATION OF THE SPECIAL EXTENSIONS OF THE KARUP-KING FORMULA USED FOR INTERPOLATION OF MORTALITY RATES AT AGES 5 TO 11 AND 26 TO 31

As explained on pages 125 and 126 of part V, the rates of mortality in the various life tables were interpolated by Jenkins' modified fifth difference interpolation formula at ages 32 and over, and by the Karup-King formula at ages 12 to 27, while the rates for ages 0 to 4 were calculated directly from detailed statistics for the individual years of age. The rates for ages 5 to 11 were interpolated from a special third degree curve determined so as to reproduce the calculated rates of mortality at ages 4, 7, and 12, and to have the same first derivative at age 12 as the Karup-King curve used for interpolation in the age interval 12 to 17. Similarly, the rates for ages 28 to 31 were interpolated from a special third degree curve determined so as to have the same ordinate and the same first derivative at age 27 as the Karup-King curve used for interpolation in the age interval 22 to 27, and the same ordinate and first derivative at age 32 as the Jenkins curve employed in the interval 32 to 37. By a suitable artificial extension of the series of pivotal rates of mortality, it was possible to simplify the numerical work by regarding these two special third degree curves as merely continuations of the interpolation by the Karup-King formula. It is the purpose of this section to explain how these artificial extensions were arrived at.

If the Karup-King formula (formula (35), p. 126) were to be used in the regular way in the age interval 7 to 12, the formula would be:

$$q_{7+t}=\frac{s}{5}q_7+\frac{s^2(s-5)}{250}\delta^2 q_7+\frac{t}{5}q_{12}+\frac{t^2(t-5)}{250}\delta^2 q_{12} \qquad (40)$$

where $s=5-t$, and the requirements as to reproduction of the calculated values of q_7 and q_{12} and equality of the derivatives at age 12 would be automatically satisfied, no matter what value of $\delta^2 q_7$ is used. Therefore, it is proposed to use instead of the actual value of $\delta^2 q_7$ an artifical value ϵ determined so that the formula will reproduce the value of q_4. Setting $t=-3$ in formula (40) then gives:

$$q_4=1.6q_7+.768\epsilon-.6q_{12}-.288\delta^2 q_{12}$$

Solving for ϵ and, at the same time, substituting $\delta^2 q_{12}=q_{17}-2q_{12}+q_7$ gives:

$$\epsilon=\frac{1}{96}(125q_4-164q_7+3q_{12}+36q_{17}) \qquad (41)$$

Formula (40), with $\delta^2 q_7$ replaced by a value of ϵ computed from formula (41) was used not only in the interval 7 to 12, but for ages 5 and 6 as well.

In deriving the special formula used between ages 27 and 32, the pivotal rates of mortality will be denoted by "Q" and the interpolated rates (including the pivotal rates at ages 22 and 27 reproduced by the Karup-King formula and the adjusted rates obtained at the pivotal ages 32, 37, and 42 by Jenkins' formula) will be denoted by "q." The special formula for interpolation between 27 and 32 can be written in the Karup-King form:

$$q_{27+t}=\frac{s}{5}q_{27}+\frac{s^2(s-5)}{250}\delta^2 q_{27}+\frac{t}{5}q_{32}-\frac{t^2(5-t)}{250}\epsilon \qquad (42)$$

where ϵ denotes an artificial value to be used instead of $\delta^2 q_{32}$. The conditions as to equality of ordinates and derivatives at age 27 and equality of ordinates at age 32 are automatically satisfied, regardless of the value of ϵ. Therefore, ϵ will be determined so as to secure equality of the derivatives at age 32. Differentiating formula (42) with respect to t and setting $t=5$ gives:

$$q_{32}'=-\frac{1}{5}q_{27}+\frac{1}{5}q_{32}+\frac{1}{10}\epsilon$$

Since $q_{27}=Q_{27}$ and

$$q_{32}=Q_{32}-\frac{1}{36}\delta^4 Q_{32}$$

this may be written:

$$q_{32}'=-\frac{1}{5}Q_{27}+\frac{1}{5}Q_{32}-\frac{1}{180}\delta^4 Q_{32}+\frac{1}{10}\epsilon \qquad (43)$$

On the other hand, the Jenkins formula to be used for interpolation between 32 and 37 may be written as

$$q_{32+t}=\frac{s}{5}\left(Q_{32}-\frac{1}{36}\delta^4 Q_{32}\right)+\frac{s(s^2-25)}{750}\left(\delta^2 Q_{32}-\frac{1}{6}\delta^4 Q_{32}\right)+\frac{t}{5}\left(Q_{37}-\frac{1}{36}\delta^4 Q_{37}\right)+\frac{t(t^2-25)}{750}\left(\delta^2 Q_{37}-\frac{1}{6}\delta^4 Q_{37}\right)$$

Differentiating with respect to t and setting $t=0$ gives:

$$q_{32}'=-\frac{1}{5}Q_{32}-\frac{1}{15}\delta^2 Q_{32}+\frac{1}{60}\delta^4 Q_{32}+\frac{1}{5}Q_{37}-\frac{1}{30}\delta^2 Q_{37} \qquad (44)$$

Equating formulas (43) and (44) and solving for ϵ gives:

$$\epsilon=2Q_{27}-4Q_{32}-\frac{2}{3}\delta^2 Q_{32}+\frac{2}{9}\delta^4 Q_{32}+2Q_{37}-\frac{1}{3}\delta^2 Q_{37}$$

Upon substituting the expressions in terms of ordinates for the differences appearing in this formula, it becomes:

$$\epsilon=\frac{1}{9}(2Q_{22}+4Q_{27}-15Q_{32}+10Q_{37}-Q_{42})$$

This gives the value of ϵ to be employed in formula (42).

C. METHOD OF COMPUTATION OF THE ACTUARIAL TABLES FOR WHITE MALES AND WHITE FEMALES

Modification of the basic life table values for use in the actuarial tables for white males and white females

In order to secure a high degree of consistency between the values shown for the various actuarial functions tabulated, so that the various mathematical relationships between commutation symbols, annuity and assurance premiums, etc., would hold as precisely as possible, the basic life tables were slightly modified by taking the l_x column as the basic column and deriving all other values from it. The use of l_x (instead of q_x) as the basic function causes numerous, but slight, differences between the life tables for white males and white females included with the actuarial tables (tables 14 and 25) and those which appear earlier in the volume (tables 5 and 6). The values of l_x are the same to age 93 for white males and to age 95 for white females. However, beyond these ages, the l_x values in the actuarial tables are shown to enough decimal places to have a total of four significant figures, in order not to impair the smoothness of the actuarial functions by excessive rounding. Nevertheless, the limiting ages of the original life tables have been retained. The d_x values were obtained by differencing the new l_x columns, and therefore differ at the old ages from the ones previously given. The new values of q_x were obtained by dividing d_x by l_x in these tables, and therefore differ slightly from the earlier values in most cases.

Calculation of the force of mortality

Although the force of mortality is not given for the general life tables in part I, it has been tabulated, for white males and females, for inclusion with the actuarial tables, because of its usefulness in various actuarial approximations. From age 3 to the last ages shown, μ_x was obtained by the usual approximate formula:[2]

$$\mu_x=\frac{8(l_{x-1}-l_{x+1})-(l_{x-2}-l_{x+2})}{12l_x} \qquad (45)$$

The original, unrounded values of l_x were used.

This formula is not applicable at ages 0 and 1, and was considered unsuitable at age 2, where it would involve l_0. Therefore μ_1 and μ_2 were calculated by making use of the l_x values at fractional ages under 1, in each case fitting fourth degree curves to five consecutive (but not equally spaced) values by means of Waring's formula.[3] The resulting equations were:

$$\mu_1=\frac{1}{l_1}(-4.74725l_{\frac{10}{12}}+21.26769l_{\frac{11}{12}}-16.50000l_1-.02198l_2+.00154l_3)$$

$$\mu_2=\frac{1}{l_2}(-3.44881l_{\frac{11}{12}}+4.33333l_1-.42308l_2-.52000l_3+.05856l_4)$$

The estimation of the force of mortality at birth presents peculiar difficulties because of the extremely rapid decrease in the death rate immediately following birth. The value has little, if any, practical utility; however, values of μ_0 have been included for the sake of completeness and because of academic interest in the results. It is believed that this is the first time a serious attempt has been made to obtain a realistic value of the force of mortality at the moment of birth. However, the result obtained must be regarded only as a general indication of the magnitude of this quantity, and in no sense an accurate computation of its value.

Previously published values of μ_0 show a wide variation as indicated in table BG. Values of the force of mortality have not appeared in the official publications of any country except Australia and Belgium. However, a value calculated from English Life Table No. 8 has been published in a text book of the Institute of Actuaries.[4] King[5] obtained a value of μ_0 (based on data for insured lives) by fitting a Makeham curve to the values of l_0, l_1, and l_2. Aside from the curious assumptions made by him in deriving his Makeham constants, it is clear that the shape of the l_x curve in the neighborhood of age 0 cannot be correctly represented without taking into account the incidence of mortality within the first year of life. King's value (.15920) is, of course, absurdly low. The Belgian figures were obtained by merely fitting a fourth degree polynomial to the values of l_x at the integral ages 0 to 4. This method is open to the same objections as King's. Spurgeon's value for England and Wales was obtained by the admittedly rough method of taking 365 times the ratio of the deaths under 1 day of age in the 3 years 1910–1912 to the number of births in the same 3 years. Spurgeon states that this method "clearly underestimates the true value of the force of mortality at the moment of birth." In connection with the Australian life tables of 1901–1910, constructed by Mr. C. H. Wickens, it is stated[6] that these values were obtained from a graduation of the rates of mortality at ages 0 to 4 by Makeham's second modification of Gompertz's formula. The method appears to have been similar to King's. In the report concerning the Australian life tables of 1920–1922, it is stated[7] that "μ_x for age 0 for each sex was determined from special data available for deaths during the first week of life." The statement

[2] Spurgeon, E. F., *Life Contingencies*, third edition, p. 14, Cambridge University Press, London, 1938.

[3] See footnote on p. 111.

[4] Spurgeon, *op. cit.*, p. 397 and 398.

[5] King, George, *Institute of Actuaries' Text Book of the Principles of Interest, Life Annuities, and Assurances, and Their Practical Application, Part II, Life Contingencies*, second edition, pp. 103–104, Charles and Edwin Layton, London, 1902.

[6] Australia Census Bureau, *Census of the Commonwealth of Australia, 3rd April, 1911, vol. I, Statisticians Report*, p. 325, McCarron, Bird and Co., Melbourne, 1917.

[7] Australia Commonwealth Bureau of Census and Statistics, *Census of the Commonwealth of Australia, 4th April, 1921*, vol. II, p. 329, Government Printer, Melbourne, 1927.

is not accompanied by any such qualification as that given by Spurgeon, although the similarity in the results suggests that a similar method was used. In the account of methods of construction of the most recent Australian life tables,[8] the method of computation of the force of mortality at birth is not stated, but it may be presumed to be similar to that employed in connection with the 1920–1922 tables.

TABLE BG.—FORCE OF MORTALITY AT THE MOMENT OF BIRTH: PUBLISHED VALUES FOR ENGLAND, AUSTRALIA, AND BELGIUM COMPARED WITH RESULTS OBTAINED FOR WHITES IN THE UNITED STATES, 1939–1941

COUNTRY, DATE, AND CLASS OF POPULATION	VALUE OF μ_0	
	Male	Female
England and Wales (total population) 1910–1912 [1]	4.70944	----------
Australia (total population) 1901–1910 [2]	.2279	0.1784
Australia (total population) 1920–1922 [3]	4.83547	3.63620
Australia (total population) 1932–1934 [4]	4.83249	3.74807
Belgium (total population) 1928–1932 [5]	.18554	.14241
United States, whites, 1939–1941, using Spurgeon's method [6]	5.51469	4.25239
United States, whites, 1939–1941 as shown in tables 14 and 25 [7]	10.29757	8.06964

[1] Spurgeon, *op. cit.*, p. 398.
[2] Australia Census Bureau, *Census of the Commonwealth of Australia, 3rd April, 1911*, vol. III, pp. 1215, 1217, McCarron, Bird & Co., Melbourne, 1914.
[3] Australia Commonwealth Bureau of Census and Statistics, *Census of the Commonwealth of Australia, 4th April, 1921*, vol. II, pp. 1838, 1840, Government Printer, Melbourne, 1925.
[4] Australia Commonwealth Bureau of Census and Statistics, *Census of the Commonwealth of Australia, 30th June, 1933, Australian Life Tables, 1932–1934*, pp. 6, 65, Government Printer, Canberra, 1937.
[5] Office Central de Statistique, *Recensement Général de la Population, au 31 Décembre 1930*, tome VII, *Tables de Mortalité de la Population Belge, 1928–1932*, pp. 57, 59.
[6] For method of calculation, see text, p. 137.
[7] See pp. 58, 69 of this volume: for method of calculation, see p. 138.

It seems highly probable that mortality is heavier in the earlier than in the later part of the first day of life, and that Spurgeon's method considerably underestimates the true value. The values for μ_0 shown in tables 14 and 25 were obtained by fitting a Gompertz curve to the l_x values at birth and at the ages of 1 day and 2 days. Taking $x=0$, $\frac{1}{h}$, and $\frac{2}{h}$, respectively (where $h=365\frac{1}{4}$), in the Gompertz formula:

$$l_x = kg^{c^x} \tag{46}$$

and equating to the corresponding l_x values gives three equations which can be solved for k, g, and c. Taking the logarithm of the expression (46) and differentiating gives:

$$\mu_x = -\frac{d}{dx}(\log_e l_x) = -c^x \log_e c \log_e g$$

Therefore,

$$\mu_0 = -\log_e c \log_e g$$

Calculation of commutation columns and net premiums

The commutation functions C_x and D_x were obtained by the usual elementary formulas:

$$D_x = v^x l_x$$

$$C_x = v^{x+1} d_x$$

[8] Australia Commonwealth Bureau of Census and Statistics, *Official Year Book of the Commonwealth of Australia*, No. 29, pp. 928–942, Government Printer, Canberra, 1936.

They were checked by the relation:

$$C_x = vD_x - D_{x+1}$$

The functions N_x and S_x, M_x and R_x were obtained by successive accumulation of the values of D_x and C_x, respectively. These were checked by the corresponding relations:

$$M_x = vN_x - N_{x+1}$$

$$R_x = vS_x - S_{x+1}$$

The functions a_x, A_x, and P_x were obtained directly from the commutation columns, and checked by the relations:

$$A_x = 1 - d(1 + a_x)$$

$$P_x = \frac{A_x}{1 + a_x}$$

The values of C_x and D_x were obtained to five significant figures throughout. In the case of the commutation values obtained by summation, the number of decimal places retained in each case was the smallest number contained in any one of the figures included in the sum.

D. PROCEDURE USED IN CARRYING OUT THE MAKEHAM GRADUATION OF THE LIFE TABLE FOR TOTAL WHITES

General considerations

It has already been stated [9] that an important reason for preparing a makehamized mortality table for the total white population rather than for the separate sexes was the fact that certain technical difficulties were encountered in attempting to graduate the white male and female data separately by Makeham's law. It is well known [10] that with distinct tables for males and females, the law of uniform seniority does not hold in connection with annuities involving combinations of male and female lives unless the Makeham constant c has the same value in both the male and female tables. Experimental calculations indicated that this could not be done without marked distortion of the rates of mortality as previously calculated for the separate sexes.

Method of graduation employed

In the belief that the makehamized table would find its chief use in the calculation of life annuity values, the graduation was performed with the specific aim of reproducing as closely as possible the values of whole life annuities as calculated from the life table already prepared for the total white population. For convenience, the latter table will be referred to in the following discussion as the "original" table. As it was planned to publish life annuity values at rates of interest ranging from 2 to 4 percent, the actual fitting

[9] See p. 94.
[10] See Transactions of the Faculty of Actuaries (Scotland), vol. 3, p. 296.

was carried out by the use of annuities calculated at the intermediate rate of 3 percent, in order to secure the closest over-all agreement for the several interest rates tabulated.

The method employed in determining the Makeham constants is that suggested by Henderson.[11] In this method, a preliminary graduation is first made, using approximate values of the Makeham constants, and life annuity values are computed on the basis of the preliminary graduation. Next, the differential calculus is employed to estimate closely the effect on the annuity values of small changes in the values of the constants; and finally, the method of least squares is used to determine precisely the small adjustments to be made in the values of the constants, in order to reproduce most closely the annuity values based on the original table.

Preliminary graduation

Under Makeham's law, the force of mortality μ_x is given by the equation:

$$\mu_x = A + Bc^x = A + Be^{\lambda x} \tag{47}$$

where A, B, and c are constants to be determined, and $\lambda = \log_e c$. This leads to the further equation: [12]

$$l_x = ks^x g^{c^x} \tag{48}$$

where $s = e^{-A}$, $g = e^{-B/\lambda}$, and k is a further constant depending on the radix of the life table. Therefore, in a life table which follows Makeham's law,

$$\frac{\mu_{x+10} - \mu_{x+5}}{\mu_{x+5} - \mu_x} = c^5$$

In making the preliminary graduation, c^5 was calculated by this formula for $x = 30, 35, 40, \ldots, 80$, using values of μ_x calculated from the original table by formula (45); and the arithmetic average of the 11 values so obtained was taken as the preliminary value of c^5. The values of g and s were then determined by fitting the curve to the values of l_{30}, l_{60}, and l_{90}, as given by the original table. The values of the constants so obtained were:

$$c = 1.091889$$

$$\lambda = .08790888$$

$$\log_{10} g = -.0004974$$

$$\log_{10} s = -.0004566$$

$$A = .0010514$$

$$B = .0001007$$

Final determination of the Makeham constants

Using accented symbols to denote values based on the preliminary graduation, and unaccented symbols to denote those based on the final graduation, and writing

$$A = A' + h,\ \log_e B = \log_e B' + \lambda j,\ \lambda = \lambda' + l \tag{49}$$

gives:

$$\mu_x = A + Be^{\lambda x} = A' + h + B'e^{(\lambda' + l)(x+j)}$$

so that, approximately

$$\bar{a}_x = \bar{a}_x' + h\frac{\partial \bar{a}_x'}{\partial A'} + j\frac{\partial \bar{a}_x'}{\partial x} + l\frac{\partial \bar{a}_x'}{\partial \lambda'} \tag{50}$$

Continuous annuities have been employed in this expression because of the difficulty of obtaining expressions for the partial derivatives of annual annuities.

From the relations:[13]

$$\bar{a}_x = \int_0^\infty v^t\, {}_tp_x\, dt$$

$$v = e^{-\delta}$$

$${}_tp_x = e^{-\int_0^t \mu_{x+r}\, dr}$$

it follows that

$$\bar{a}_x = \int_0^\infty e^{-(A+\delta)t - B\int_0^t e^{\lambda(x+r)}\, dr}$$

whence

$$\frac{\partial \bar{a}_x}{\partial A} = \frac{\partial \bar{a}_x}{\partial \delta} = -\int_0^\infty t v^t\, {}_tp_x dt = -(I\bar{a})_x \tag{51}$$

where $(I\bar{a})_x$ denotes the present value of a continuous increasing life annuity in which the payment at exact time t, if the annuitant is then alive, is tdt. An approximate formula for $(I\bar{a})_x$ is obtained from the approximate relation: [14]

$$\bar{a}_x = a_x + \frac{1}{2} - \frac{1}{12}(\mu_x + \delta) \tag{52}$$

upon differentiating with respect to δ. First, it may be noted that

$$\frac{d}{d\delta}v^t = \frac{d}{d\delta}e^{-t\delta} = -te^{-t\delta} = -tv^t.$$

Since

$$a_x = \sum_{t=1}^{\infty} v^t\, {}_tp_x, \text{ it follows that } \frac{\partial a_x}{\partial \delta} = -\sum_{t=1}^{\infty} tv^t\, {}_tp_x = -(Ia)_x$$

Therefore,

$$(I\bar{a})_x = -\frac{\partial \bar{a}_x}{\partial \delta} = (Ia)_x + \frac{1}{12} \tag{53}$$

approximately. The other partial derivatives are given by the equations:[15]

$$\frac{\partial \bar{a}_x}{\partial x} = \bar{a}_x(\mu_x + \delta) - 1 \tag{54}$$

and

$$\lambda\frac{\partial \bar{a}_x}{\partial \lambda} = \left(x - \frac{1}{\lambda}\right)\frac{\partial \bar{a}_x}{\partial x} + (A + \delta)(I\bar{a})_x - \bar{a}_x \tag{55}$$

[11] Henderson, Robert, *Mathematical Theory of Graduation (Actuarial Studies No. 4)*, second edition, pp. 97–99, Actuarial Society of America, New York, 1938.

[12] For the derivation of this formula, see Spurgeon, *op. cit.*, pp. 191–192.

[13] Spurgeon, *op. cit.*, pp. 133, 16; Rietz, H. L., Crathorne, A. R., and Rietz, J. Chas., *Mathematics of Finance*, second edition, p. 31, Henry Holt and Co., New York, 1939

[14] Spurgeon, *op. cit.*, p. 133.

[15] Spurgeon, *op. cit.*, p. 134; Henderson, *op. cit.*, p. 99.

Values of μ_x and l_x (based on an arbitrary radix) were calculated from the constants obtained in the preliminary graduation, and, from the latter, values of a_x and $(Ia)_x$ were computed. These results were used in calculating the values of the partial derivatives by equations (51) to (55); and h, j, and l were then determined by the method of moments (equivalent in this case to the method of least squares). As the sole purpose of the graduation was to reproduce as closely as possible the annuity values based on the original table, it was decided to assign equal weight to all the individual ages. Under these conditions, the method of moments was most easily carried out by means of a process of successive accumulation applied to the terms of equation (37), the equations for the determination of h, j, and l being:

$$\sum_{x=\alpha}^{\beta}(a_x''-a_x')=h\sum_{x=\alpha}^{\beta}\frac{\partial\bar{a}_x'}{\partial A'}+j\sum_{x=\alpha}^{\beta}\frac{\partial\bar{a}_x'}{\partial x}+l\sum_{x=\alpha}^{\beta}\frac{\partial\bar{a}_x'}{\partial\lambda'}$$

$$\sum_{x=\alpha}^{\beta}{}^{2}(a_x''-a_x')=h\sum_{x=\alpha}^{\beta}{}^{2}\frac{\partial\bar{a}_x'}{\partial A'}+j\sum_{x=\alpha}^{\beta}{}^{2}\frac{\partial\bar{a}_x'}{\partial x}+l\sum_{x=\alpha}^{\beta}{}^{2}\frac{\partial\bar{a}_x'}{\partial\lambda'}$$

$$\sum_{x=\alpha}^{\beta}{}^{3}(a_x''-a_x')=h\sum_{x=\alpha}^{\beta}{}^{3}\frac{\partial\bar{a}_x'}{\partial A'}+j\sum_{x=\alpha}^{\beta}{}^{3}\frac{\partial\bar{a}_x'}{\partial x}+l\sum_{x=\alpha}^{\beta}{}^{3}\frac{\partial\bar{a}_x'}{\partial\lambda'}$$

where the double accent denotes values based on the original table, and where:

$$\sum_{x=\alpha}^{y}{}^{2}f(x)=\sum_{x=\alpha}^{y}\sum_{z=\alpha}^{x}f(z)$$

and

$$\sum_{x=\alpha}^{y}{}^{3}f(x)=\sum_{x=\alpha}^{y}\sum_{z=\alpha}^{x}{}^{2}f(z)$$

Some study was given to the question of the exact range of ages to be employed in the determination of h, j, and l, the ultimate decision being in favor of the ages 10 to 80, inclusive. Although the Makeham curve was not actually used down to age 10, it was found that the use of values for this and subsequent ages in the fitting facilitated obtaining a smooth junction with the values from the original table, which were to be used for the ages under 10. A close fit at the ages above 80 was not considered important, and it was found that it could be secured only at the cost of accepting much less satisfactory agreement at the younger ages. The resulting adjusted values of the constants, obtained by substituting in equations (49) were:

$$c=1.0924931$$
$$\lambda=.08846246$$
$$\log_{10} g=-.000474834$$
$$\log_{10} s=-.000461004$$
$$A=.0010615$$
$$B=.00009672$$

Junction with original values at very young ages

From about age 90 down to about age 17, the final Makeham graduation provides a close fit to the original table, but between ages 10 and 17 the Makeham curve produces rates of mortality which are much too high. Accordingly, it was decided to use the Makeham formula only at ages 17 and over. It was desired to retain the mortality rates from the original table from birth to age 10, in order to preserve the minimum in the rate of mortality which occurs at age 10. It was also desired to have the values of whole life annuities under the original table exactly reproduced at ages 11 and under. This was accomplished by the following process. First, blended annuity values were obtained for ages 12 to 16 by the formula:

$$a_x=\frac{1}{6}[(x-11)a_x^M+(17-x)a_x^O]$$

where a_x denotes the blended annuity value; a_x^O, the value according to the original table; and a_x^M, the value according to the Makeham curve. The blended values were taken as the final graduated annuity values, and rates of mortality at ages 11 to 16 were obtained by the formula:

$$q_x=1-\frac{(1+i)a_x}{1+a_{x+1}}$$

Completion of the mortality table

In order to secure the consistency among the various actuarial functions which results from regarding l_x as the basic function of the mortality table, and yet retain the full smoothness of the Makeham graduation, the radix of the table was taken as 1,000,000 rather than 100,000. The values of l_x up to and including age 11 were those calculated for the original table, but retaining one significant figure in addition to those shown in table 4. From age 11 to age 17, inclusive, the values of l_x were computed by the formulas (37), employing the values of q_x obtained from the blended annuity values, as described above. The value of l_{17} determined in this manner was then equated to the Makeham formula (48) in order to determine the constant k. The values of l_x at all the remaining ages were then calculated from this formula. All values were rounded to the nearest integer, except that at the older ages, sufficient decimal places were retained, for the sake of smoothness, to have six significant figures in all cases. The table was terminated at the point where l_x first became 0 to the nearest integer on the conventional 100,000 radix: that is, when it became less than 5, on the basis of the radix of 1,000,000 actually used.

The values of d_x, p_x, and q_x were obtained from the l_x column in the conventional manner. From birth to age 16, μ_x was calculated by the same formulas[16] which were used in the case of white males and white females. At ages 17 and over, it was calculated in accordance with Makeham's law by formula (47).

Tests of the graduation

A graduation by means of a mathematical formula such as Makeham's law, of course, does not need to be tested for smoothness. As the graduation was specif-

[16] See p. 137.

ically designed to reproduce life annuity values as closely as possible, the most obvious test of the "fit" of the graduation is a comparison of annuity values based on the original and makehamized tables. This comparison is made in tables BH and BJ for both whole life and temporary life annuities at selected ages, with interest at 2, 3, and 4 percent. Up to age 80, the agreement is seen to be extremely close. Table BH also compares the rates of mortality under the two tables. A further comparison showing joint life annuity values on both tables for selected combinations of ages at 3 percent interest is given in table BK.

In table BL, the expected deaths according to the makehamized table are compared with the reported deaths. As the fit is much less close than in the case of the life tables graduated by osculatory interpolation, the precise method to be used in calculating the expected deaths was not a matter of great moment. Accordingly, the simplest method was chosen: that of computing an average value of m_x for each 5-year age group, by formula (36),[17] and applying it to three times the estimated July 1, 1940, population in the age group.

[17] See p. 127.

In view of the rigid character of the Makeham curve, it is to be expected that the differences would be much greater than in the other cases where the more flexible osculatory method was used, and table BL shows this to be the case. However, between ages 25 and 90, the difference never exceeds 3 percent of the reported deaths except by a very small margin in the age group 35 to 39. The expected deaths are deficient by more than 9 percent at ages 20 to 24, and are in excess by more than 11 percent at ages 10 to 14. At ages 11 to 14, in particular, the rates of mortality in the makehamized table are much too high. However, these discrepancies would have little effect on the values of life annuities, even temporary annuities at young ages, because the actual level of mortality at the ages concerned is very low. The only common financial functions which would be seriously affected are premiums and values for short term assurances at young ages, and there would be little occasion to use this table for such calculations. All things considered, it is believed to be a highly satisfactory table for the purpose it was mainly intended to serve: that of approximating the values of single and joint life annuities by the original table.

TABLE BH.—COMPARISON OF RATES OF MORTALITY AND VALUES OF IMMEDIATE WHOLE LIFE ANNUITIES BY ORIGINAL AND MAKEHAMIZED MORTALITY TABLES: TOTAL WHITES IN THE UNITED STATES, 1939–1941

AGE (x)	RATE OF MORTALITY (1,000 q_x)		PRESENT VALUE OF IMMEDIATE WHOLE LIFE ANNUITY (a_x)					
			Interest at 2 percent		Interest at 3 percent		Interest at 4 percent	
	Original table	Makehamized table	Original table	Makehamized table	Original table	Makehamized table	Original table	Makehamized table
0	43.15	43.15	34.2881	34.2889	26.7047	26.7047	21.5432	21.5428
5	1.24	1.24	34.7444	34.7452	27.3545	27.3545	22.2342	22.2336
10	.85	.85	33.3438	33.3451	26.5553	26.5553	21.7637	21.7630
15	1.20	1.30	31.7710	31.7888	25.6078	25.6216	21.1747	21.1852
20	1.78	1.65	30.1115	30.1372	24.5754	24.5966	20.5151	20.5323
25	2.12	1.98	28.3457	28.3470	23.4380	23.4392	19.7648	19.7664
30	2.49	2.49	26.4200	26.4109	22.1427	22.1352	18.8759	18.8695
35	3.20	3.29	24.3388	24.3340	20.6834	20.6798	17.8345	17.8306
40	4.41	4.53	22.1207	22.1288	19.0674	19.0744	16.6363	16.6429
45	6.46	6.46	19.7977	19.8174	17.3112	17.3295	15.2905	15.3070
50	9.64	9.45	17.4190	17.4333	15.4517	15.4660	13.8192	13.8333
55	14.43	14.08	15.0288	15.0219	13.5226	13.5186	12.2476	12.2451
60	21.40	21.25	12.6741	12.6398	11.5648	11.5350	10.6060	10.5800
65	31.64	32.30	10.3861	10.3516	9.6057	9.5745	8.9182	8.8897
70	48.39	49.26	8.2190	8.2234	7.6983	7.7029	7.2311	7.2357
75	75.83	75.06	6.2870	6.3151	5.9560	5.9845	5.6542	5.6827
80	115.99	113.82	4.6845	4.6713	4.4817	4.4730	4.2941	4.2891
85	171.09	170.94	3.4257	3.3147	3.3039	3.2026	3.1899	3.0971
90	238.74	252.61	2.5009	2.2437	2.4273	2.1840	2.3575	2.1273
95	312.88	363.99	1.8461	1.4355	1.8003	1.4057	1.7566	1.3770
100	388.11	505.25	1.3824	.8486	1.3532	.8349	1.3252	.8216

TABLE BJ.—COMPARISON OF VALUES OF TEMPORARY IMMEDIATE LIFE ANNUITIES BY ORIGINAL AND MAKEHAMIZED MORTALITY TABLES: TOTAL WHITES IN THE UNITED STATES, 1939–1941

AGE (x)	INTEREST AT 2 PERCENT		INTEREST AT 3 PERCENT		INTEREST AT 4 PERCENT	
	Original table	Makehamized table	Original table	Makehamized table	Original table	Makehamized table
	VALUE OF TEMPORARY IMMEDIATE LIFE ANNUITY FOR 10 YEARS ($a_{x:\overline{10}\vert}$)					
0	8.5134	8.5134	8.0856	8.0856	7.6891	7.6891
10	8.9332	8.9295	8.4843	8.4806	8.0679	8.0646
20	8.8874	8.8945	8.4412	8.4479	8.0277	8.0339
30	8.8442	8.8417	8.4010	8.3987	7.9901	7.9879
40	8.7183	8.7157	8.2837	8.2811	7.8802	7.8781
50	8.4085	8.4208	7.9944	8.0059	7.6101	7.6209
60	7.7744	7.7633	7.4021	7.3917	7.0560	7.0465
70	6.4527	6.4508	6.1655	6.1632	5.8978	5.8953
80	4.3577	4.3825	4.1940	4.2180	4.0404	4.0636
90	2.4719	2.2358	2.4016	2.1769	2.3347	2.1209
	VALUE OF TEMPORARY IMMEDIATE LIFE ANNUITY FOR 20 YEARS ($a_{x:\overline{20}\vert}$)					
0	15.4188	15.4158	14.0343	14.0317	12.8249	12.8228
10	16.1380	16.1353	14.6913	14.6885	13.4272	13.4245
20	15.9922	16.0060	14.5626	14.5752	13.3136	13.3248
30	15.7713	15.7615	14.3710	14.3623	13.1464	13.1388
40	15.1879	15.1946	13.8630	13.8682	12.7021	12.7067
50	13.9357	13.9562	12.7675	12.7864	11.7410	11.7585
60	11.6212	11.5886	10.7360	10.7069	9.9514	9.9255
70	8.0958	8.1139	7.5999	7.6151	7.1524	7.1652
80	4.6807	4.6703	4.4787	4.4721	4.2917	4.2884

TABLE BK.—COMPARISON OF VALUES OF IMMEDIATE WHOLE LIFE ANNUITIES ON TWO JOINT LIVES, BY ORIGINAL AND MAKEHAMIZED MORTALITY TABLES: TOTAL WHITES IN THE UNITED STATES, 1939–1941, INTEREST AT 3 PERCENT

AGE OF OLDER LIFE	AGE OF YOUNGER LIFE									
	0		10		20		30		40	
	Original table	Makehamized table	Original table	Makehamized table	Original table	Makehamized table	Original table	Makehamized table	Original table	Makehamized table
0	23.9240	23.9239								
10	24.1577	24.1290	24.6108	24.6107						
20	22.6244	22.6431	23.2590	23.2789	22.2554	22.2944				
30	20.5645	20.5563	21.2977	21.2896	20.6410	20.6526	19.4834	19.4714		
40	17.8244	17.8300	18.5555	18.5609	18.1711	18.1939	17.4670	17.4692	16.0579	16.0714
50	14.5186	14.5309	15.1644	15.1764	14.9581	14.9846	14.5935	14.6030	13.7701	13.7906
60	10.9107	10.8820	11.4205	11.3895	11.3169	11.2079	11.1567	11.1247	10.7547	10.7322
70	7.2874	7.2915	7.6381	7.6409	7.5905	7.6020	7.5318	7.5350	7.3716	7.3773
80	4.2545	4.2466	4.4609	4.4516	4.4417	4.4377	4.4232	4.4161	4.3694	4.3644
90	2.3096	2.0792	2.4203	2.1784	2.4130	2.1745	2.4071	2.1687	2.3898	2.1549

AGE OF OLDER LIFE	AGE OF YOUNGER LIFE									
	50		60		70		80		90	
	Original table	Makehamized table	Original table	Makehamized table	Original table	Makehamized table	Original table	Makehamized table	Original table	Makehamized table
50	12.2387	12.2674								
60	9.9212	9.9124	8.4701	8.4341						
70	7.0082	7.0248	6.3114	6.3065	5.0708	5.0732				
80	4.2391	4.2426	3.9739	3.9753	3.4360	3.4458	2.5597	2.5860		
90	2.3454	2.1218	2.2518	2.0439	2.0487	1.8760	1.6575	1.5534	1.1729	1.0593

TABLE BL.—COMPARISON OF REPORTED DEATHS AND EXPECTED DEATHS ON THE BASIS OF THE MAKEHAMIZED MORTALITY TABLE: TOTAL WHITES IN THE UNITED STATES, 1939–1941

AGE	Reported deaths	Expected deaths	EXCESS OF EXPECTED OVER REPORTED DEATHS	
			+	−
5–9	28,825	28,712		113
10–14	28,396	31,647	3,251	
15–19	47,647	48,544	897	
20–24	60,997	55,236		5,761
25–29	66,636	64,735		1,901
30–34	76,114	77,315	1,201	
35–39	93,059	95,855	2,796	
40–44	123,291	125,718	2,427	
45–49	173,259	171,217		2,042
50–54	229,300	224,904		4,396
55–59	280,242	275,463		4,779
60–64	337,151	339,033	1,882	
65–69	401,551	412,691	11,140	
70–74	428,703	434,553	5,850	
75–79	398,670	393,201		5,409
80–84	316,588	313,053		3,535
85–89	164,966	168,572	3,606	
90–94	55,065	61,200	6,135	
95 and over	13,451	19,598	6,147	
Total 5 and over	3,323,911	3,341,307	45,332	27,936
Total of absolute values			73,268	
Net total			−17,396	

Calculation of other tables derived from the makehamized mortality table

The values of the Makeham constants and their logarithms (given in table 36) were either obtained in the process of graduation or followed readily from values so obtained. Values in the table of uniform seniority (table 37) were calculated by the formula:[18]

$$w-x=\frac{\log\,(1+c^{y-x})-\log 2}{\log c}$$

where $y-x$ denotes the difference between the ages of the two lives and $w-x$ denotes the addition to the younger age.

The annuity values shown in tables 39 to 42 were obtained by division from values of D_x and N_x, D_{xx} and N_{xx}, etc., calculated for that purpose. The "D's" were obtained by successive multiplication by l_x: thus $D_x=v^x l_x$, $D_{xx}=D_x l_x$, $D_{xxx}=D_{xx} l_x$, and so on. The "N's" were obtained by summing the "D's." Enough significant figures were retained in both "D's" and "N's" to obtain annuity values correct to four decimal places.

[18] Spurgeon, *op. cit.*, p. 258.

INDEX

INDEX

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